THE MECHANICS OF BUILDING CONSTRUCTION

BY

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PREFACE

During the summer of 1906 the author was requested by the Board of Education to give a course of lectures at South Kensington upon "The Mechanics of Building Construction" to a number of Science teachers selected by the Department from all parts of the Kingdom, with the object of perfecting their knowledge of the subject and at the same time illustrating the manner in which it should be taught. The course was so well appreciated by the teachers that a repetition was asked for in 1907, and was duly given to another selected group. Those lectures form the basis of the present work; but in preparing his notes for the press the author has made some alteration in the division of the sections and many additions to the text and drawings, and has revised the remainder where expedient. The details have been expanded, and explained more fully than they were in the first instance, so that what was originally prepared for teachers only is now available also for students, and forms a complete text-book of all the more important branches of the subject. The matter is divided up into thirty chapters so that, if used as lectures, each would occupy not more than one and a half hours in delivery, including the time taken up in sketching the diagrams on the blackboard.

The want of a good general work upon the principles of Structural Engineering, which covers the whole ground that a student needs to traverse before he begins to specialise, has often been felt by young architects and engineers, although there are many books dealing with some of the elementary branches which leave little to be desired. This work is intended to supply the deficiency. It must not, however, be

thought that it is merely a student's book; the information in it is in such a form as to make it useful as a book of reference for Architects and Engineers who need at any time to look up the theory of their constructions. A glance at the list of contents will show the wide range that is covered.

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CONTENTS

LECTURE I

LECTURE II

LECTURE III

LECTURE IV

LECTURE V

LECTURE VI

LECTURE VII

LECTURE VIII

LECTURE IX

LECTURE X

LECTURE XI

LECTURE XII

LECTURE XIII

LECTURE XIV

LECTURE XV

LECTURE XVI

LECTURE XVII

LECTURE XVIII

LECTURE XIX

LECTURE XX

LECTURE XXI

LECTURE XXII

LECTURE XXIII

LECTURE XXIV

LECTURE XXV

LECTURE XXVI

LECTURE XXVII

LECTURE XXVIII

LECTURE XXIX

LECTURE XXX

LABORATORY WORK

THE MECHANICS OF BUILDING CONSTRUCTION

LECTURE I

Matter and Force—Statics—Pressures and Reactions—Specification of a Force—Parallelogram of Forces—Triangle of Forces—Polygon of Forces—Forces in more than one Plane—Leverage—Moments—Parallel Forces—Couples.

It will be useful to commence with an explanation of the terms that will be adopted in dealing with this subject. All materials come under the general term of matter, which may be defined as "the element of resistance in the sensible world." Even the greatest philosophers are unable to say what matter really is in itself, but we all know that it has certain essential properties. They are impenetrability, extension, and figure or form. Impenetrability means that two given portions of matter, or bodies, cannot occupy the same space at the same time. Extension is the fact of occupying space, expressed by the three dimensions of length, breadth, and thickness. By figure or form is meant the presentation of a definite shape at a given instant. Then there are certain accessory properties, and an illustration of each will render its nature clear. Divisibility—one grain of iodide of potassium dissolved in 480,000 grains of water, when mixed with a little starch, will tint every drop of the fluid blue on the addition of a solution of chlorine. Flexibility—the property of permitting change of shape without disintegration or fracture, as indiarubber. Tenacity or toughness—a steel wire may bear a direct load of 100 tons per square inch without failure. Brittleness—depends largely upon molecular arrangement; steel when heated and suddenly cooled in water may be very brittle, but this condition may be removed by again heating and cooling slowly. Elasticity—is the property of recovering its shape after distortion; a bent spring tends to regain its original form immediately the distorting pressure is removed. In a perfectly elastic body the force of restitution would be equal to the impressed force, but no body is perfectly elastic. Malleability—is the property of allowing the shape to be changed by forging or hammering without severance of the constituent molecules; soft charcoal iron, gold, and lead are typical of malleable metals. Ductility—is the capability of being drawn into wire; platinum, silver, iron, and copper are particularly ductile metals. Hardness—may be associated with brittleness, but not necessarily so; hard wrought-iron is more tenacious than soft iron, but breaks more readily under a sudden blow; the diamond is the hardest substance known. Then there are three conditions of matter—solid, liquid (including viscous or semi-liquid), and

gaseous—depending upon the comparative intensity of the molecular forces of attraction and repulsion, and influenced largely by temperature. In solids, the molecules are relatively fixed, in liquids they are coherent but not fixed, in gases they are repellent to each other. Hence, solids press downwards only, liquids press downwards and sideways, gases press in all directions.

Force is as difficult to define strictly as matter is. Force is that which produces or destroys motion, or which tends to produce or destroy it; or which alters or tends to alter its direction. By Newton's "Laws of Motion," we learn (1) that every change of state is due to external force; (2) that every force produces its own result; and (3) that action and reaction are equal. If only one force act upon a body, motion must ensue, but two or more forces may be in equilibrium and the body is then at rest. Forces at rest are usually called pressures or reactions; pressure is a force balanced by a resistance. The science of forces in equilibrium is called Statics, and it is the branch specially applying to the Mechanics of Building Construction. It forms part of the larger group of Applied Mechanics, which is that branch of applied science explaining the principles upon which machines and structures are made, how they act, and how their strength and efficiency may be tested and calculated.

It is necessary to obtain a clear idea of pressures and reactions. When a surface supports a load the pressure or active force is caused by the attraction of gravitation pulling down the load, but action and reaction being equal, there is a passive force, or resistance, or reaction, tending to push the load upwards with exactly the same force that gravity is pulling downwards. This would be self-evident on a surface of indiarubber, but it is equally true for all surfaces, although the actual compression and tendency to regain its original form are not visible to the naked eye with ordinary materials.

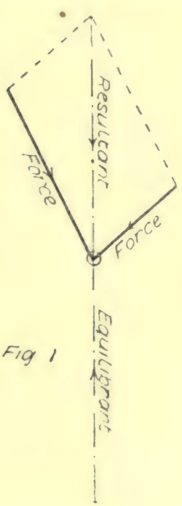
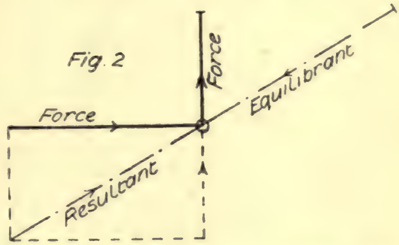


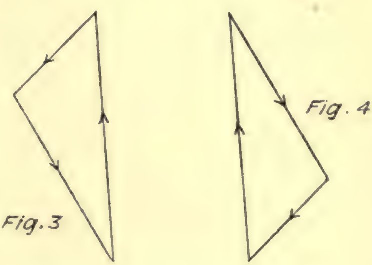
Fig 1

Forces may be represented graphically by straight lines whose position upon the paper gives their line of action or direction; the length of each line to any given scale represents the magnitude of the force; an arrow-head placed anywhere upon the line gives its sense, or the direction in which it presses; and if a force acts upon a body, the point at which it touches is called the point of application. The statement of these various particulars for any given force, pictorially or verbally, is called the specification of the force. If two forces act upon a point, Fig. 1, each tends (by the second law of motion) to produce its own result, but as the point cannot move in two directions at the same time, it takes an intermediate direction more nearly coinciding with that of the stronger force. By forming a parallelogram, of which the two forces make two adjacent sides, the diagonal of the parallelogram meeting the same point represents a single force that could be substituted for the two forces to produce exactly the same effect. This is called their resultant, and it is self-evident that an

equal and opposite force to this would exactly balance it. This latter is called the equilibrant, and a brief consideration will show that the equilibrant may be considered as a third force that will exactly balance the first two. The converse of all this is equally true, and by working backwards a single force may be resolved into two others, of which either both directions may be given, leaving the magnitude to be determined, or both magnitudes may be given, leaving the direction to be found; or the direction and magnitude of one force may be given (within practicable limits), leaving the magnitude and direction of the other to be determined. The first proceeding by which two forces are converted into one is called the composition of forces, and the latter proceeding by which one force is converted into two is called the resolution of forces. Two given forces can have only one resultant, but one given force may be resolved into any number of forces. A force may be considered to act anywhere in its line of direction, and to compound two forces when one is acting towards the point and the other is acting away from it as in Fig. 2, one of the forces must be transferred to the opposite side of the point as shown, before the parallelogram can be drawn and the resultant and equilibrant found.



When three forces acting upon a point are in equilibrium they may be used to form the three sides of a triangle; they may be taken in any order so long as the arrow-heads all run the same way round, called "circuitally." The shape of the triangle will be the same as one or other half of the parallelogram, as seen in Figs. 3 and 4, both based upon the forces shown in Fig. 1. The fact of the lines forming a perfectly closed triangle is the test of their equilibrium, and, therefore, if any two forces be used for two sides of a triangle the third side will represent the equilibrant, or reversing the arrow head will represent the resultant. This is called the triangle of forces. The conditions of equilibrium for three forces acting on a point, are (1) they must be in the same plane; (2) the resultant of any two of them must be equal and opposite to the third; (3) if lines be drawn representing the forces in position, magnitude and direction, these lines taken in order, with the senses concurrent, will form a perfect triangle.



When several forces meet in a point they may be compounded in pairs, and then the resultants compounded to obtain the final resultant, as in Fig. 5. Or the resultant of two adjacent forces may be found and this compounded with the next force, and so on, as in Fig. 6, where

the forces are the same as shown in Fig. 5. When the equilibrant is found, the whole of the forces put together will form a closed polygon (Fig. 7), and if the forces be placed with the arrow heads to run circuitally, the equilibrant will be represented by the line necessary to complete the polygon. When only two forces act upon a point they are necessarily in the same plane, but when more than two forces act upon a point they may be in one or more planes. The preceding examples have all been in one plane, but cases arise in practice, as for example in connection with shear legs, where the forces are in more than one plane. In such cases a knowledge of practical geometry is necessary because each pair of forces, whether original forces or resultants, must be compounded in their own plane. The method of doing this must be left until shear legs are being dealt with.

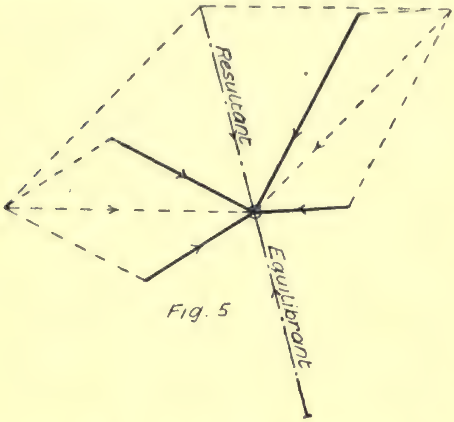


Fig. 5

Parallel forces offer difficulties of their own, and before these are considered the principles of leverage must be studied. It is generally stated that there are three kinds of lever according to the relative position of the fulcrum, weight, and power, but there is really only one

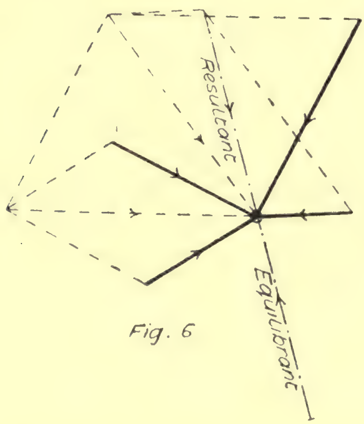


Fig. 6

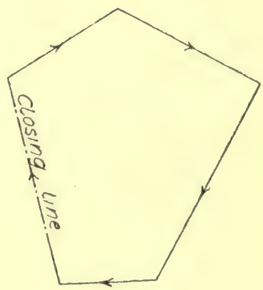


Fig. 7

kind of lever with various arrangements of the components. In every case there is a force acting with a certain length of arm to balance another force acting with its length of arm. Generally there is an

active force called the power [1] and a resisting force called the weight. If the power be marked P and its arm x, the weight W and its arm y, and the fulcrum or pivot upon which the lever works F, the various arrangements will be as shown in Figs. 8, 9, and 10, and the following equation holds good for them all:

$$Px = Wy$$

in which if any one factor be an unknown quantity, it can be found by dividing the other side of the equation by its companion letter, viz.

$$P = \frac{Wy}{x},\ x = \frac{Wy}{P},\ W = \frac{Px}{y},\ y = \frac{Px}{W}.$$

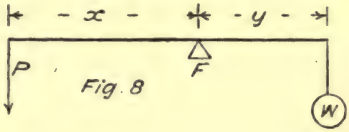


Fig. 8

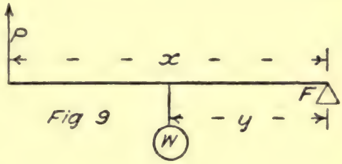


Fig 9

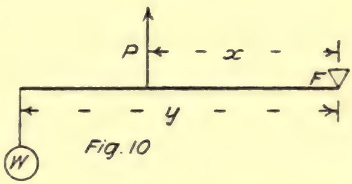


Fig. 10

Px is called the moment of the power and Wy the moment of the weight, "moment" being the name given to a force multiplied by a leverage. These equations may be called algebra, but when the values are inserted instead of the letters, it is only common arithmetic, and no student should be afraid of a formula because it has an algebraical look about it.

When two parallel forces act upon a point they must be in the same straight line, either acting together or opposed to each other. In the former case the resultant is their sum, and in the latter their difference. When the parallel forces are not in the same straight line, they may be considered as acting at different points along a lever. Suppose two parallel forces of 4 and 8 lbs. respectively be acting in the same direction at a distance of 15 in. as in Fig. 11, the resultant is equal to their sum, and must act at such a point that 4 multiplied by its arm equals 8 multiplied by its arm. Thus the sum of the forces is to the distance between them as either force is to the arm on the opposite side, or 12 : 15 : : 4 : 5 and 12 : 15 : : 8 : 10, or the short arm for the larger force $= \frac{15 \times 4}{12} = 5$, and the longer arm for the smaller force $= \frac{15 \times 8}{12} = 10$. The proof of accuracy is that $4 \times 10 = 8 \times 5$, or *the moments are equal.* The same results may be obtained graphically as in Fig. 12. Add equal

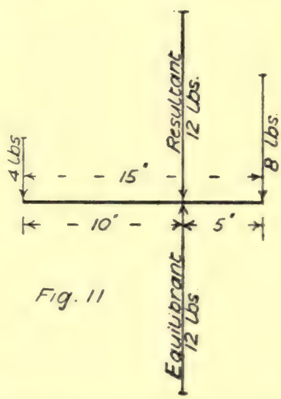


Fig. 11

[1] Power is hardly a proper term to use, as it really involves "the action of a force through a given distance in a given time," but custom authorizes its use in this instance, when force only should have been sufficient.

forces at opposite ends of the lever acting towards each other, which will not alter the balance; complete the parallelogram and produce the diagonals to intersect, which will give a point in the line of action of the resultant or equilibrant, the magnitude of which will

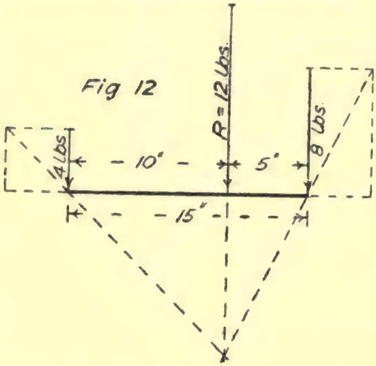


Fig 12

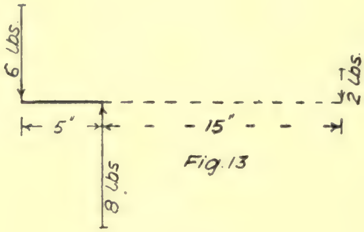


Fig. 13

equal the sum of the forces. If two unequal parallel forces be taken, acting in opposite directions as in Fig. 13, the equilibrant force must be equal to their difference, and must act at such a distance beyond the greater force, that the moment is equal to the moment of the first force about the point of application of the second. Thus, with parallel forces of 6 and 8 lbs. acting in opposite directions at a distance apart of 12 in., the equilibrant force must be $8 - 6 = 2$ lbs., acting at a distance of $\frac{6 \times 5}{2} = 15$ in. The two forces cannot quite be said to have a resultant, and the reason of this will appear presently.

When two equal parallel forces act in opposite directions, and not in the same straight line, they form what is called a "couple," and they

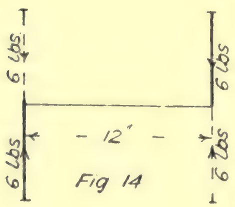


Fig 14

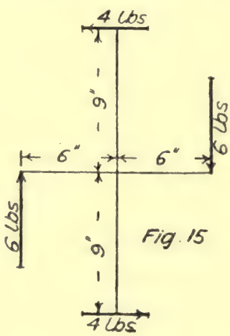


Fig. 15

have no single resultant, but they may be balanced by two equal and opposite forces as shown by dotted lines in Fig. 14. Or if two levers be imagined joined together at right angles as in Fig. 15, then the two forces, forming the couple, tending to turn the system in a clockwise

direction, may be balanced by two other forces giving equal moments and tending to turn the system anti-clockwise. In calculating the moments we may either multiply each force by its distance from the fulcrum, or, as many cases arise where there is no apparent fulcrum, we may multiply one of the forces of the couple by the distance between them. The latter is the usual method, the distance between the forces being called the arm of the couple. When several forces in one plane, whose lines of action do not pass through a point, act upon a body, there may of course be numberless variations of magnitude, position, etc., but certain general principles may be applied for elucidating the results. In Fig. 16 four forces are shown acting at the points of a square. Upon inspection it will be seen that two couples of 5 lbs. can be formed tending to turn the square in opposite directions, leaving two forces at right angles of 4 and 7 lbs. giving a resultant of 8·06 lbs. acting through one corner of the square. Now let the four forces act at the corner of the square, but all towards the square in the direction of a side, as in Fig. 17. Then they may be combined into two resultants of 13 and 10·3 respectively. The resultant of these resultants will be obtained by producing them as shown, and constructing a parallelogram of which the diagonal gives the final resultant of 8·06 lbs. acting away from the square in the position shown. It will be evident that in forming a polygon from these forces, as in Fig. 18, the value of the final resultant would be given, but there would be nothing to fix its position; another condition must be fulfilled to prove that the forces will be in equilibrium, and that is that the funicular polygon must also close. This introduces a new subject, the consideration of which will be better postponed until Graphic Statics in

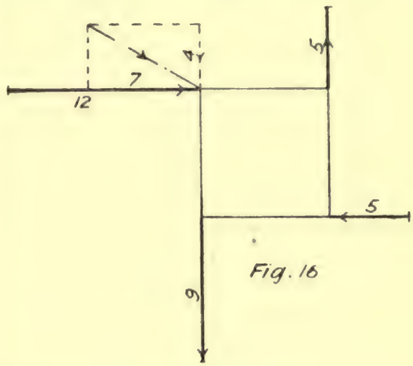


Fig. 16

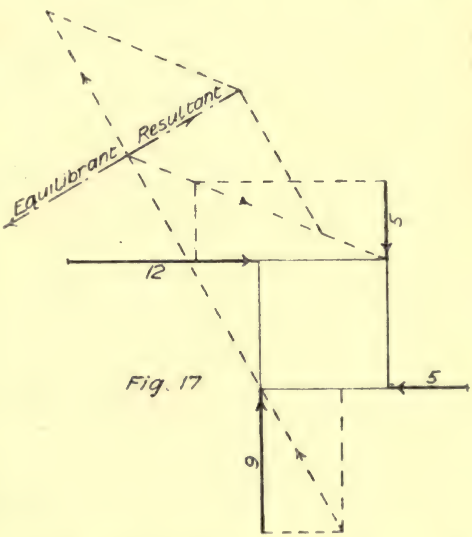


Fig. 17

general are dealt with, but one other matter may be referred to now that moments have been explained. It is that when any forces are in equilibrium the sum of their moments about any given point is zero. Repeating in Fig. 19 the forces shown in Fig. 1 which are in equilibrium,

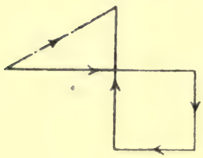


Fig. 18

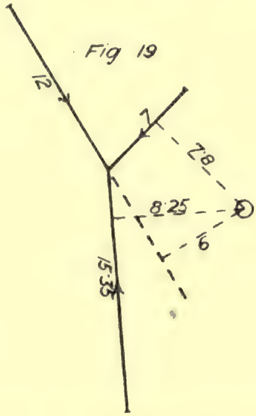


Fig 19

their moments round point O will be found by dropping perpendiculars on to the directions of each of the forces and scaling the distances, then multiplying each force by its distance and taking the algebraical sum, clockwise moments being $-$ and anti-clockwise $+$. Thus $+ (7 \times 7{\cdot}8) + (12 \times 6) - (15{\cdot}35 \times 8{\cdot}25) = +54{\cdot}6 + 72 - 126{\cdot}6 = 0$

EXERCISES ON LECTURE I

Q. 1. **Forces of 10 lbs. and 5 lbs. act upon a point with an included angle of 60 degrees. Find the resultant and equilibrant.**

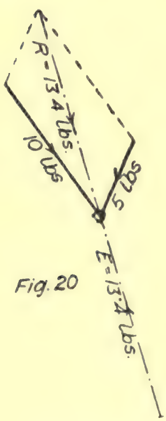


Fig. 20

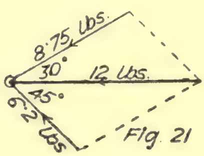


Fig. 21

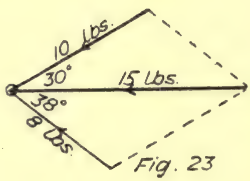


Fig. 23

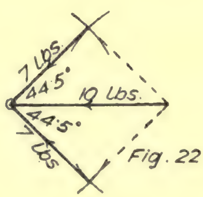


Fig. 22

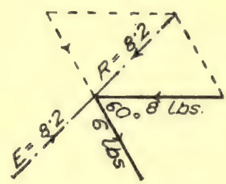


Fig. 24

For answer, see Fig. 20.

Q. 2. A force of 12 lbs. acts upon a point. Resolve this into two forces, making respectively 30 degrees and 45 degrees with the given force.

For answer, see Fig. 21.

Q. 3. A force of 10 lbs. acts upon a point. Resolve this into two forces of 7 lbs. each.

For answer, see Fig. 22.

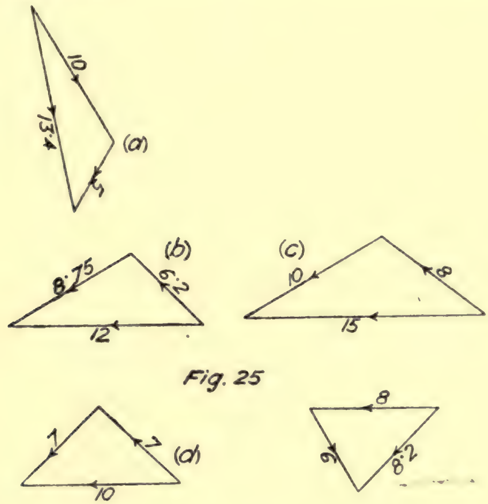


Fig. 25

Q. 4. A force of 15 lbs. acts upon a point. Resolve this into a force of 10 lbs. at an angle of 30 degrees, and another force to be determined in magnitude and direction.

For answer, see Fig. 23.

Q. 5. A force of 8 lbs. is acting towards a given point, and another of 6 lbs. making an angle of 60 degrees with the first is acting away from the point. Find the resultant and equilibrant.

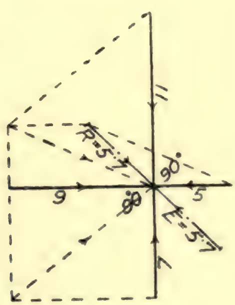


Fig 26

Fig. 27

For answer, see Fig. 24.

Q. 6. Show a force triangle for each of the above cases.

For answer, see Fig. 25.

Q. 7. Forces of 5, 7, 9, and 11 lbs. acting upon a point are separated by angles of 90 degrees. Find their resultant and equilibrant, and construct the corresponding polygon.

For answer, see Figs. 26 and 27.

Q. 8. Parallel forces of 5 and 7 lbs. act in the same direction at a distance apart of $17\frac{1}{2}$ in. Find the position and magnitude of their resultant.

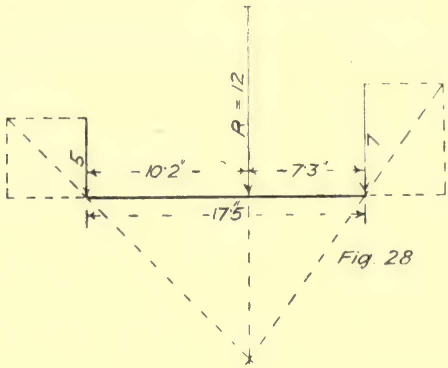
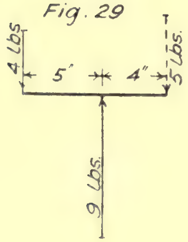


Fig. 28

For answer, see Fig. 28.

Q. 9. Parallel forces of 4 and 9 lbs. act in opposite directions at a distance apart of 5 in. Find the magnitude and position of their equilibrant.

Fig. 29



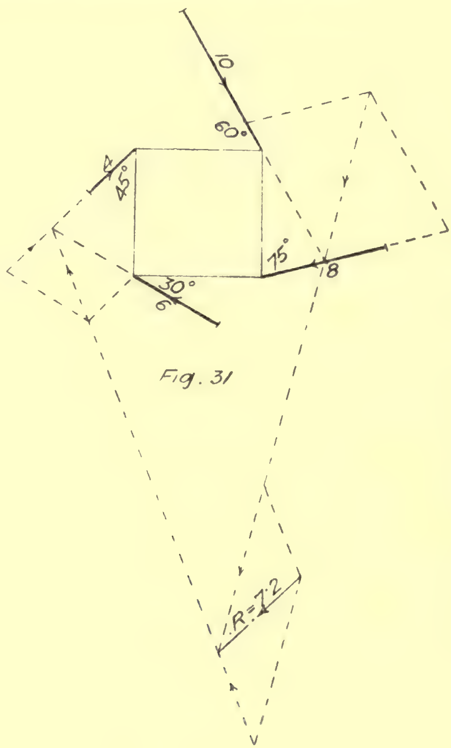
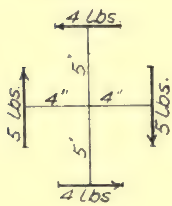


Fig. 31

Fig 30



For answer, see Fig. 29.

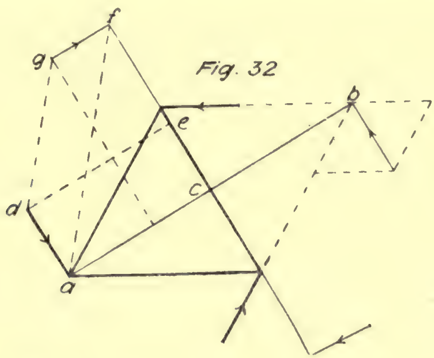
Q. 10. Two parallel forces of 5 lbs. each act in opposite directions at a distance apart of 8 in. Find the balancing couple whose arm is 10 in.

For answer, see Fig. 30.

Q. 11. Forces are applied at the four angles of a square as follows: 4 lbs. at 45 degrees, 10 lbs. at 60 degrees, 8 lbs. at 75 degrees, and 6 lbs. at 30 degrees. Find the magnitude, line of action, and sense of the resultant.

For answer, see Fig. 31.

Q. 12. Three equal forces are applied at the angles of an equilateral triangle, and act anti-clockwise parallel with the opposite side. Find an equilibrant force or forces.



For answer, see Fig. 32.

NOTE.—If two of the three given forces are combined, the resultant taken with the remaining force will form a couple whose arm is *a b*, and can only be balanced by another couple of equal value acting in the opposite direction. Then taking moments about the centre of arm *c*, construct the parallelogram of moments *a d e c*. Assume the half-arm of the new couple as *c f*. Join *a f*, and parallel to this through *d* draw *d g* to meet a line through *f* parallel with *a c*. Then *g f* will be the value of one of the forces of the equilibrant couple.

LECTURE II

Force of Gravity—Weight—Centre of Gravity—Centroid—Neutral axis—Moment of Inertia—Section Modulus—Bending Moment.

Sir Isaac Newton was the first to show that all bodies attract each other with a force proportional directly to their masses and inversely to the squares of the distances between them. "The reason of these properties of gravity," he said, "I have not yet been able to deduce; and I frame no hypotheses." As the attraction is proportional to the mass, or quantity of matter in the body, that of the earth practically overwhelms all others; and as the attraction is towards the centre of the mass, the force of attraction, which we call weight, acts vertically downwards towards the centre of the earth. A falling body travels with an increasing velocity because the force is constantly acting upon it. It falls 16 ft. in the first second, but as it starts from rest the velocity at the end is 32 ft. per second, with which it begins the next second, and it gets another 32 ft. per second added during the second interval of time, and so on. The force of gravity, or the acceleratrix of gravity, is therefore described as 32 ft. per second per second, and is denoted by the letter $g = 32$. We are not concerned with this in statics beyond the fact that the attraction produces the effect known as weight. Mass is the quantity of matter in a body of any volume or temperature, and is constant at all heights and in all latitudes. Weight = mass × gravity, and is constant only at the same level and same latitude. The centre of the mass is called the centre of gravity; it is that point through which the resultant of the gravities (or weights) of all its parts passes, in every position the body can assume, and is such that if it be supported the whole body will be supported in equilibrium; but the centre of gravity is not necessarily situated in the solid portion of a body, or enclosed by its surfaces, as, for instance, in the case of a wire bent into a semicircle. The centre of gravity of a number of bodies in a straight line may be found by taking moments about a point. Let

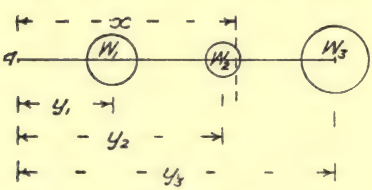


Fig. 33

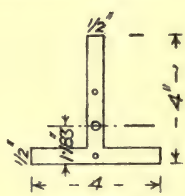


Fig. 34

a series of bodies, W_1 W_2 W_3, etc., be situated at distances of y_1 y_2 y_3, etc., from a given point A, as in Fig. 33, then the mean centre of

gravity x will be found by the equation $Ax = \frac{W_1 y_1 + W_2 y_2 + W_3 y_3, \text{etc.}}{W_1 + W_2 + W_3, \text{etc.}}$

The centre of gravity of an irregular surface is frequently spoken of, such as the section of a tee iron; but a surface has no weight, and it is therefore more correct to call it the centroid, or centre of form; no practical difficulty is, however, likely to arise by using the term "centre of gravity," and it may be justified by assuming a slight thickness for the section under consideration. The centre of gravity of a tee iron, as Fig. 34, may be found by the rule just given, the area of each part being taken as a weight, and the distance of its centre of gravity from the base being seen by inspection. Then $\frac{(4 \times \frac{1}{2}) \times \frac{1}{4} + (3\frac{1}{2} \times \frac{1}{2}) \times 2\frac{1}{4}}{(4 \times \frac{1}{2}) + (3\frac{1}{2} \times \frac{1}{2})}$ $= \frac{\cdot 5 + 3 \cdot 9375}{2 + 1 \cdot 75} = \frac{4 \cdot 4375}{3 \cdot 75} = 1 \cdot 183$ in. above the base line. By formula, if a = area of flange, A = area of web, d = total depth, t = thickness, H = height of centre of gravity from outer edge of flange. $H = \frac{1}{2}\left(\frac{at + At + Ad}{a + A}\right)$, or $H = \frac{1}{2}\left(d + t - \frac{ad}{a+A}\right)$, but it is always better to rely upon general principles than upon formulæ.

The neutral axis of a section is a line passing through its centre of gravity. When a beam is bent, the concave surface is compressed and the convex surface is stretched, but there is always an intermediate layer which is unaltered in length; this is the *neutral layer*, and coincides with a line through the centre of gravity of every section.[1]

The moment of inertia is the value of a section as regards its power of resistance due to its area and the disposition of its parts. It is the summation of the areas of all its individual parts multiplied by the squares of their distances from the neutral axis. This is generally expressed as $I = \Sigma ay^2$. It is necessary to divide it up into very small portions, otherwise we could take the whole area on one side of the neutral axis of a beam, as in Fig. 35, and multiply its area by the square of the distance of its centre of gravity from the neutral axis, and the same for the other half, giving $2(6 \times 6 \times 3^2) = 648$ for the moment of inertia; this, however, would be too small a value. The result would be a little closer if smaller divisions were taken, as in Fig. 36, where $2(6 \times 2 \times 5^2 + 6 \times 2 \times 3^2 + 6 \times 2 \times 1^2) = 840$; but this again would be insufficient. Doubling the number of divisions, as in Fig. 37, we should have $2(6 \times 1 \times 5 \cdot 5^2 + 6 \times 1 \times 4 \cdot 5^2 + 6 \times 1 \times 3 \cdot 5^2 + 6 \times 1 \times 2 \cdot 5^2 + 6 \times 1 \times 1 \cdot 5^2 + 6 \times 1 \times \cdot 5^2) = 858$, which would still be too small; but as the number has only increased from 840 to 858 by doubling the number of divisions, it is clear that we are getting much closer to the true result. If the divisions could in this way be caried to infinity, we should find the value come out at 864. It can be arrived at very readily in a rectangular beam by the formula $I = \frac{bd^3}{12} = \frac{6 \times 12^3}{12} = 864$ in. units.

[1] An indiarubber beam 12 in. × 1½ in. × 1½ in. was here exhibited. One face was left plain, the second was marked off in transverse parallel lines ¼ in. apart, the third was covered with squares of ⅜ in. side, and the fourth with circles ½ in. diameter touching each other. The effects of compression, tension, torsion, and shearing were thus rendered visible in a very interesting manner.

The formulæ for this and other sections will be found at p. 92 of my "Engineers' Handbook" (Cassell, 7*s*. 6*d*. net). The way this formula is made up is instructive. Draw diagonals on the section, as Fig. 38 ; the horizontally shaded portions represent what is called the inertia area. The length of each horizontal line may be looked upon as the measure of the intensity of stress in that layer, the whole width being equal to the

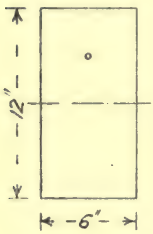


Fig 35

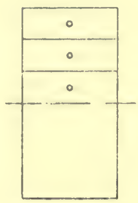


Fig 36

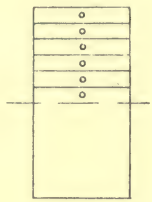


Fig 37

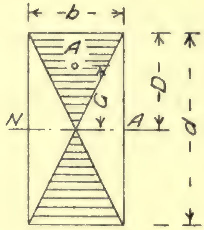


Fig. 38

maximum stress on the outer fibres ; or upon another view, the shaded area may be looked upon as the virtual section if each component fibre were under the full stress. Let A be the area of shaded part on one side of neutral axis, G the distance of its centre of gravity from the neutral axis, and D the depth from outer surface to neutral axis, then the moment of inertia $I = 2DAG$; but $D = \frac{1}{2}d$, $A = \frac{bd}{4}$, $G = \frac{2}{3} \times \frac{1}{2}d$, therefore $I = 2 \times \frac{1}{2}d \times \frac{bd}{4} \times \frac{2}{3} \times \frac{1}{2}d$, whence $I = \frac{bd^3}{12}$.

In bygone days it was customary always to estimate the breaking stress of any structure, and then employ an assumed factor of safety to arrive at the working stress ; the more direct method is now employed of fixing a working stress for given conditions, and designing directly from that. In a similar way the moment of inertia was used in determining the moment of resistance of a section, and dividing by the depth from the neutral axis to the furthest edge of the section to obtain the section modulus $\frac{I}{y} = Z$; now we determine the section modulus for any given section and work from that. The word "modulus" may be looked upon as simply meaning multiplier. Turning again to Fig. 38, the shaded area on each side of the neutral axis may be looked upon as a collection of resisting forces, compression above and tension below, the centre of gravity of each being the centre of effort, so that they form a couple as in Fig. 39, acting lengthways of the beam with an arm equal to two-thirds the total depth ; so that the value of the couple becomes —

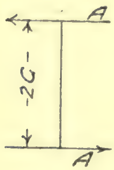


Fig. 39

$$A \times 2g = \frac{bd}{4} \times 2 \times \frac{2}{3}\left(\frac{1}{2}d\right) = \frac{bd^2}{6},$$

which is the section modulus.

The bending moment on a beam, however supported or loaded, is

the moment of the force by its virtual leverage which tends to bend or break the beam. It is denoted by the letter M, is the full measure of the active force, and is quite independent of the cross-section or the strength of the material. There are various definitions of bending moment; perhaps the most concise is that it is "the moment of the external forces on one side of a transverse section estimated relatively to the section." For example, with a cantilever as in Fig. 40, the maximum bending moment is $M = W \times L$, and the bending moment at any other point, at x distance from support, is $W(L - x)$. With a beam supported at both ends and loaded in the centre, as in Fig. 41,

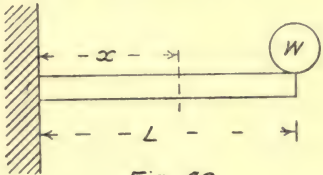


Fig. 40

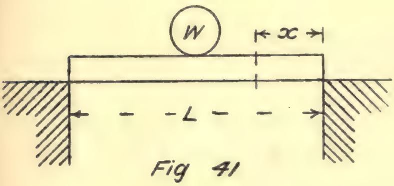


Fig. 41

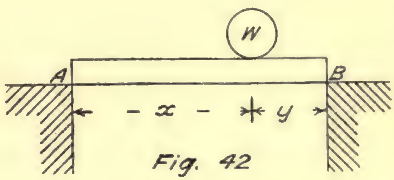


Fig. 42

it is not at first clear how the moment is to be measured, until it is observed that the reactions can be taken as forces, then the maximum bending moment will be seen to be $\frac{1}{2}W \times \frac{1}{2}L$, or $\frac{WL}{4}$; and at any intermediate point, distant x from one support, it will be $\frac{1}{2}Wx$. With a concentrated load out of the centre, as in Fig. 42, we must first find the reaction by leverage; reaction at $A = W \times \frac{y}{x+y}$, and at $B = W \times \frac{x}{x+y}$. Then the maximum bending moment $=$ reaction at $A \times x = W \times \frac{xy}{x+y}$, or reaction at $B \times y = W \times \frac{xy}{x+y}$ as before. With more than one concentrated load, as in Fig. 43, the same method must be adopted. Reaction at $A = W\frac{y+z}{x+y+z} + w\frac{z}{x+y+z}$, and the bending moment under $W =$ reaction at $A \times x$. Similarly, the reaction at $B = w\frac{x+y}{x+y+z} + W\frac{x}{x+y+z}$, and the bending moment under $w =$ reaction at $B \times z$. At distance a the bending moment will be reaction at $A \times a$. At distance b it will be reaction at $A \times b - W(b - x)$, or it will be reaction at $B \times c - w(c - z)$. This ought to be clear without further explanation, as it is only a question of clockwise or anti-clockwise moments about the point under each load.

An extension of the last example brings us to a uniformly distributed load throughout the length of the beam, as in Fig. 44. Judging by the

examination work of students, many teachers have adopted the objectionable method of treating distributed loads as single independent weights upon each separate foot of the length. There is no reason why dis-

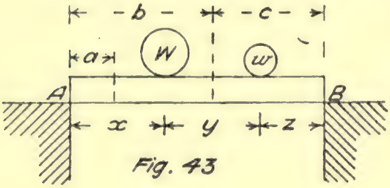


Fig. 43

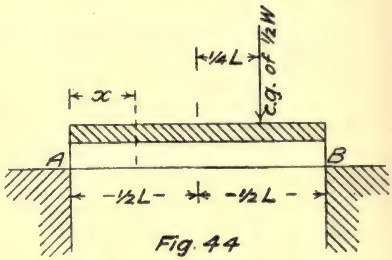


Fig. 44

tributed loads should not be treated as absolutely continuous and indivisible into separate bodies. In the present example the reaction at each end will be $\frac{1}{2}$W, and the maximum bending moment occurring in the centre will be $\frac{1}{2}\text{W} \times \frac{1}{2}\text{L} - \frac{1}{2}\text{W} \times \frac{1}{4}\text{L} = \frac{\text{WL}}{4} - \frac{\text{WL}}{8} = \frac{\text{WL}}{8}$. At any intermediate point at a distance x from support, the bending moment will be $\frac{1}{2}\text{W} \times x - \frac{x}{\text{L}}(\text{W}) \times \frac{1}{2}x = \frac{\text{W}x}{2} - \frac{\text{W}x^2}{2\text{L}} = \frac{\text{W}}{2}\left(x - \frac{x^2}{\text{L}}\right)$. Graphic diagrams for all these bending moments will be given later on.

EXERCISES ON LECTURE II

Q. 13. A weight of 3 lbs. is applied at 2 in. from A, and in the same line 4 lbs. at 5 in. from A, and 5 lbs. at 9 in. What distance will the mean centre of gravity be from A?

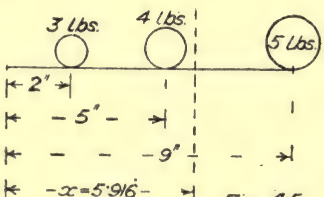


Fig 45

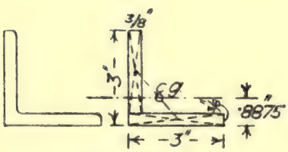


Fig. 46

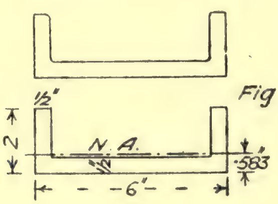


Fig. 47

A. (See Fig. 45) $\frac{3 \times 2 + 4 \times 5 + 5 \times 9}{3 + 4 + 5} = \frac{6 + 20 + 45}{12} = \frac{71}{12} = 5{\cdot}916$ in.

Q. 14. Find the centre of gravity of a 3 in. by 3 in. by $\frac{3}{8}$ in. angle iron.

A. (See Fig. 46) $\dfrac{3 \times \frac{3}{8} \times \frac{3}{16} + 2\frac{5}{8} \times \frac{3}{8} \times 1\frac{11}{16}}{3 \times \frac{3}{8} + 2\frac{5}{8} \times \frac{3}{8}} = \dfrac{\frac{27}{128} + \frac{1701}{1024}}{\frac{9}{8} + \frac{63}{64}} = \dfrac{1917 \times 64}{1024 \times 135} = \cdot 8875$ in. above base.

Q. 15. Find the neutral axis of a 6 in. by 2 in. by $\frac{1}{2}$ in. channel iron.

A. (See Fig. 47) $\dfrac{6 \times \frac{1}{2} \times \frac{1}{4} + 2 \times 1\frac{1}{2} \times \frac{1}{2} \times 1\frac{1}{4}}{6 \times \frac{1}{2} + 2 \times 1\frac{1}{2} \times \frac{1}{2}} = \dfrac{\cdot 75 + 1\cdot 875}{3 + 1\cdot 5} = \dfrac{2\cdot 625}{4\cdot 5} = \cdot 583$ in. above base.

Q. 16. What is the moment of inertia of a fir joist 9 in. by 3 in.?

A. $\mathrm{I} = \dfrac{bd^3}{12} = \dfrac{3 \times 9^3}{12} = 182\cdot 25$ inch units.

Q. 17. Find the section modulus of an oak beam 9 in. by 6 in.

A. $\mathrm{Z} = \dfrac{bd^2}{6} = \dfrac{6 \times 9^2}{6} = 81$ inch units.

LECTURE III

Moment of Resistance—Modulus of Rupture—Stress and Strain—Permanent Set—Modulus of Elasticity—Deflection.

THE next matter to be dealt with is the "moment of resistance," which is the value of the resisting forces brought into play by the bending moment. It is the section modulus multiplied by the modulus of transverse rupture. The section modulus, which has been already explained, does not depend wholly upon the magnitude of the sectional area. It depends very largely upon the arrangement of the parts of that area; the further they are removed from the neutral axis the greater the value. Thus the modulus of section (Z) is the element of resistance that is given by the size and arrangement of the resisting area. The modulus of transverse rupture (C) is theoretically the extreme fibre stress, but it is by no means the same thing in reality. For the present we will assume them to be identical; then the section modulus being made up (*inter alia*) of the shaded area in Fig. 38 (Lecture II) all of which may be considered as under maximum stress, we have only to determine what the maximum stress shall be to obtain the moment of resistance. We may either take the "modulus of rupture" (C) from a table of ultimate strength, according to the material, and obtain the ultimate resistance of the beam; or we may divide the tabular value by a factor of safety to obtain the safe working resistance; or we may assume a working intensity of stress, according to the material and the circumstances of its use. Then in any question of the strength of a beam we have the following equations:—

Effort = Resistance

Bending moment M = Moment of resistance R.

And for (say) a distributed load we may amplify the equation thus:—

$$\frac{Wl}{8} = ZC$$

Whence we obtain $W = \frac{8ZC}{l}$, and $C = \frac{Wl}{8Z}$. It should be noted that W and C must be in the same units, lbs., cwts. or tons, and Z and l must be in the same units, either inches or feet.

The modulus of rupture should apparently be the same as the ultimate tensile or compressive strength, whichever is the one that gives way first, or be made up from both of them. It is, however, from some causes which are not clearly understood, generally in excess of the tensile strength, and has to be found by experiment for each material. For this purpose a unit beam is taken 1 in. square, resting on supports 1 foot apart. The load in the centre which will just break the beam may be called the *coefficient of transverse strength* (c) to distinguish it

from the *modulus of rupture* (C). The latter is 18 times the former, or in other words the modulus of rupture is 18 times the central load that will break the unit beam. This must not be misunderstood as 18 times the tensile stress. The 18 is made up as follows :

$$\frac{Wl}{4} = ZC, \qquad \frac{Wl}{4} = \frac{bd^2}{6}C, \qquad C = \frac{Wl6}{4bd^2},$$

$$\text{but } l = 12 \text{ in., } bd^2 = 1 \times 1 \times 1 = 1, \; C = \frac{W \times 12 \times 6}{4 \times 1} = 18W.$$

C is the same as K of Molesworth, k of Tredgold, and f of other writers.[1] The mean values of C in lbs. may be taken as oak 10,000, teak 12,000, greenheart 16,000, Baltic fir 7500, but very much depends upon the quality of the specimen. In "Harmsworth's Self Educator," Part 9, a very complete list will be found giving the maximum and minimum values. The formula generally used by architects for the strength of beams is $W = \frac{cbd^2}{L}$; in this formula W is breaking weight in cwts. in centre, and small c is the central load in cwts. that would break a unit beam, therefore b and d are kept in inches and L in feet. The coefficient c may be taken as 8 for greenheart, 6 for ash, 5 for oak and teak, 4·5 for beech, 4 for pitch pine, Memel and Danzig, 3·5 for Riga and spruce fir, 3 for English elm. For fir beams the formula may be simplified to $W = \frac{bd^2}{L}$, where W is the uniformly distributed safe load in cwts. The simplification is brought about from $W = \frac{cbd^2}{L}$ in this way ; Allow 3·5 for c, multiply by 2 for distributed load and divide by 7 for factor of safety ; then $W = \frac{2 \times 3{\cdot}5bd^2}{7L} = \frac{bd^2}{L}$. Upon the same basis, the safe load in cwts. per foot super on a floor supported by fir joists, s inches centre to centre, will be $W = \frac{12bd^2}{L^2s}$, because the area in square feet supported by each joist will be $L \times \frac{s}{12}$, and the total safe load on the joist divided by the supported area will give the safe load on the floor in cwts. per foot super, thus

$$W = \frac{bd^2}{L} \div \left(L \times \frac{s}{12}\right) = \frac{12bd^2}{L^2s}.$$

[1] The following comparisons may be noted :

$$C = f_0, \frac{Wl}{4} = ZC, \text{ but } ZC = \frac{bd^2}{6} f_0 = \frac{f_0}{6} bd^2 = Kbd^2.$$

$$\therefore W = \frac{4Kbd^2}{l} \text{ (so-called rational formula) and } K = \frac{f_0}{6}.$$

$$\text{Also } c = \frac{f_0}{18}, \; W = \frac{4Kbd^2}{l} = \frac{4 \times 3c \times bd^2}{12L} = \frac{cbd^2}{L} \text{ (so-called empirical formula).}$$

In 1907 at the warehouse of Messrs. Blundy & Co., North Street, York, a floor 13 ft. span formed of 9 in. by 3 in. fir joists, 16 in. apart, and covered with 1 in. floor boards, was loaded with sacks of cement two tiers high, and collapsed, killing a man accidentally (?). The safe load was about $\frac{9}{10}$ cwt. per foot super, and the actual load about 6 cwts. per foot super.

Before leaving this preliminary part of the subject it will be necessary to distinguish clearly between the terms "stress" and "strain." Any load, however it may be applied to a body, induces a resistance which is called stress, and the alteration in form which is produced is called strain. There is, therefore, no load without stress, and no stress without strain. This will perhaps be more fully apprehended if the statement be amplified as follows: By a *load* on any piece or member of structure is meant the aggregate of all the external forces acting upon it, including the weight of the piece itself and of other pieces supported by it. By a *strain* is meant the change of form produced in a piece by the action of a load, and by a *stress* is meant the resistance set up in the material, by its molecular forces opposing the action of a load in producing a strain. Thus a load of so many *pounds* upon a piece having so many *square inches area* produces a *stress* of so many *pounds per square inch*, the result being a *strain*, or change of form of a certain amount, whether temporary or permanent, and when large enough, appearing as stretching, shortening, bending, crumpling or twisting. The *strength* of a piece is measured by its power of resisting stress. The three chief kinds of stress are *tension*—stretching, pulling or tearing; *compression*—crushing, pushing or squeezing; *shearing*—cutting, nipping or sliding; called also *detrusion* when acting along the grain of wood. There are three other varieties, *transverse stress*—cross strain, bending or deflection, which is made up of tension and compression on opposite sides, together with shear increasing towards the ends; *torsion*—twisting or wrenching, which is a variety of shearing; *buckling*—crumpling, corrugating, twisting, which merges into crushing when the length is short enough. Any change from the original dimensions that remains after the removal of the stress is called *permanent set*. There is no very precise limit, compared with the ultimate strength of the material, at which permanent set commences, but, approximately, it begins to be appreciable when the piece is loaded to half its ultimate capacity.

The *elastic limit*, or *limit of elasticity*, is the point up to which the change of length in a piece under test is sensibly proportional to the force applied, and from which the piece will return to its original dimensions when the force is removed. In other words, the elastic limit is the maximum stress per square inch sectional area which any material can undergo without receiving a visible permanent set. As the elastic limit is the highest stress that can be put upon any piece without causing actual damage to it, the factor of safety should be a ratio determined by the elastic limit rather than by the ultimate strength. Generally speaking, the working stress should not exceed half and the proof stress two-thirds of the elastic limit.

The modulus of direct elasticity, or *stress-strain modulus*, known also by many other names, particularly that of *Young's modulus*, is the

ratio obtained by experiment of the stress per unit of section to the strain per unit of length, up to the limit of elasticity. It is fairly constant for each material within this limit, but beyond it the strain increases more rapidly than the stress. It was at first described as the height in feet to which a body would have to be piled in order that any small addition to its top, of its own substance, might compress the rest to an extent equal to the bulk of that added quantity. This is strictly Young's modulus and is expressed in feet, but out of compliment to his memory his name is still associated with the present stress-strain modulus. An intermediate stage, which, however, is numerically equivalent to the stress-strain modulus, was reached by Hooke, who defined the modulus of elasticity as the load which would increase a bar to twice its original length, assuming the elasticity to remain perfect so long.

The modulus of elasticity (indicated always by the letter E) affects chiefly the extension of material under direct tensile stress, and shortening under direct compressive stress. It also affects the resistance to bending or deflection, which is made up of extension on one side and compression on the other. This modulus is used in calculations of the strength of struts where the maximum working stress, or proof stress, must be below the limit where a tendency to bend commences. The modulus of elasticity in lbs. for the principal materials is—

Cast steel tempered	40,000,000
Mild steel	29,000,000
Wrought-iron bar	26,000,000
" plate	25,000,000
Cast iron	18,000,000
Oak and teak	2,000,000
Fir timber	1,500,000

Deflection is the displacement of any point in a loaded beam from its position when the beam is unloaded. Camber is an upward curvature given to a beam or girder, or some line in it, in order to ensure its horizontality when fully loaded. The general formula for deflection of beams is $d = k\frac{Wl^3}{48EI}$, where k = coefficient, d = deflection in centre in inches, W = load in lbs., l = span in inches, E = modulus of elasticity in lbs., I = moment of inertia in inch units. The coefficients according to the method of loading and supporting are—

Fixed one end loaded the other	= 16
" " load distributed	= 6
Supported both ends load central	= 1
" " distributed	= $\frac{5}{8}$

EXERCISES ON LECTURE III

Q. 18. Calculate the moment of resistance of an oak beam 9 in. by 6 in., when when C = 10,000.

A. $ZC = \frac{6 \times 9^2}{6} \times 10{,}000 = 81 \times 10{,}000 = 810{,}000$ lb.-inches.

Q. 19. Determine the maximum bending moment on a beam 12 ft. span with loads of 2 and 3 tons respectively at the points of trisection.

A. See Fig. 48. Reaction A $= \frac{2\times8}{12} + \frac{3\times4}{12} = 2\cdot\dot{3}$.

$$\text{Reaction B} = \frac{3\times8}{12} + \frac{2\times4}{12} = 2\cdot\dot{6}.$$

Total $2 + 3 = 2\cdot\dot{3} + 2\cdot\dot{6} = 5$ tons. Bending moment under 2 ton load $= 2\cdot\dot{3} \times 4 = 9\cdot\dot{3}$ ton-ft. Bending moment under 3 ton load $= 2\cdot\dot{6} \times 4 = 10\cdot\dot{6}$ ton-ft., which is therefore the maximum.

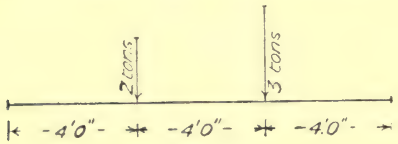


Fig. 48

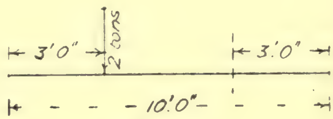


Fig. 49

Q. 20. A beam 10 ft. span carries a load of 2 tons at 3 ft. from one end; what is the bending moment at 3 ft. from the other end?

A. See Fig. 49. $\frac{2\times3}{10} \times 3 = 1\cdot8$ ton-ft.

Q. 21. A fir beam is required to carry 2 tons distributed over an 8 ft. span; what should be its scantling?

A. $W = \frac{bd^2}{L} \therefore bd^2 = WL = 2 \times 20 \times 8 = 320$. Let breadth be 4 in., then $b \times d^2 = 320$, or $d^2 = \frac{320}{4} = 80$, and $d = \sqrt{80}$ = say 9, making beam 9 in. by 4 in.

Q. 22. What will be the deflection of a fir beam 1 in. square on supports 3 ft. apart under a distributed load of 1 cwt.?

A. $d = \frac{Wl^3}{48EI} \times \frac{5}{8} = \frac{112 \times (3\times12)^3 \times 5}{48 \times 1{,}500{,}000 \times \frac{1}{12} \times 8} = 0\cdot544$ in.

LECTURE IV

Graphic Statics—Reciprocal Diagrams—Bow's Notation—Funicular Polygons—Bending Moment Diagrams—Shearing Forces—Vertical Shear—Horizontal Shear—Shear Diagrams—Combined Diagrams.

GRAPHIC Statics is the name given to the method of ascertaining stresses in structures by means of geometrical diagrams, whether by parallelograms of forces or reciprocal diagrams. Generally speaking, the latter is much shorter, and more likely to lead to accurate results. The system is due to Prof. Clerk Maxwell, and the best mode of lettering is that known as "Bow's notation" by Mr. R. H. Bow, an architect of Edinburgh, whose book on "The Economics of Construction," was published in 1873. A practical modification introduced by the writer was the substitution of figures for letters. In this system the spaces separated by the external forces are numbered in order, and then the internal spaces of the frame in the case of trussed structures. A few examples will make this clear and explain the principles. The simplest possible case will be that of three forces in equilibrium acting upon a point as in Fig. 50. The diagram of the forces acting upon the point will be what is called in more complex cases the frame diagram, and the corresponding triangle of forces, Fig 51, will be the reciprocal diagram. The spaces in the frame diagram are numbered in order clockwise, each force being known by the numbers of the spaces it separates, as force 1–2, force 2–3, and force 3–1. The reciprocal diagram is formed by drawing to scale, line 1–2 parallel to force 1–2, 2–3 parallel to force 2–3, and 3–1 parallel to force 3–1. It will be observed that lines meeting in a point in the frame diagram make a closed figure in the reciprocal diagram.

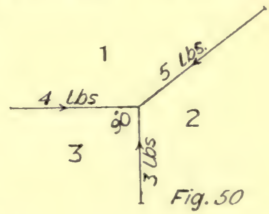


Fig. 50

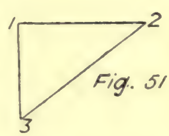


Fig. 51

Now take a more complicated case. Let the forces in Fig. 52 be in equilibrium, draw the reciprocal diagram, Fig. 53. Take any point inside the reciprocal diagram as a pole, and draw polar lines or vectors to the angles. Then across the frame diagram, commencing at any point in space 1, draw a line parallel to the vector 0–1 ; continue across space 2 parallel to vector 0–2, and so on. These lines in the frame diagram may be looked upon as actual bars or links hinged at their ends, and hence this is called the "link polygon." With the forces acting as shown the bars would all be in compression, but if the sense of the forces was reversed they would all be in tension, and might be

replaced by a continuous cord. The dotted polygon might then be called a "funicular polygon," from the Latin word *funis*, a cord. If one of the forces were missing from the frame diagram, Fig. 52, its magnitude would be found by its being the closing line of the reciprocal diagram, Fig. 53, and *a point in its line of action* would be found by the intersection of the vector parallels, so that for a number of forces acting on a body to be in equilibrium, the reciprocal polygon must close, and their funicular polygon must also close.

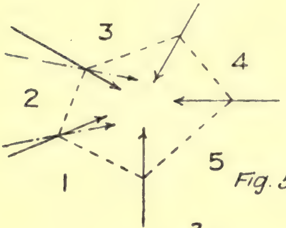


Fig. 52

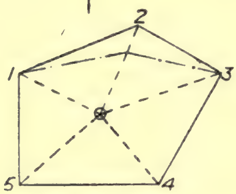


Fig. 53

Although in the last diagram all the forces are acting towards one point, the method of working, and the results, equally apply when the directions of the forces do not pass through one point. As a proof of this the stroke and dot lines in Fig. 53 may be taken as new directions and magnitudes for forces 1–2 and 2–3, when the corresponding force lines in Fig. 52 will be seen not to pass through the same point as the other forces.

Next take a series of parallel forces such as concentrated loads on a beam, with their reactions, as in Fig. 54. The reciprocal polygon, Fig. 55, will appear as a straight line, but point 5 cannot yet be fixed, because the respective values of the reactions 4–5 and 5–1 are unknown. Take any pole O in Fig. 55 and draw vectors; parallel to these draw a funicular polygon, Fig. 56, across the lines of action of the forces in Fig. 54, the closing line being obtained by joining the extremities across space 5. Parallel to this closing line draw a vector from the pole, which will give point 5 on the line of loads, and therefore the reactions 4–5 and 5–1. If the funicular polygon were formed of a loose cord with the ends supported, the given loads being upon it in their respective lines of action would make the cord take the shape shown. If any other point be taken for the pole in Fig. 55 a

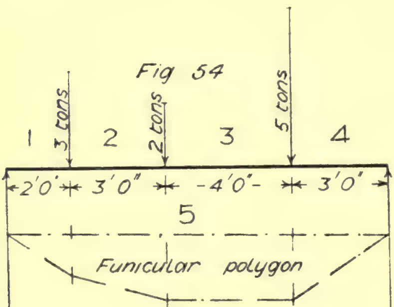


Fig. 56

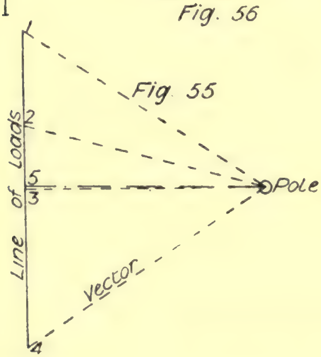
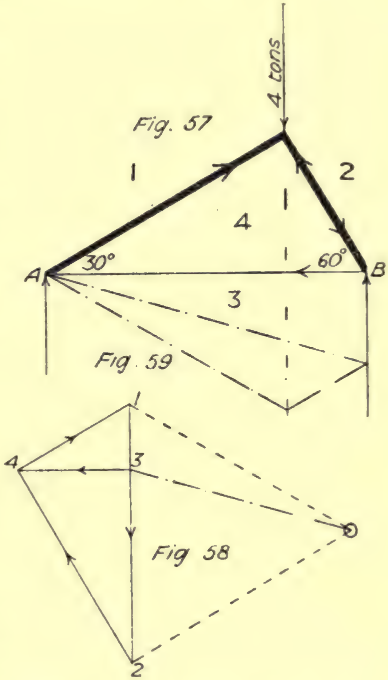


Fig 55

different funicular polygon would be drawn, but the closing line would still give the true direction for the vector to point 5.

Now take a triangular frame, as Fig. 57, draw the load line of the reciprocal diagram Fig. 58, then parallel to 1–4 and 2–4 draw lines from points 1 and 2 to intersect in 4, and draw 4–3 parallel to 4–3 of the frame diagram. Here the reactions are obtained without a funicular polygon, but one may be drawn as in Fig. 59 to show the application. No vector is required to point 4 as it is only the external forces that are dealt with by the funicular polygon. Some other useful information not yet mentioned is given by the reciprocal diagram. The load line having been drawn to scale in Fig. 58 the length of line 1–4 gives the magnitude of the stress in bar 1–4 of the frame, line 2–4 the stress in bar 2–4, and line 3–4 the stress is bar 3–4. Besides this the nature of the stress, whether tension or compression, can be ascertained. Take the parts meeting together at the apex of the frame; the force 1–2 is acting downwards, therefore place an arrow head in that direction upon 1–2 in the reciprocal diagram. Then, working clockwise, the stress in bar 2–4 is found by putting an arrow head on line 2–4 in the reciprocal diagram to run circuitally, and transferring it in the same direction to the frame diagram as shown. It will be seen that this force acts towards the joint and therefore indicates compression. In the same way bar 4–1 will be found to be in compression, and it is convenient to indicate all compression bars by thick lines. With a little practice the stresses can be equally well discovered without actually marking the arrow heads on the paper, and it is better to omit them because, in considering the other joints, it will be found that some of the same lines want the arrow heads the other way on. Take for instance the joint at B; bar 2–4 being in compression will have the arrow head towards the joint, force 2–3 acts upwards, and putting these arrow heads on the reciprocal diagram (or imagining them there) it will be seen that, to run circuitally, 3–4 acts away from the joint and therefore indicates tension. One other fact to observe is that lines meeting in a point in the frame diagram form a closed figure in the reciprocal diagram, and *vice versâ*, lines forming a closed figure in the frame diagram radiate from a point in the reciprocal diagram.



Applied to beams the funicular polygon forms a bending moment diagram, but before these are further investigated it will be well to consider the shearing forces in a beam. When a beam supported at the ends carries a load there is a tendency to cut through vertically at all points, which is represented by Fig. 60. This is vertical shear. There is at the same time a tendency to cause the layers to slide upon one another horizontally as represented in Fig. 61, and as would be shown visibly if separate planks were superposed. This is horizontal shear. Horizontal shear is only taken account of in determining the pitch of rivets, but vertical shear has often to be considered, particularly when a heavy load is carried upon a short beam. The shearing force at any point is equal to the amount of load passing through that point to the nearest abutment; at the abutment the shearing force is therefore equal to the reaction. For instance, in Fig. 62 with central load, if the reaction is measured upwards from A in Fig. 63 to indicate the shearing force, it will be seen that it remains constant until the load is reached. Then it is lessened by the amount of load passed, so that at the centre it changes from

Fig. 60

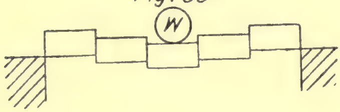


Fig. 62

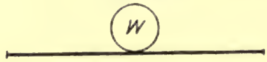
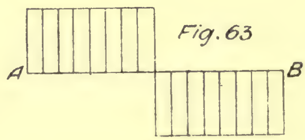


Fig. 61

Fig. 63

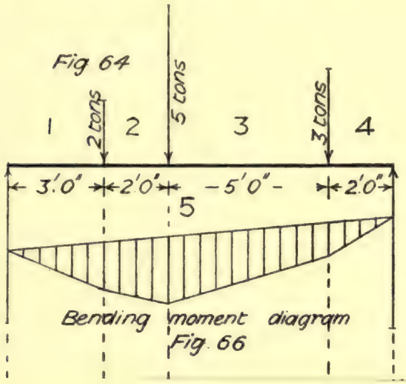


$+\frac{1}{2}$W to $-\frac{1}{2}$W, and remains $-\frac{1}{2}$W up to the abutment B. This is a very simple mode of determining shear and applies even to the most complex cases. Now, applying these principles, a combined diagram of bending moments and shearing forces can be obtained. Let Fig. 64 represent a loaded beam to a scale of $\frac{1}{4}$ in. to 1 ft. Upon the line of direction of the reaction B, draw the line of loads Fig. 65 to a scale of $\frac{1}{4}$ in. to 1 ton. Select a pole O at 10 ft. distance from the line of loads, and draw vectors. Parallel to these in the space between the line of loads and the beam draw the funicular polygon Fig. 66, and parallel to the closing line draw the vector to give point 5. Now draw horizontal lines from the divisions on the load line across the lines of direction of the loads, and the complete shear diagram will be as shown in Fig. 67. The funicular polygon forms the bending moment diagram to the same horizontal scale as the beam and to a vertical scale of 4 tons to 1 in. $\times$ 10 ft. distance of pole = 40 ton-ft. to 1 in. The vertical ordinates of both diagrams give the value at intermediate points.

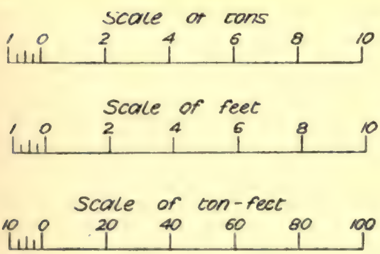
An important point to observe is that the shear is a minimum where the bending moment is a maximum, also that the bending moment at

any point is equal to the area of the shearing force diagram from the nearest abutment to that point.

As the parabola is frequently required in connection with stress diagrams and in designing girders, the simplest methods of constructing it may be given. Fig. 68 shows the method for ordinary cases, where



NOTE.—The figures 66 given in the bracket after weight per foot run at middle of page should be 96, a consequent error runs through the remainder of the chapter.



the semi-base and the height are divided into the same number of equal parts; lines are drawn from the side divisions to the apex and their intersection with vertical lines from the base gives the curve. The method is equally applicable to a vertical parabola upon an inclined line as in Fig. 69. Fig. 70 applies best when the height is less than one-eighth of the span. A triangle is drawn with a height equal to twice

the required height of parabola, and equal divisions are made on the two sides. The lines joining similarly numbered points give tangents

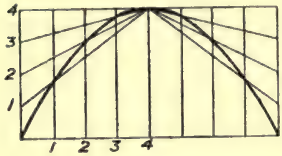


Fig. 68

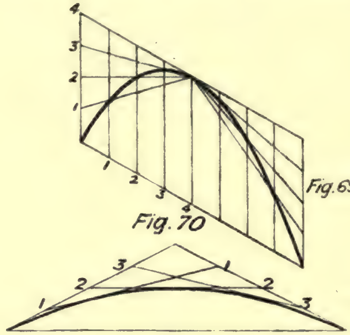


Fig. 69

Fig. 70

to the required curve, and when the curve has little rise the tangents may be used to represent the curve.

EXERCISES ON LECTURE IV

Q. 23. Draw any four forces acting on a body, find the closing force and its

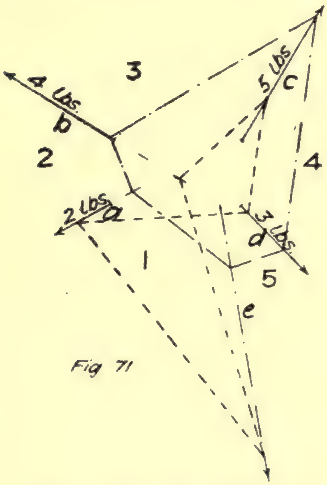


Fig 71

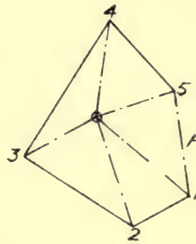


Fig. 72

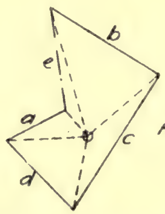


Fig. 73

line of action. Check the result by making a second reciprocal diagram with the forces in a new order.

For answer, see Figs. 71, 72 and 73.

Q. 24. A beam AB 12 ft. long has forces applied to it at 3 ft. intervals from A, as follows, 7 tons at angle 60 degrees, 3 tons at angle 135 degrees, 5 tons at angle 90 degrees, all measured clockwise. Find the reactions, the bending moments, the shearing forces and the thrusts.

For answer, see Figs. 74, 75 and 76.

Q. 25. A triangular frame having sides 6, 8 and 10 ft. long is supported vertically on its longest side; a force of 3 tons is applied to the apex at an angle of 30 degrees from the vertical in the plane of the frame and towards the long

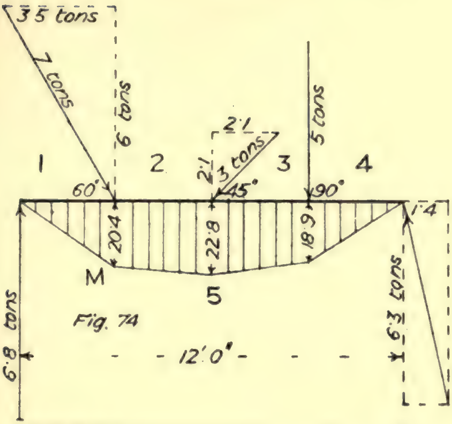


Fig. 74

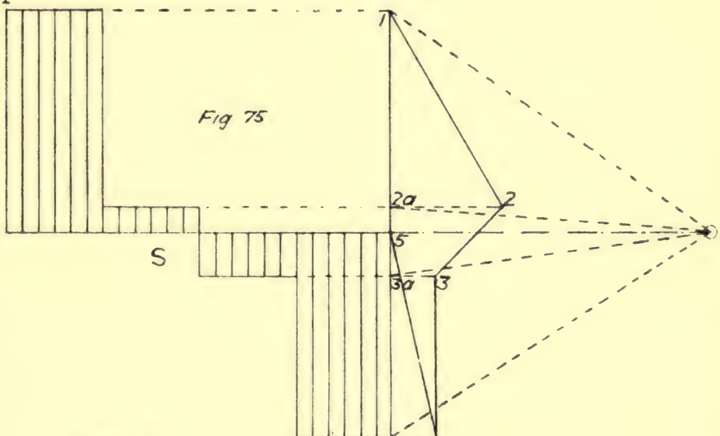
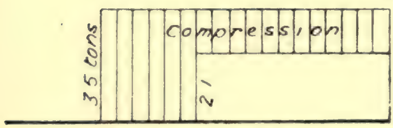


Fig. 75

Fig. 76



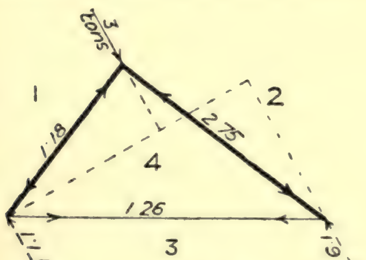


Fig. 77

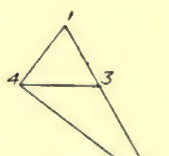
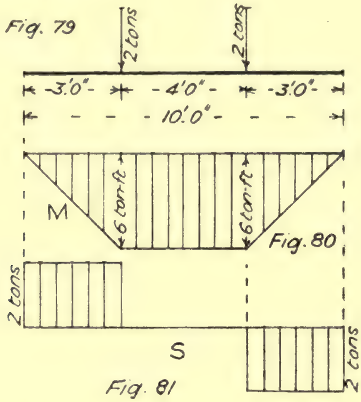


Fig. 78

end; find the reactions and the nature and amount of the stresses in the bars of the frame.

For answer, see Figs. 77 and 78.



Q. 26. A girder 10 ft. span has two concentrated loads of 2 tons, each 2 ft. from the centre of span; show the bending moment and shear diagrams.

For answer, see Figs. 79, 80 and 81.

LECTURE V

Construction of Bending Moment and Shear Diagrams, for Beams with Standard Loading and Supporting.

A CANTILEVER, loaded at the end as Fig. 82, has bending moments varying directly as the load and the distance from the load to the point considered. The maximum bending moment for span l and load W will be Wl, and the intermediate moments will be ordinates to the triangle Fig. 83. The shear will, upon the principles previously laid down, be uniform throughout and equal to W, as in Fig. 84. A cantilever with a uniformly distributed load of w per foot run, lbs., cwts., or tons, as Fig. 85, will have a maximum bending moment of $wl \times \frac{1}{2}l = \frac{1}{2}wl^2$, and the intermediate moments will be as ordinates to a semi-parabola with the vertex at the end of cantilever, as Fig. 86. The maximum shear

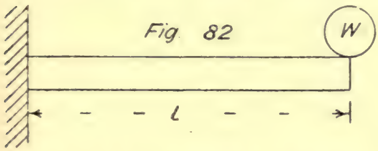


Fig. 82

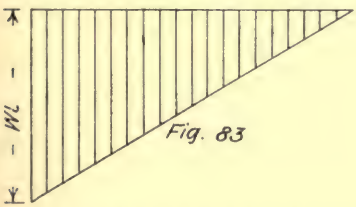


Fig. 83

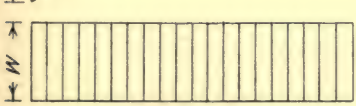


Fig. 84

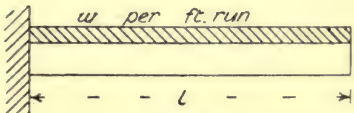


Fig. 85

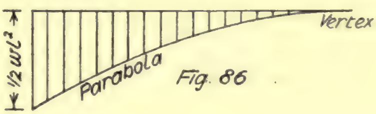


Fig. 86

Fig. 87

will be wl, and diminishing as the load is passed it will vary as ordinates to a triangle and be nil at the end, as Fig. 87.

When the beam or girder is supported at both ends and loaded in the centre, as Fig. 88, the maximum bending moment will be, reaction $\frac{1}{2}$W $\times$ leverage $\frac{1}{2}l = \frac{Wl}{4}$, immediately under the load, the intermediate moments giving ordinates to a triangle, as Fig. 89. The shear diagram commencing equal to the reaction $+\frac{1}{2}$W will remain constant until the load is reached, when it will become $-\frac{1}{2}$W through to the other support, as Fig. 90. With a uniformly distributed load of w per foot run, as

Fig. 91, the maximum bending moment in the centre will be, reaction $\frac{1}{2}wl \times \frac{1}{2}l - \frac{1}{2}wl \times \frac{1}{4}l = \frac{wl^2}{8}$, and the intermediate moments will be as ordinates to a parabola, as Fig. 92. The shear stress commencing $+\frac{1}{2}wl$ reduces towards the opposite end at the rate of w per foot run, being nil in centre and $-\frac{1}{2}wl$ at the far end, as in Fig. 93.

With a concentrated load dividing the span l into the segments x and y, as Fig. 94, the maximum bending moment, calculated by leverage as before, will be $\mathrm{W}\frac{xy}{x+y}$ under the load, and the remainder as ordinates to the triangle, as Fig. 95. The shearing force at the left abutment will be $\mathrm{W}\frac{y}{x+y}$, dropping under the load to $-\mathrm{W}\frac{x}{x+y}$, as Fig. 96, Whether a shear stress is plus or minus it makes no difference in the calculations of strength, the signs merely indicate whether the forces are measured upwards or downwards from the line of origin.

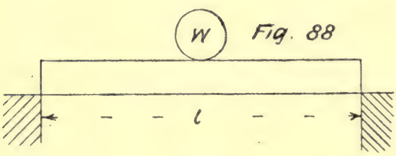


Fig. 88

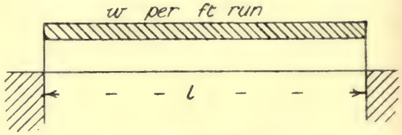


Fig 91

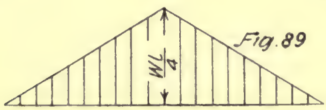


Fig. 89

Fig 92

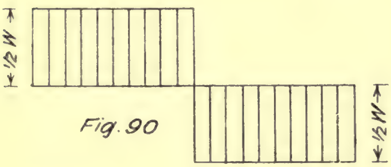


Fig. 90

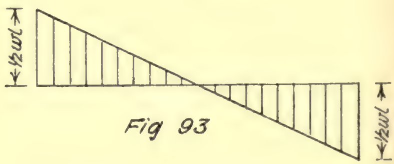
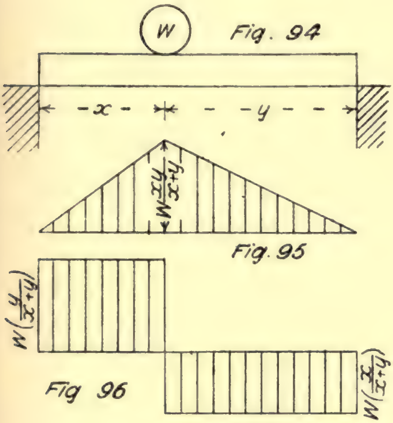


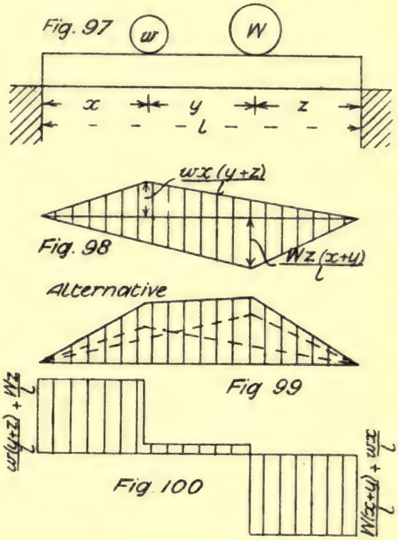
Fig 93

With two unequal concentrated loads dividing the beam into three segments x, y, z, as Fig. 97, the bending moment triangle from each load is conveniently placed on opposite sides of the line of origin and the ordinates measured right through, as Fig. 98, but it is equally correct to put them both on the same side, and add on to the outside the overlapping parts, as Fig. 99. The shear diagram, determined as before, will be as shown in Fig. 100.

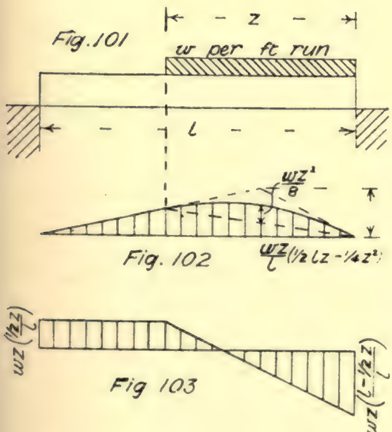
When a girder is only partially loaded with a uniformly distributed load, as Fig. 101, the principle of the bending moment diagram Fig. 102 can be very easily explained. First the triangle of moments is drawn as if the load were concentrated at its centre of gravity, then the triangle being intersected by lines dropped from the ends of the load, the points of intersection are joined by a straight line, upon which a parabola is

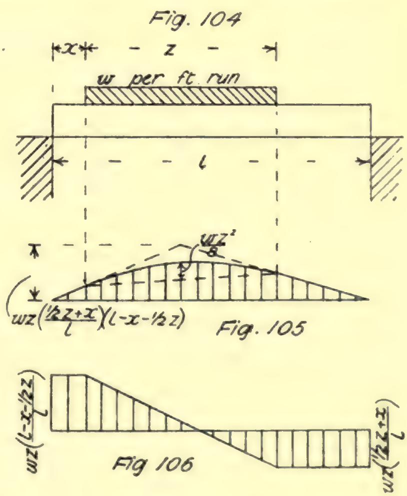
constructed as if it were the whole span of a girder. The shear diagram Fig. 103 is constructed upon the same principles as before, and is now a





convenient illustration of the statement that the bending moment is a maximum where the shear is a minimum, the point where the shear is





nil is opposite the ordinate to be measured for maximum bending moment. In Fig. 104 the distributed load does not reach the supports, but the method of finding the bending moment diagram, Fig. 105, and

shear diagram, Fig. 106, is the same as the last case, the formulæ, however, come out a little more complex.

EXERCISES ON LECTURE V

Q. 27. A rolled joist 12 ft. span carries concentrated loads of 1 ton at 2 ft.

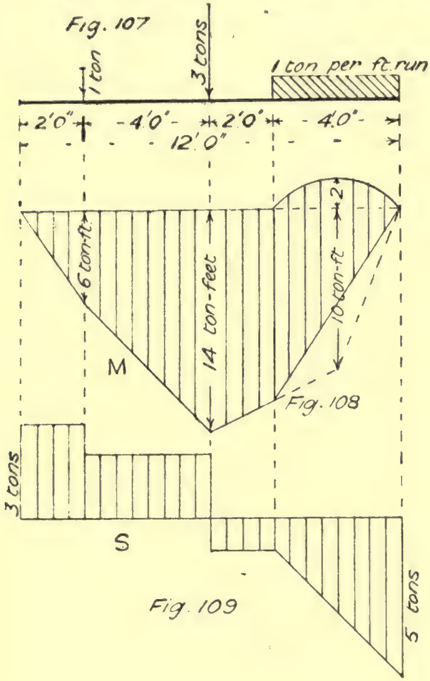


Fig. 107

Fig. 108

Fig. 109

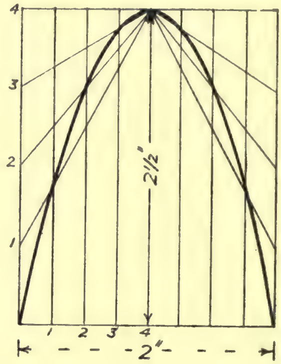


Fig. 110

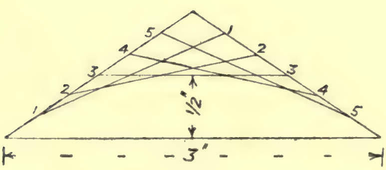


Fig. 111

Fig. 112

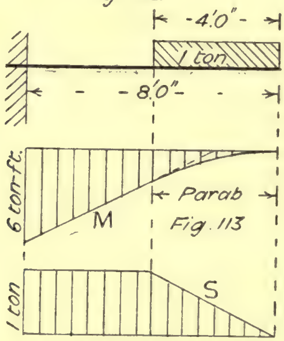


Fig. 113

Fig. 114

from A, 3 tons at 6 ft. from A, and a distributed load of 1 ton per foot run for 4 ft. from B. Construct the bending moment and shear diagrams.

For answer, see Figs. 107, 108, and 109.

Q. 28. Draw a parabola with a base of 2 in. and height of 2½ in.

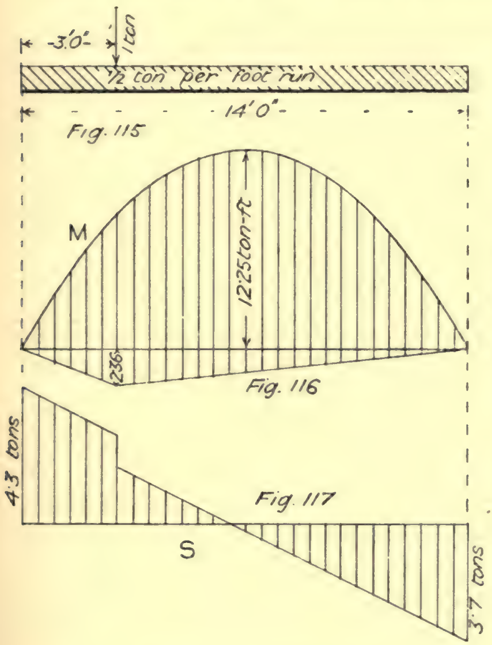
For answer, see Fig. 110.

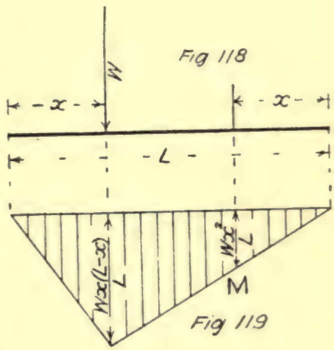
Q. 29. Draw by another method a parabola with a base of 3 in. and height of ½ in.

For answer, see Fig. 111.

Q. 30. A cantilever 8 ft. span carries a load of 1 ton distributed over the outer half; draw the bending moment and shear diagrams.

For answer, see Figs. 112, 113, and 114.





Q. 31. A girder, 14 ft. span, carries a uniformly distributed load of ½ ton per foot run, and a concentrated load of 1 ton at 3 ft. from one end. Find the bending moment and shear diagrams.

For answer, see Figs. 115, 116, and 117.

Q. 32. A girder of L span supported at the ends carries a non-central load of W tons at x feet from one support; what will be the bending moment at x feet from the other support?

For answer, see Figs. 118 and 119.

LECTURE VI

Rolled Joists—Finding Moment of Inertia Graphically—Shear Stress in Rolled Joists—Deflection of Rolled Joists—Compound Girders.

ALTHOUGH a solid rectangular beam contains the greatest possible amount of material for the dimensions, it is not all equally effective, those parts lying near the neutral axis being of very little value in resisting the bending stresses, because they have so short a leverage. It is clear, therefore, that if a considerable portion of the material could be removed from the neighbourhood of the neutral axis, and placed as far as possible away from it, a much stronger beam would be obtained with a given amount of material. This is the principle of flanged beams, and rolled steel joists represent the most efficient kind. Now that published tables of what are called the "mechanical elements" of rolled joists are so readily obtainable, there is not the necessity that formerly existed for being able to calculate the section modulus from the dimensions; approximately it is the area of one flange in square inches multiplied by the distance in inches from centre to centre of the flanges. For example, B.S.B. 21, that is, British Standard Beam No. 21, 12 ins. by 6 ins. by 44 lbs. per foot run, has a mean flange thickness of 0·717 in. and web thickness of 0·4 in., as in Fig. 120. Then the area of one flange will be $6 \times 0{\cdot}717 = 4{\cdot}302$ sq. ins., and the mean depth $12 - 0{\cdot}717 = 11{\cdot}283$ ins., making the section modulus $4{\cdot}302 \times 11{\cdot}283 = 48{\cdot}54$ inch units. The true section modulus, according to the published tables, is 52·55, and this takes the web into account. The formula for calculating the section modulus of a rolled joist made up of rectangles, as in Fig 121, is $Z = \frac{BD^3 - bd^3}{6D}$. Applied to the same joist, this gives

$$\frac{6 \times 12^3 - (6 - 0{\cdot}4)(12 - 2 \times 0{\cdot}717)^3}{6 \times 12} = \frac{10368 - 6605{\cdot}7}{72} = 52{\cdot}25,$$

which is very nearly the full amount. The same section may be used to illustrate the two geometrical methods of finding the moment of inertia, and from it the section modulus. The first is shown in Fig. 122; it is on the same principle as the rectangular beam, Fig. 38. Every point on the outline may be considered as brought in towards the centre line as far as the line joining the projection of the point on the upper surface with the centre of the neutral axis. The area of the shaded figure must be obtained by a planimeter, and then the piece must be cut out and suspended from two points to find the centre of gravity, which makes it rather a troublesome method. Let D = the depth from neutral axis to extreme fibres, A = area of shaded figure or inertia area, G = distance of centre of gravity of shaded area from neutral axis;

then this being one-half of the whole section the moment of inertia will be 2DAG, or the section modulus will be 2AG. The least moment of inertia will be found by taking the neutral axis in the opposite direction and working similarly, but it is only wanted when the rolled joist is to be used as a stanchion. The other method is shown in Figs. 123, 124, and 125. Divide up the section into portions at each change of width as before, and project a horizontal line from the centre of gravity of each portion. Take the area of these portions as if they were weights or forces and number the spaces between them. Then set off distances

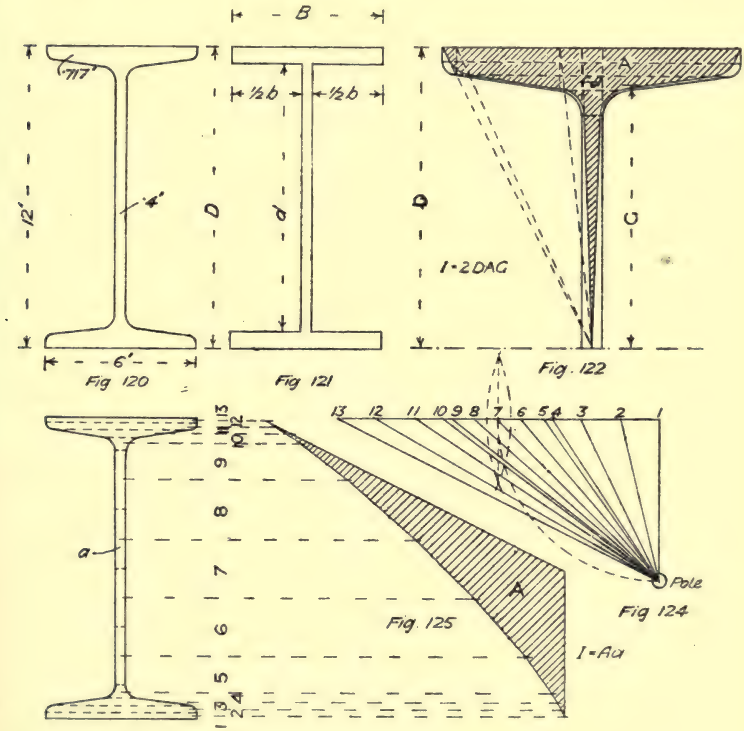


Fig 120 Fig 121 Fig. 122

Fig 124 Fig. 125 Fig. 123

from the point 1, Fig. 124, to any given scale to represent these areas, and draw a vertical line from 1 equal to half the length of the horizontal line, making the lower end a pole. Then draw vectors from the pole to the divisions on the horizontal line, and construct the funicular polygon, Fig. 125, by drawing lines across the spaces parallel to the vectors. Then the area of the shaded funicular polygon, A, multiplied by the whole area of the section, *a*, gives the moment of inertia, I, and this, divided by the distance from neutral axis to edge of section, or half the depth, gives the section modulus Z. Thus $I = Aa$, $Z = \frac{I}{\frac{1}{2}d}$.

The common formula in use by architects for the strength of a

rolled steel joist is $W = \frac{10ad}{L}$, where W = breaking weight in tons in centre, a = sectional area of one flange in square inches plus one-sixth of the web area, d = total depth of joist in inches, L = clear span in feet. Applied to the 12 ins. by 6 ins. by 44 lbs. r.s.j. for a span of 12 ft., this will give $W = \frac{10 \times (4 \cdot 302 + 1 \cdot 76 \times 0 \cdot 4) \times 12}{12} = 50$ tons; allowing a factor of safety of 4, the safe central load would be 12·5 tons, or safe distributed load 25 tons. By the table of safe loads in Dorman, Long & Co.'s catalogue (1906) the safe load on this section at 12 ft. span is 22 tons.

With ordinary loads and spans the web of a rolled joist has usually ample strength to resist the shear stresses, but the load-carrying capacity, as measured by the moment of resistance, increases as the span reduces until a proportion is reached where the web is insufficient to resist the shear stress at the supports. It is sufficiently accurate to take two-thirds of the full depth of a rolled joist as the depth of web free to take the shear stress. In the last example the effective depth of web will be $\frac{2}{3} \times 12 = 8$ ins., and the thickness is 0·4 in. With a distributed load of 22 tons the shear at the support will be 11 tons, or $\frac{11}{8 \times 0 \cdot 4} = 3 \cdot 44$ tons per sq. in. With half the span the load would be doubled to give the same bending moment, but this load would double the shear stress, and a stiffening plate should then be riveted on the side of the web. The maximum working stresses for rolled steel joists in tension and compression are 6, $7\frac{1}{2}$ or 10 tons, according to circumstances, being $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{3}$ of the ultimate strength. The maximum safe shear stress is generally taken as 6 tons per sq. in., but it might be more satisfactory to adopt the formula $s = 8 - \frac{1}{6}d$, where d = depth of web in inches, and s = tons per sq. in. maximum safe shear stress in rolled steel joists.

Examination questions that involve the determination of the size of a rolled joist are always difficult because there are 30 standard sizes, and it is impossible to carry all these in the memory. There are, however, certain approximate rules to bear in mind that will facilitate the work. The depth in inches should not be less than half the span in feet nor more than the whole span. The maximum bending moment having been found, the formula $M = fad$ may be used, where f = maximum stress per square inch, say $7\frac{1}{2}$ tons, a = area of each flange in square inches, d = mean depth of joist. Or the following rules may be adopted. Let d = total depth of joist in inches, M = maximum bending moment in ton-ft., a = sectional area of *one* flange in square inches, b breadth of flange in inches, t = mean thickness of flange in inches, w = weight of joist in lbs. per foot run. Then approximately $d = 2\sqrt{M}$, $a = \frac{M}{\cdot 78d}$, $t = \sqrt{\frac{a}{6}}$, $b = 6t$, $w = 12a$. The weight of a rolled steel joist in lbs. per foot run is 3·4 times the sectional area in square inches, and is approximately $= \frac{WL}{d} \times 2$. When rolled joists are used side by side it

is advisable to connect them by cast-iron separators, spreaders, or distance pieces, with two bolts to each, passing through the webs at intervals of 4 to 6 ft.

A rolled joist built firmly into the wall at each end for a distance of not less than four times its depth, with a stone template under the face of each bearing, and another over each end of joist with a good top load, may be looked upon as fixed at the ends, and may be loaded with one-third more than if merely supported at the ends. If built in at each end a distance equal to one-fourth of the span it may be loaded with half as much again as if merely supported.

When a rolled joist is supported in the centre as well as at both ends it will carry four times the load that it will when merely supported at the ends.

Two rolled joists placed side by side, with distance pieces between, or placed one over the other without riveting, will have together only twice the strength of a single joist; but two rolled joists placed one over the other and sufficiently riveted together will have $2\frac{1}{2}$ times the strength of one of the joists alone. Sufficient riveting will be obtained when the sectional area of the rivets per foot run equals the shear stress divided by the total depth in feet.

Rolled steel joists follow the same rules for deflection as other beams, which have already been given. The usual case may be repeated here, viz. $D = \frac{Wl^3}{48EI} \times \frac{5}{8}$, where W = distributed load in tons, l = span in inches, E = modulus of elasticity, say 12,000 to 13,000 tons, I = vertical or greatest moment of inertia in inch units. The ratio of span to depth ought to govern the factor of safety used in the case of long or shallow joists. A suitable factor is given by the formula $\frac{\text{span ins.}}{3\cdot5 \text{ depth ins.}}$ and the weight of the joist itself must always be taken into account. Or the maximum bending moment in ton-inches must not exceed 100 times the ratio of depth to span multiplied by the section modulus. In ordinary cases when the depth is not less than $\frac{1}{24}$ span the section modulus must be at least $1\frac{1}{2}$ times the maximum bending moment in ton-feet.

Compound girders are rolled joists with one or more flange plates riveted on. The simplest consists of a single joist with a plate riveted on the top flange, where the rivet holes being through the compression flange do not weaken the girders, and the neutral axis is raised to pass through the centre of gavity of the whole section. When compound girders were introduced in 1867 by Messrs. Homan & Phillips, a fictitious value was assigned to them owing to an error in the method of calculating their strength. We now know that they follow the same laws as any other section, and their advantage over built-up plate girders consists only in having a solid connection between the web and angles. It is more economical to use two or three plain rolled joists side by side than to have a compound girder, and there is less likely to be delay in delivery.

EXERCISES ON LECTURE VI

Q. 33. A 10-in. by 5-in. by 30-lb. rolled steel joist (B.S.B.17) with a mean flange thickness of ·552 in. and web thickness of ·36 in. has a 10 in. by ½ in. plate riveted on the top flange. Find the neutral axis and the greatest moment of inertia.

For answer, see Figs. 126, 127, and 128. The neutral axis is found by constructing *ab* and drawing a vertical to intersect line from *a* in *c*.

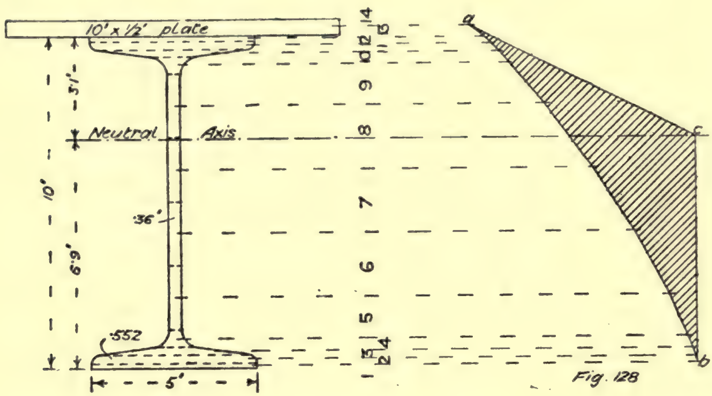


Fig. 126

Fig. 128

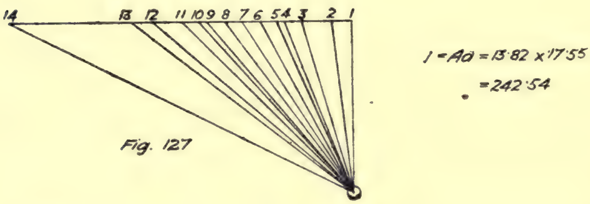


Fig. 127

Q. 34. A rolled joist supported at the ends, on a span of 20 feet, is required to carry a distributed load of 9 tons. What section should be adopted?

A. The maximum bending moment will be $\frac{9 \times 20 \times 12}{8} = 270$ ton-inches. Say depth $= \frac{1}{20}$ span $= 12$ in. Then $M = fad$, whence $a = \frac{M}{fd} = \frac{270}{7·5 \times 12} = 3$ sq. in. area of flange, say 5 in. wide and $\frac{3}{5} = 0·6$ in. mean thickness, and $w = 3 \times 12 = 36$ lbs. per ft. run, making the whole section 12 in. by 5 in. by 36 lbs. Or,

$$d = 2\sqrt{M} = 2 \times \sqrt{(\tfrac{270}{12})} = 2 \times \sqrt{22·5} = 2 \times 4·74 = 9·48, \text{ say 10 in.,}$$

$$a = \frac{M}{·78d} = \frac{22·5}{·78 \times 10} = 2·88 \text{ sq. in.,}$$

$$t = \sqrt{\frac{a}{6}} = \sqrt{\frac{2·88}{6}} = \sqrt{·48} = 0·7,$$

$$b = 6t = 6 \times 0·7 = 4·2, \text{ say 4 in.,}$$

$$w = 12a = 12 \times 2·88 = 34·56,$$

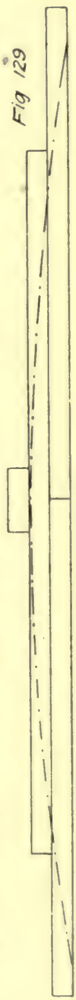
making the whole section 10 in. by 4 in. by 35 lbs. Upon reference to a list the proper section will be found to be B.S.B.20, 12 in. by 5 in. by 32 lbs.

LECTURE VII

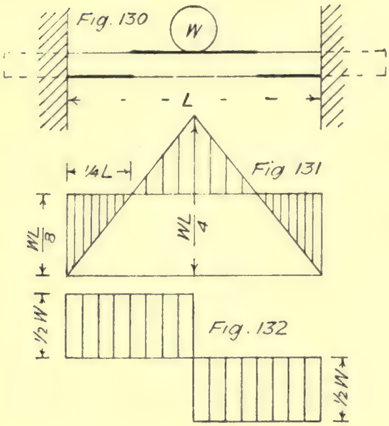
Plate Girders—Continuous Beams—Comparative Strength of Structures—Safe Load on Structures

PLATE girders are those made up of plates and angles riveted together; they are now only used for comparatively long spans and heavy loads, and appertain more to engineering than to building construction. Many precautions have to be taken in their design, but there are only one or two points that can be mentioned here. The span taken in the calculations is the effective span, measured from centre to centre of the bearing surfaces, and the depth is the mean depth from centre to centre of the flanges. The weight of the girder itself has to be allowed for, say w in tons $= \frac{\mathrm{WL}}{400} \times \sqrt{\frac{\mathrm{L}}{d}}$, where W = external load in tons, L = clear span in feet, d = total depth in inches. When more than one plate is required in each flange the outer plates are kept short, so that the sectional area of flange at any point is proportional to the bending moment, and the girder is made of uniform strength instead of uniform section. Owing to the length of the girders it frequently happens that joints have to be made in some of the inner flange plates, and the strength thus lost is made up by cover plates on the outside, of a thickness equal to that of the plate cut through, and of such length that the sectional area of the rivets on each side of the joint multiplied by the safe shear stress is equal to the sectional area of the cover plate multiplied by the safe tensile stress. The practice varies with regard to cover plates on the compression flange, but generally speaking they are made the same length as on the tension flange. In determining the length of the outer plates a parabola is drawn, as in Fig. 129, of a depth equal to the calculated thickness of plates, and the plates are continued half-cover length beyond the outline of the curve. With a distributed load the shear stress is nil at the centre of the girder, but no plate is made less than $\frac{1}{4}$ in. thick under any circumstances. The proper pitch for the rivets is a question that puzzles many young designers. It is not a matter of choice, but depends upon the horizontal shear, found in the same way as described for the rolled joists riveted together. Stiffeners are required at intervals of 4 ft. to 6 ft., and over the edge of each bearing surface, these may be of simple angle or tee sections, or built up with gusset plates and angles according to circumstances.

Fig 129



When a beam is sufficiently anchored down, or built in at the ends, or continued over more than one span, the stresses are considerably modified, and the girder carries an increased load for a given sectional area. The simplest case is that of a single span with the ends built in.



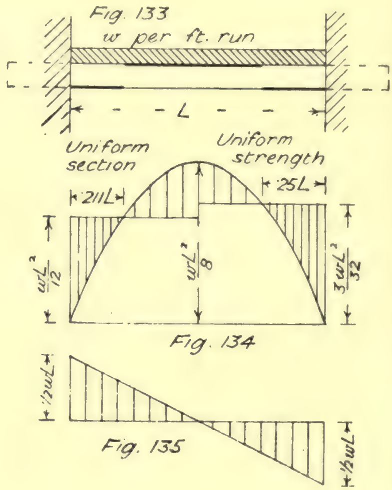
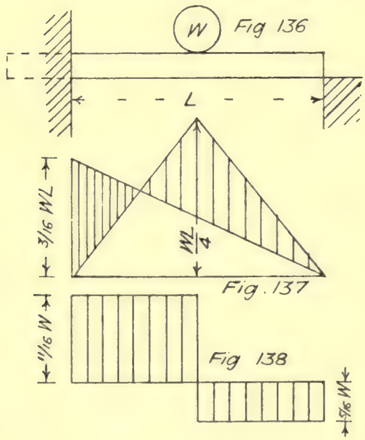


Fig. 130 shows the general effect of the fixed ends, the thick lines of the girder indicating compression and the thin lines tension, the change of stress occurring at the points of contrary flexure. The bending moment diagram for a beam of uniform section with a central load and fixed ends is shown in Fig. 131, and the shear diagram by Fig. 132. When the load is distributed, as in Fig. 133, the bending moment diagram will be as Fig. 134, and the shear diagram as Fig. 135. When fixed at one end and freely supported at the other with central load, as Fig. 136, the bending moment diagram will be as Fig. 137, and the shear diagram as Fig. 138. When fixed at one end and freely supported at the other, with a distributed load, as Fig. 139, the bending moment diagram will be as Fig. 140, and the shear diagram as Fig. 141.



A girder fixed at one end and freely supported at the other, with an intermediate support, as in Fig. 142, will have the bending moment diagram shown in Fig. 143, and the shear diagram as in Fig. 144. When

a beam is continued beyond two simple supports it receives stresses in one or other of the forms shown.

An important point to note in connection with continuous beams is that the load on the supports is not proportioned as would appear to

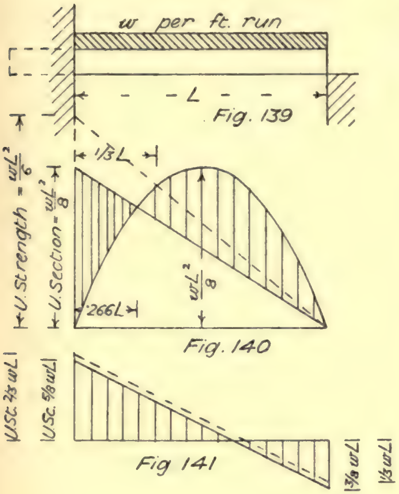


Fig. 139

Fig. 140

Fig 141

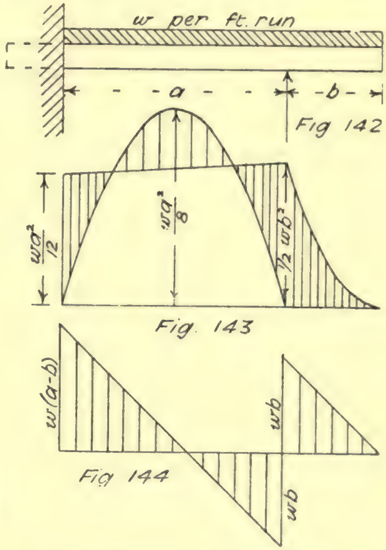


Fig 142

Fig. 143

Fig 144

be the case from inspection. When continuous over two equal spans it looks as if the pressure on the supports from a uniformly distributed load W would be $\frac{1}{4}$W, $\frac{1}{2}$W, $\frac{1}{4}$W, but if the supports are level the pressures will be $\frac{3}{16}$W, $\frac{5}{8}$W, $\frac{3}{16}$W, and the bending moment and shear diagram will be the same as Figs. 140 and 141 repeated on each side of the central support. Over three equal spans, as Fig. 145, the distribution of pressure on the supports will be $\frac{4}{30}$W, $\frac{11}{30}$W, $\frac{11}{30}$W, $\frac{4}{30}$W, the bending moment diagram will be as Fig. 146, and the shear diagram Fig. 147.

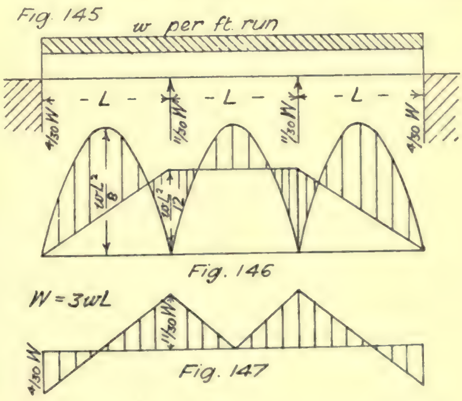


Fig. 145

Fig. 146

Fig. 147

The proof of this distribution of load depends upon "The Theorem of Three Moments," but the following is a fairly simple explanation of the case where the beam has two equal spans. Let R = reaction at central support in lbs., W = total load in lbs., l = span in inches, E = modulus of elasticity in lbs., I = moment of inertia in inch units. Then

if the central support were lowered and the beam rested upon the end bearings only, the deflection under a uniformly distributed load would be $D = \frac{Wl^3}{48EI} \times \frac{5}{8}$. But the upward reaction at central support has to balance this deflection and acts like a concentrated load at the centre of beam, the upward deflection of which would be $D = \frac{Rl^3}{48EI}$. With the supports on the same level the deflection = 0, therefore

$$\frac{Rl^3}{48EI} = \frac{Wl^3}{48EI} \times \frac{5}{8}, \text{ whence } R = \tfrac{5}{8}W.$$

"The strength of structures varies as the square of the linear dimensions of similar parts, excluding the effect of weight, but the weight varies as the cube of the linear dimensions. The strength of a structure of any kind is not, therefore, to be determined by that of its model, which will always be much stronger in proportion to its size. All works, natural and artificial, have limits of magnitude which, while their materials remain the same, they cannot surpass" (Lardner).

This is an important statement that should be stored in the memory, but there is not time to give any illustrations of its application.

Common ratios for safe working load to breaking load are as follows :—

Cast-iron columns	$= \frac{1}{4}$
Cast-iron girders for tanks	
Wrought-iron structures	
Mild steel structures	
Cast iron for bridges and floors	$= \frac{1}{6}$
Stone and bricks	$= \frac{1}{8}$
Timber under live loads (permanent structures)	$= \frac{1}{10}$
Timber under dead loads (permanent structures)	$= \frac{1}{7}$
Timber under live loads (temporary structures)	
Timber under dead loads (temporary structures)	$= \frac{1}{5}$

The safe loads allowed on floors, including the floor itself when of timber, but excluding it when of concrete and steel, are—

Dwelling-houses	1 cwt. per sq. ft.
Churches and public buildings	$1\frac{1}{2}$,, ,,
Warehouses	$2\frac{1}{2}$,, ,,

EXERCISES ON LECTURE VII

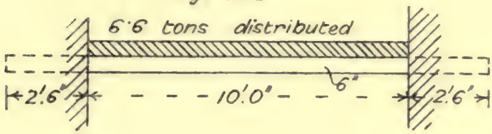
Q. 35. A rolled joist cantilever 6 ft. span carries a load of 10 tons at the extremity and has a moment of inertia of 315·3. What will be its deflection?

A. $$d = \frac{Wl^3}{48EI} \times 16 = \frac{Wl^3}{3EI} = \frac{(10 \times 2240) \times (6 \times 12)^3}{3 \times 29{,}000{,}000 \times 315{\cdot}3} = 0{\cdot}305 \text{ in.}$$

Q. 36. A rolled joist 6 in. deep and 10 ft. span will carry 4·4 tons uniformly distributed when supported at the ends. It has to be built in at the ends so as to carry 6·6 tons. Show the elevation of the girder with the built-in ends, and also the bending moment and shear diagrams.

For answer, see Figs. 148, 149, and 150.

Fig 148



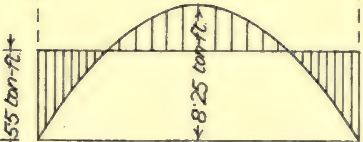


Fig. 149

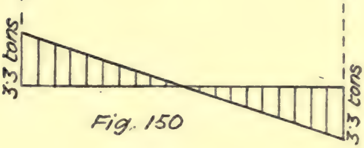
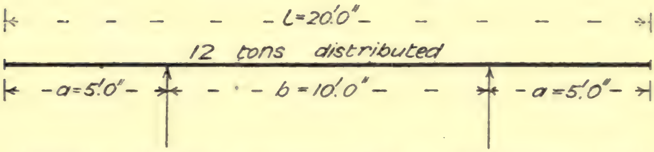


Fig. 150

Fig. 151



Negative bending moments due to overhanging load

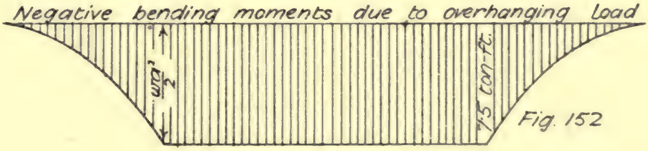


Fig. 152

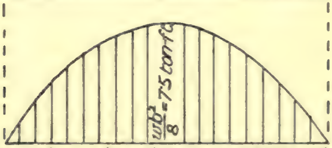


Fig. 153

Positive bending moments due to central span

Combined bending moments

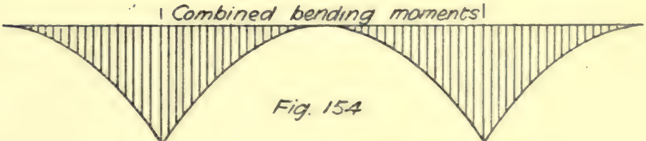
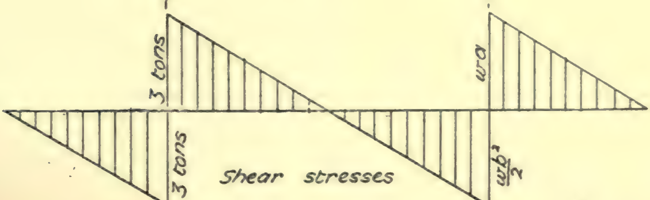


Fig. 154

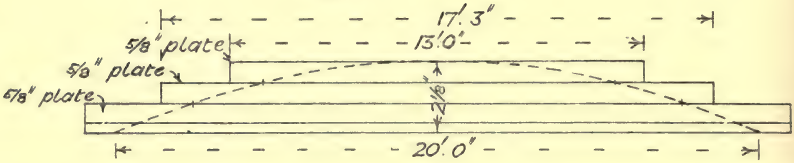


Shear stresses

Q. 37. A rolled joist 20 ft. long is supported at two points 10 ft. apart and carries a uniformly distributed load of 12 tons over the whole length. Draw the bending moment and shear diagrams.

For answer, see Figs. 151 to 155.

Fig. 156



Q. 38. A plate girder 20 ft. span requires a flange thickness of $2\frac{1}{8}$ in. Find the length and thickness of the plates when the angle irons are equivalent to $\frac{1}{4}$-in. plate. Half-cover length may be taken as 12 in.

For answer, see Fig. 156.

LECTURE VIII

Bent Girders for Staircases—Cambered Girders—Cast Iron Cantilever—Gallery Cantilevers—Painters' Hangers.

STONE staircases are frequently supported on bent girders, either actually bent or formed of separate pieces connected together by plates at the bends. The bending moments and shear stresses are precisely the

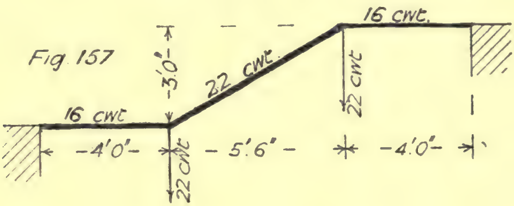


Fig. 157

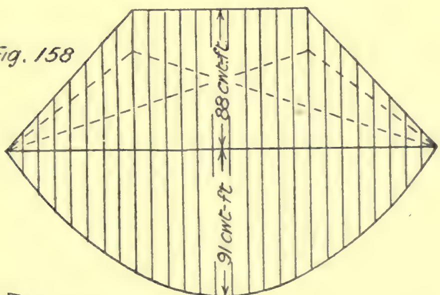


Fig. 158

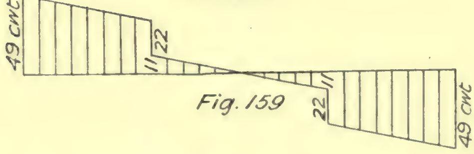


Fig. 159

same as for a straight girder. Fig. 157 shows the frame diagram of one of a pair of bent girders carrying two square landings and a short flight of steps, together with the top end of a lower flight and the bottom end of an upper flight. Fig. 158 shows the bending-moment diagram, and Fig. 159 the shear diagram.

When a girder is bent with a plain camber, a very interesting problem is produced. Fig. 160 shows a girder 20 ft. span, cambered

with a rise of 5 ft., and carrying a concentrated load of 4 tons in the centre, the ends being rigidly fixed by bolting down or otherwise. Join the point of application of the load with the points of support, and these lines represent the direct course of the forces. Under the load draw W to scale equal to the load, and complete the parallelogram; then H will be the horizontal thrust at each end, and T the direct thrust from load to abutment. To ascertain the effect of this thrust, draw a radial line through the centre of the arc at *a* to meet the line of thrust in *b*, draw a tangent through the intersection *a* parallel with the thrust, and from the radial line set off a length *bc* equal to the thrust to complete the parallelogram, the area of which gives the maximum bending moment on each half of the bent girder. This may be converted

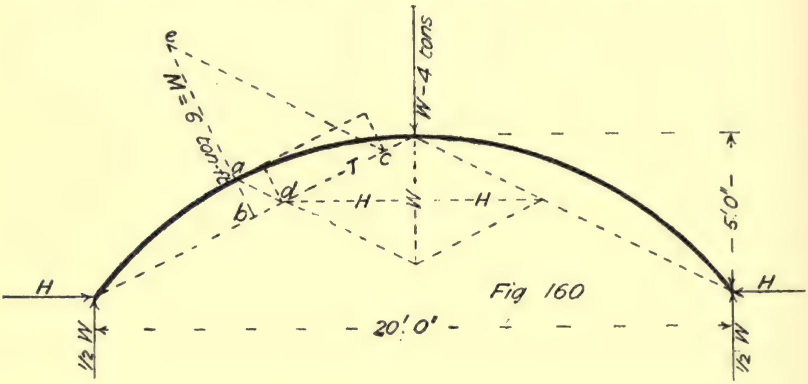


Fig 160

graphically into a line which may be scaled to give the bending moment. To the same scale as W was drawn, take unity from *b* along *bc*, giving point *d*; join *da*, and parallel to *da* draw line *ce* to meet the radial line first drawn. Then *eb* gives the bending moment. By calculation, let W = the load in tons in centre, F = working stress in tons per square inch, say 7 for steel, T = thrust found graphically, A = area of girder in square inches, M = bending moment in ton-inches found graphically, Z = section modulus of section to be tried. Then $\frac{T}{A} + \frac{M}{Z} = F$, or $\frac{4{\cdot}5}{A} + \frac{6 \times 12}{Z} = 7$. There are thus two unknown quantities, but on referring to a table of sections, it appears that a B.S.B.9, 6 in. by 4½ in. by 20 lbs. rolled joist would be suitable. Inserting the values for this section, we have $\frac{4{\cdot}5}{5{\cdot}88} + \frac{72}{11{\cdot}54} = 0{\cdot}76 + 6{\cdot}24 = 7$, as required. If the ends of the girder were not rigidly fixed, the maximum bending moment would be $\frac{WL}{4} = \frac{4 \times 20}{4} = 20$ ton-ft. instead of 6 ton-ft., and would require a B.S.B.20, 12 in. by 5 in. by 32 lbs. joist.

A cast-iron cantilever is an instructive subject to study. The load being distributed, the bending-moment diagram will be a semi-parabola,

as Fig. 86, and with uniform width of section this would be the theoretical elevation to give to the cantilever. Practical considerations, however, lead us to the modification of shape shown in Fig. 161. The section of the cantilever has unequal flanges like a cast-iron girder inverted, and very unlike a rolled joist with equal flanges and symmetrical section. Cast iron being six times as strong in compression as in tension, the lower or compression flange would theoretically be only one-sixth the area of the upper or tension flange; but practical considerations, again, usually limit it to one-fourth, making the section as in Fig. 162, and end view as Fig. 163. The span of a cantilever built into a brick wall should, as a rule, be taken at least $4\frac{1}{2}$ inches more than the external projection, to reach the centre of effective bearing surface, and at the back end the cantilever should have a stone template, carefully fitted down on to it to resist the upward thrust. Taking the distributed load on the cantilever as 2 tons, the span $6 \times 12 + 4{\cdot}5 = 76{\cdot}5$ in., the maximum bending moment will be 76·5 ton-inches. Taking the effective distance from centre of pressure to centre of pressure of the bearing surfaces as 14 in., the upward thrust at the back will be $\frac{76{\cdot}5}{14} = 5{\cdot}5$ tons, and there must be this amount of load provided above, or the cantilever must be anchored down. The load on the template at face of wall will be $2 + 5{\cdot}5 = 7{\cdot}5$ tons, and sufficient bearing area must be given for this load.

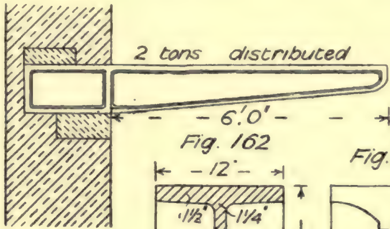


Fig 161

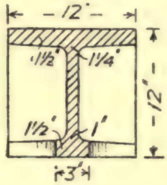


Fig. 162

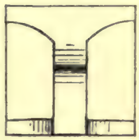


Fig. 163

Gallery cantilevers require very careful consideration, as the case in Fig. 164 will show. The bending moment due to the load on the overhanging portion will be $6 \times \frac{20}{2} = 60$ ton-ft. Now, at times the front portion only will be loaded, as Fig. 165, therefore the back end must be anchored down sufficiently to balance the front load, or $\frac{60}{11} = 5{\cdot}45$ tons, making the load on the column $6 + 5{\cdot}45 = 11{\cdot}45$ tons. The bending-moment diagram will then be as Fig. 166, and the shear diagram as Fig. 167. When the whole of the cantilever is loaded as Fig. 168 the holding down force needed at the wall end will be

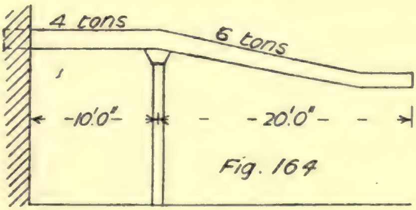


Fig. 164

$$\frac{60 - 4 \times 5}{11} = 3{\cdot}63 \text{ tons,}$$

and the load on the column $6 + 4 + 3{\cdot}63 = 13{\cdot}63$ tons. The bending-moment diagram will be as Fig. 169 and the shear diagram as Fig. 170.

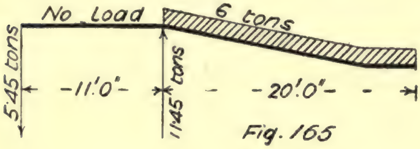


Fig. 165

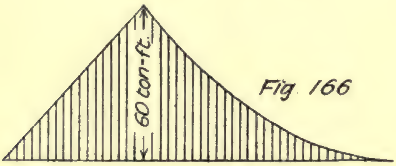


Fig. 166

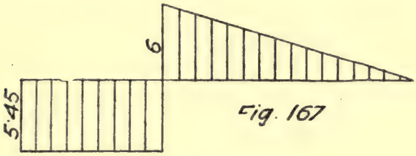


Fig. 167

The wrought iron hangers used by painters for supporting the plank upon which they stand while painting the outside of a railway bridge are cantilevers. A hanger, as Fig. 171, having failed in use and killed a man, an investigation was made. The bending moment at the time of the accident was, according to the evidence, $168 \times 9 = 1512$ lb.-ins., while the moment of resistance of a wrought iron bar $1\frac{1}{2}$ in. diameter would be

$$\frac{\pi}{32}d^3f = {\cdot}0982 \times 1{\cdot}5^3 \times 20 \times 2240 = 14847{\cdot}84 \text{ lb.-ins.},$$

so that if the evidence could be relied upon it failed with a factor of safety of

$\frac{14847{\cdot}84}{1512} = 9{\cdot}82$. The verdict was that the accident was due to a flaw, which the coroner remarked was unobservable. It is more probable that the accident was due to the great projection of the hanger allowing the plank to be too far from the upright; it was possible, even without jumping down on to the plank from the bridge, to produce a bending moment of

$$168 \times 27 = 4536 \text{ lb.-ins.},$$

which would reduce the factor of safety to $\frac{14847{\cdot}84}{4536} = 3{\cdot}27$, still insufficient to cause fracture. The fellow hanger to the one that failed was tested subsequently and was stated to have showed signs of failure with a bending moment of 69090 lb.-ins., but as this would be $\frac{69090}{14847{\cdot}84} = 4{\cdot}65$ times the ultimate resistance it seems waste of time to consider the further statement that it failed

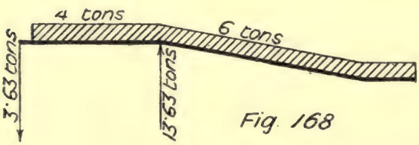


Fig. 168

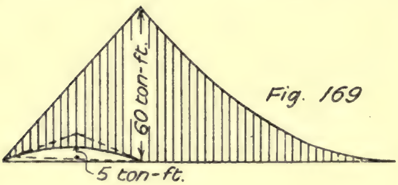


Fig. 169

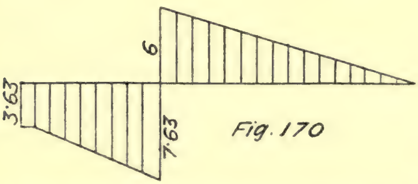
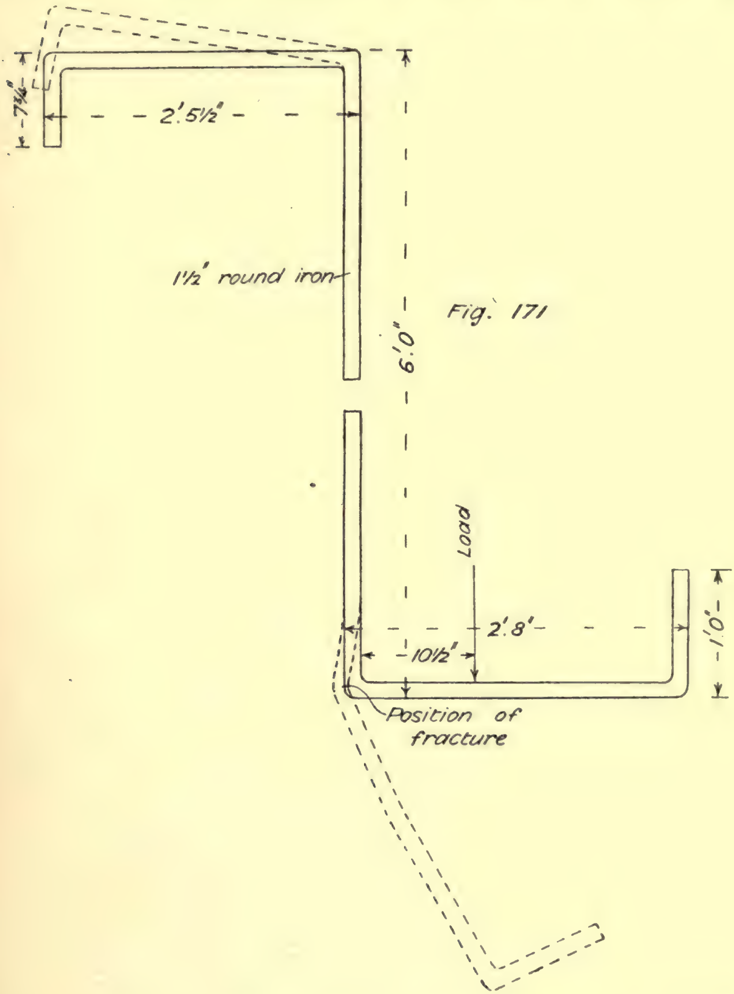


Fig. 170

completely at a bending moment of 79086 lb.-ins. This test was not made under expert supervision and a further test was therefore ordered. This new test was made with a $1\frac{7}{16}$ in. diameter hanger, as Fig. 172, and bending commenced when a load of $14\frac{1}{2}$ cwt. was



applied at $10\frac{1}{2}$ in. from the upright. This gave a bending moment of $14{\cdot}5 \times 112 \times 10{\cdot}5 = 17052$ lb.-ins. The moment of resistance was $\cdot0982 \times 1{\cdot}4375^3 \times 20 \times 2240 = 13068{\cdot}12$ lb.-ins., or the first sign of failure occurred at $\dfrac{17052}{13068} = 1{\cdot}3$ times the calculated ultimate resistance. As this test was carried out under proper supervision it goes to prove

very conclusively that in solid beams the extreme fibre stress is an unknown quantity, the apparent strength being generally in excess of

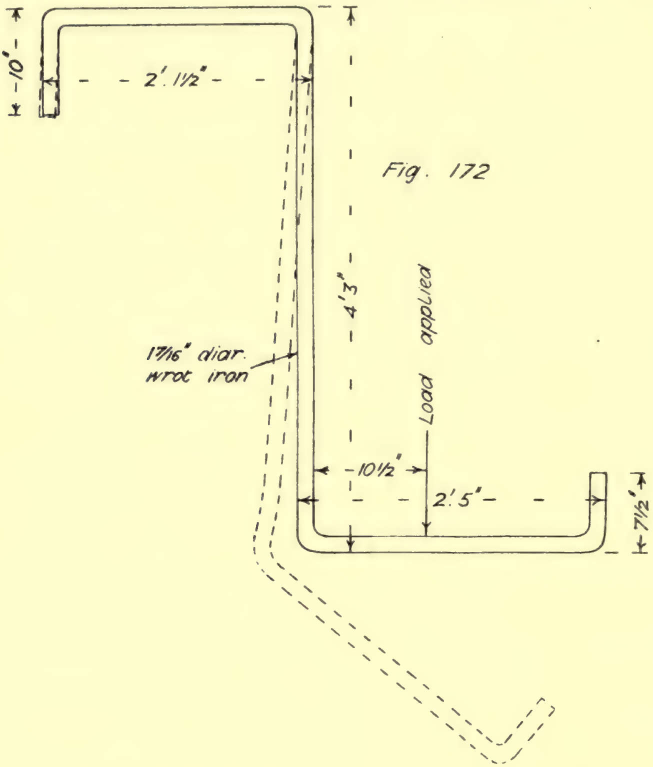


Fig. 172

the actual tensile strength, as mentioned in Lecture II. The modulus of rupture for wrought iron would be the more correct value to employ, but as this is given as varying from 18·4 to 23 tons it will not help us. The conclusions to be drawn from the above calculations are that a pair of hangers $1\frac{7}{16}$ in. diameter, with not more than 12 in. clear projection to take one 11-in. plank, will be safe for two men; and that for two 9-in. planks a pair of hangers $1\frac{1}{2}$ in. square with a projection not exceeding 19 in. clear will be required.

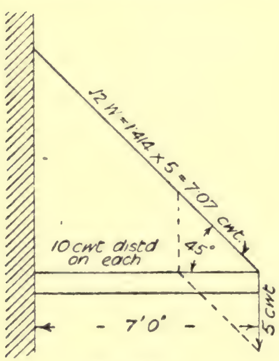


Fig 173

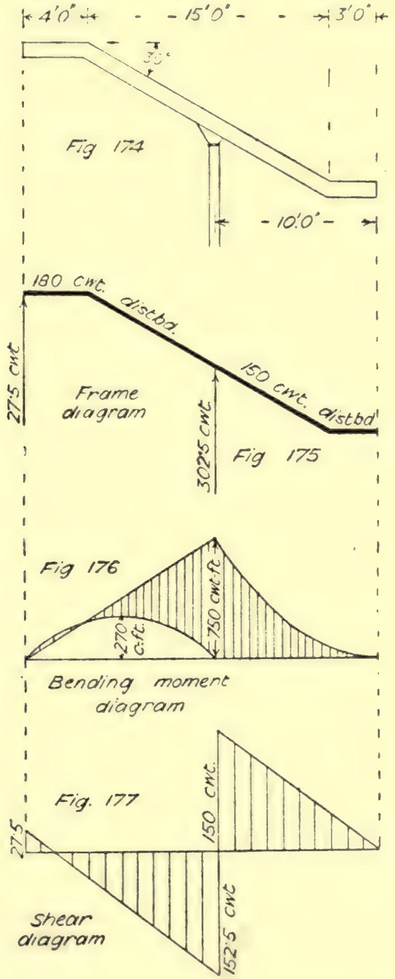
EXERCISES ON LECTURE VIII

Q. 39. A canopy for hotel entrance projects 7 ft. and weighs 1 ton. It is supported over the doorway and by two tie-rods at the outer end

making an angle of 45 degrees with the wall. Find graphically the stress in each tie. Scale $\frac{1}{4}$ in. to 1 ft. and $\frac{1}{8}$ in. to 1 cwt.

For answer, see Fig. 173.

Q. 40. A gallery cantilever, as Fig. 174, has a distributed load of 15 cwt. per



foot run of horizontal distance, and is supported at 10 ft. from outer end. Draw the frame diagram and the bending-moment and shear diagrams. Scales $\frac{1}{8}$ in. to 1 ft., $\frac{1}{8}$ in. to 20 cwt., $\frac{1}{8}$ in. to 100 cwt.-ft.

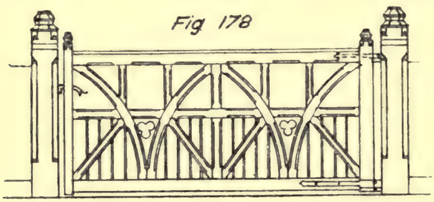
For answer, see Figs. 175, 176, and 177.

LECTURE IX

Gates and Doors—Overhanging Steps—Framed Cantilevers and Brackets.

A GATE, Fig. 178, is a cantilever with a distributed load due to its own weight, and when a boy is swinging on it there will be the addition of a concentrated load at the outer end where the longest swing can be obtained. The hinges will be the points of support and the frame diagram will be as Fig. 179. The imaginary bars 6–7 and 7–8 are what are called "substituted members," they are used instead of the real lines of the gate framing and represent the direct course of the loads to the supports. The weight of the gate may be taken level with the top hinge instead of at the centre of gravity, to show the effect of a clearance on the pin under the top hinge, the hinge being therefore incapable of taking any portion of the vertical load. The stress diagram will be as Fig. 180.

Fig 178



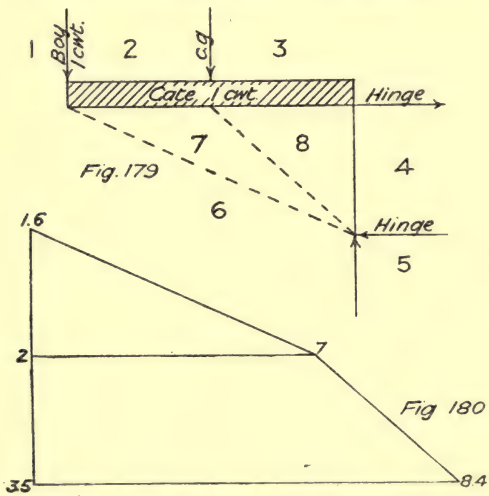


Fig. 179

Fig 180

Another kind of cantilever involving special difficulties is given by the step of an overhanging stone stair to a public building shown in Fig. 181. As this is a practical case of considerable importance it is worth while to go into the question rather fully. Suppose the staircase to be 5 ft. wide, and it is desired to find the margin of safety allowed when each step is carrying three persons of 12 stone each, and the tensile strength of the material is 1000 lbs. per sq. in.[1] First find centre of

[1] It must be remembered that the so-called extreme fibre stress made equal to the tensile strength does not represent the true state of the case, the extreme fibre

gravity of section, through which the neutral axis is assumed to pass. Divide up the section as in Fig. 182, then

$$\frac{4 \times 10 \times 2 + 2 \times 4 \times 6 + 8 \times 4 \times \frac{1}{2} \times 5\frac{1}{3}}{4 \times 10 + 2 \times 4 + 8 \times 4 \times \frac{1}{2}} = \frac{80 + 48 + 85\frac{1}{3}}{40 + 8 + 16}$$
$$= \frac{213\frac{1}{3}}{64} = 3\frac{1}{3} \text{ ins.}$$

from upper surface.

Rankine's rules to find moment of inertia of irregular figure—(1) Divide the figure into a number of simple figures. (2) Find the

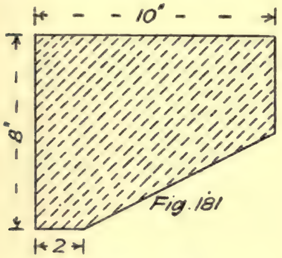


Fig. 181

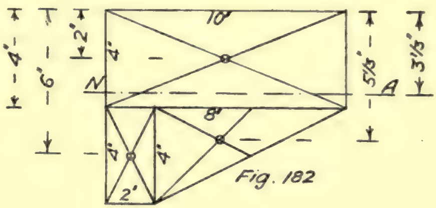


Fig. 182

moment of inertia of each of the simple figures about an axis traversing its centre of gravity parallel to the neutral axis of that complex figure. (3) Multiply the area of each simple figure by the square of the distance from the centre of gravity of the whole figure. (4) Add the results so found for the moment of inertia (I) of the whole figure.

$$\text{I of large rect.} = \frac{bh^3}{12} = \frac{10 \times 4^3}{12} = \tfrac{160}{3} = 53\cdot3\dot{3}$$
$$\text{,, small ,,} = \frac{bh^3}{12} = \frac{2 \times 4^3}{12} = \tfrac{32}{3} = 10\cdot6\dot{6}$$
$$\text{,, triangle} = \frac{bh^3}{36} = \frac{8 \times 4^3}{36} = \tfrac{128}{9} = 14\cdot2\dot{2}$$
$$\overline{78\cdot2\dot{2}}$$

$$\text{Area large rect.} \times \text{dist.}^2 \text{ c.g. from N.A.} = 10 \times 4 \times (3\tfrac{1}{3} - 2)^2 = 71\cdot\dot{1}$$
$$\text{,, small ,,} \times \text{ ,, ,, ,, ,,} = 2 \times 4 \times (6 - 3\tfrac{1}{3})^2 = 56\cdot\dot{8}$$
$$\text{,, triangle} \times \text{ ,, ,, ,, ,,} = 8 \times 4 \times \tfrac{1}{2} \times (5\tfrac{1}{3} - 3\tfrac{1}{3})^2 = 64\cdot0$$
$$\overline{192\cdot0}$$
$$\text{Add previous result} \quad \dots \quad \dots \quad 78\cdot\dot{2}$$
$$\overline{270\cdot\dot{2}}$$

Moment of inertia of whole figure, say $270\cdot2$, and modulus of

$$\text{section} = \frac{I}{y} = \frac{\text{Moment of inertia}}{\text{Distance of extreme lamina from N.A.}} = \frac{270\cdot2}{3\frac{1}{3}} = 81.$$

Moment of resistance = modulus of section × ultimate tensile stress
$= 81 \times 1000 = 81000$ lb.-ins.

stress is in all cases actually less than the formulæ appear to give, as before mentioned.

Allowing for weight of stone at 150 lbs. per cubic ft. the weight of each step will be $\frac{5(10 \times 4 + 284 + 8 \times 4 \times \frac{1}{2}) \times 150}{144} = 333\frac{1}{3}$ lbs.

Then moment of load

$$= \frac{(W + w)l}{2} = \frac{(333\frac{1}{3} + 3 \times 12 \times 14)(5 \times 12)}{2} = 25120 \text{ lb.-ins.},$$

and factor of safety $= \frac{81000}{25120} = 3{\cdot}225$.

The nature of the reactions produced by a cantilever can be best shown by a braced structure. Fig. 183 shows the frame diagram of a

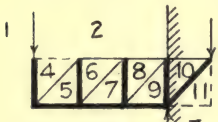


Fig 183

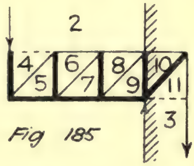


Fig 185

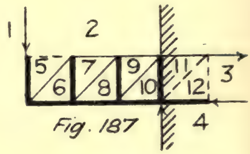


Fig. 187

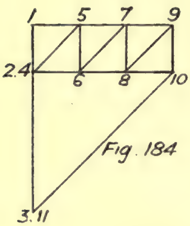


Fig. 184

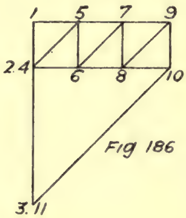
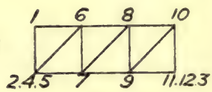


Fig 186

Fig 188


braced cantilever carrying a load at the free end and held down by a top load on the short end. Fig. 184 shows the stress diagram. Fig. 185 shows the same cantilever with the short end anchored down, for

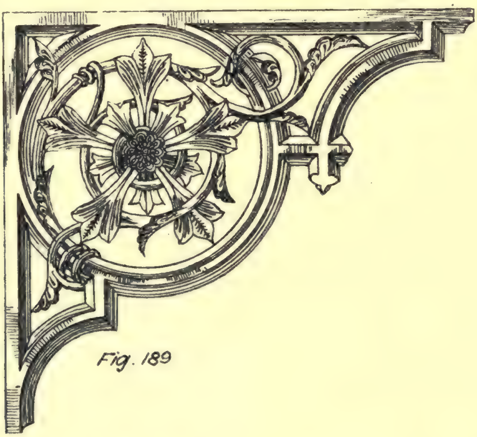


Fig. 189

which Fig. 186 is the stress diagram. It is, however, not essential to hold down the wall end at all. If the wall is sufficiently substantial to

resist the couple caused by a direct pull from the top flange, and push from the bottom flange, the same cantilever, Fig. 187, will give the stress diagram shown in Fig. 188. In practice this result may partially

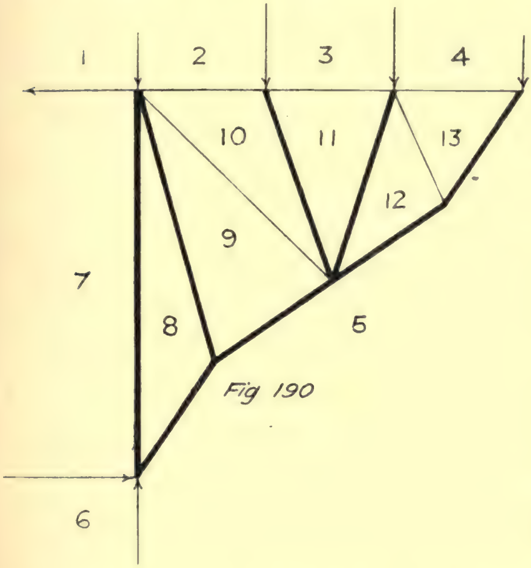
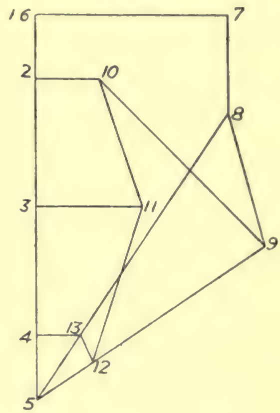


Fig 190

Fig 191



occur when the end is held down. An ogee bracket of any pattern, as Fig. 189, carrying a distributed load may have substituted members inserted to give the frame diagram, Fig. 190, and will then have the peculiar stress diagram shown in Fig. 191, and other examples of cantilevers may be found bringing in new points for consideration.

EXERCISES ON LECTURE IX

Q. 41. A door 3 ft. by 7 ft. weighs 100 lbs., the hinges equidistant from top and bottom are 5 ft. 6 in. apart. Find graphically the stresses on the hinges, assuming the centre of gravity of the door to be 3 ft. from the bottom. Scales ¼ in. to 1 ft. and 1 in. to 100 lbs.

For answer, see Figs. 191, 193, and 194.

Q. 42. A rolled joist inclined at an angle of 40 degrees from the horizontal and 20 ft. long, carries one side of a staircase, producing a distributed load of 2½ cwt. per foot run along the joist. Draw the frame diagram, and diagrams of thrusts and bending moment.

For answer, see Figs. 195, 196, and 197.

(NOTE.—This is a case in which it is important to find the maximum stress due to the combined thrust and bending moment. Assume a B.S.B.11, 7 in. × 4 in. × 16 lbs. joist for which $A = 4{\cdot}7$ and $Z = 11{\cdot}2$. Then at the lower end the stress will be $\frac{W}{A} = \frac{1{\cdot}56}{4{\cdot}7} = 0{\cdot}33$ tons per sq. in., and $\frac{M}{Z} = 0$. At 5 ft. from bottom $\frac{W}{A} + \frac{M}{Z} = \frac{1{\cdot}17}{4{\cdot}7} + \frac{45}{11{\cdot}2} = 0{\cdot}25 + 4 = 4{\cdot}25$ tons per sq. in. At the centre the stress will be $\frac{0{\cdot}78}{4{\cdot}7} + \frac{58}{11{\cdot}2} = 0{\cdot}16 + 5{\cdot}18 = 5{\cdot}34$ tons per sq. in. and 15 ft. up

$\frac{0\cdot39}{4\cdot7} + \frac{45}{11\cdot2} = 0\cdot08 + 4 = 4\cdot08$ tons per sq. in., whilst the stress at top will be nil. Plotting these points gives a curve, as in Fig. 196, and the point of maximum stress given by the highest point occurs at 9·4 ft. from lower end, where $W = 0\cdot83$

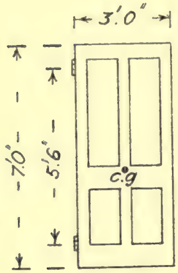


Fig. 192

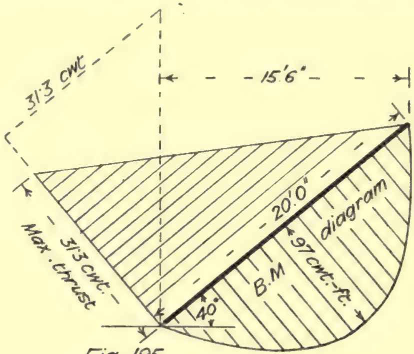


Fig. 195

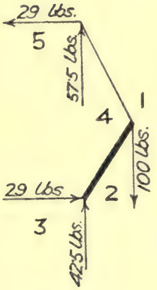


Fig. 193

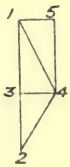


Fig. 194

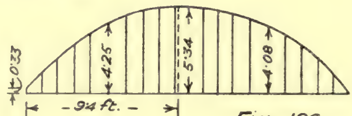


Fig. 196

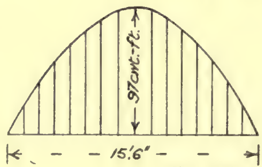


Fig. 197

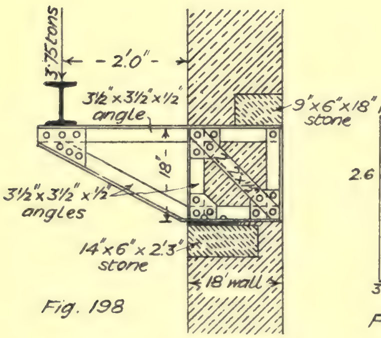


Fig. 198

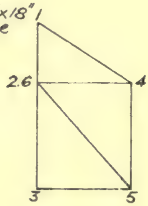


Fig. 200

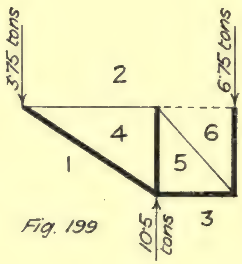


Fig. 199

tons and $M = 57\cdot8$ ton-ins., giving a maximum stress of $\frac{0\cdot83}{4\cdot7} + \frac{57\cdot8}{11\cdot2} = 5\cdot341$ tons per sq. in., against 7 tons per sq. in. maximum safe load. By the ordinary method, taking the same distributed load and horizontal span, the maximum bending moment will be $\frac{WL}{8} = \frac{50 \times 15\cdot5}{8} = 97$ cwt.-ft. $= \frac{97 \times 12}{20} = 58$ ton-ins., as in

Fig. 198. Then taking C = 7 tons, $Z = \frac{58}{7}$ = say 8·3, which will be covered by the joist found above. It will be seen that as the maximum stress in Fig. 197 occurs at less than half way, the bending moment will be less than the maximum found in the usual way, but the difference is made up by the direct thrust which has to be added in the first method.)

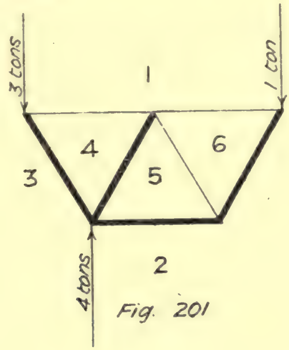


Fig. 201

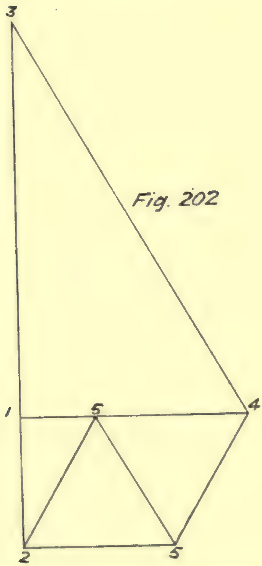


Fig. 202

Q. 43. Draw a frame and stress diagram for the bracket shown in Fig. 198. For answer, see Figs. 199 and 200.

Q. 44. A braced cantilever consists of three equilateral triangles making two bays on top and one on bottom. It carries an end load of 1 ton. Draw the stress diagram to a scale of 1 in. to 1 ton, and state the magnitude of the reactions.

For answer, see Figs. 201 and 202.

LECTURE X

Warren Girders variously loaded—Lattice Girders—Continuous Lattice Girder—Bent Lattice Girder.

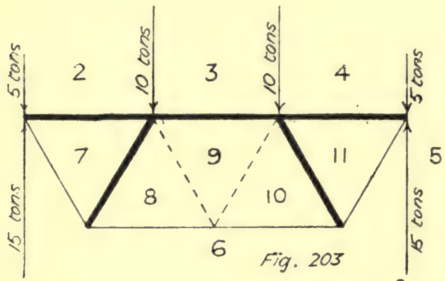


Fig. 203

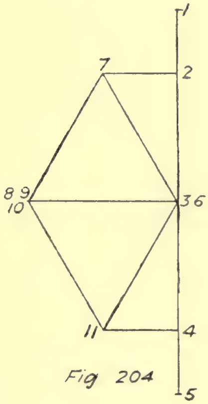


Fig 204

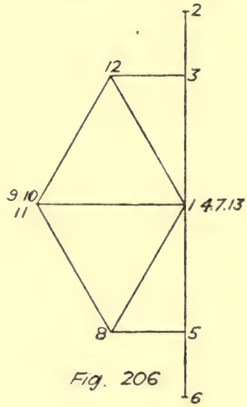


Fig. 206

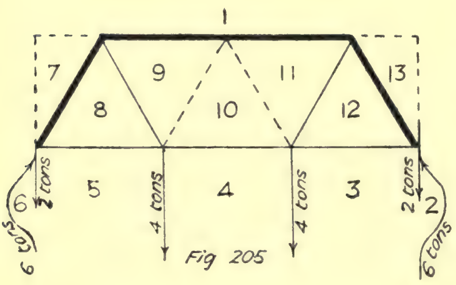


Fig 205

A Warren girder composed of a simple series of equilateral triangles is the simplest kind of lattice girder. It is a form that was much used at one time for Indian railways, but it is not well suited for a rolling load, as some of the lattice bars towards the middle of the span are put in tension and compression alternately. It is a suitable and cheap form for carrying the ends of roof trusses, especially for workshops and top floors of warehouses owing to the small obstruction it offers to the light. As a railway girder the load was applied to the top flange as in Fig. 203, where three bays only are shown to illustrate the principle, the usual number being eight or ten. The stress diagram is given in Fig. 204. Two girders of this kind, with cross ties and steel bracing, formed a bridge of the kind called a "deck span," a type which is not allowed on English railways. When the load is carried on the bottom flanges by cross girders, and the trains run between the main girders,

it is called a "through bridge." One such girder reduced to three bays,

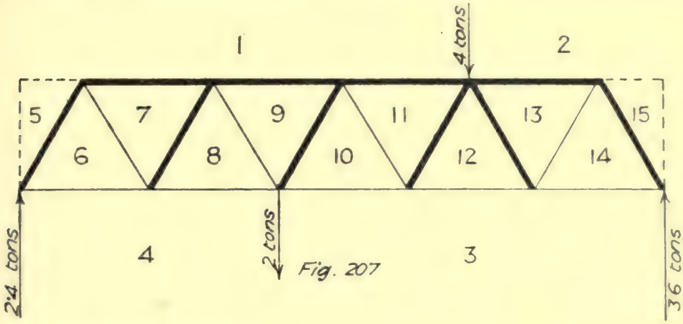


Fig. 207

merely to show the principle, is given in Fig. 205. It will be seen that the load on the end bay requires a force line separate from the reaction, and it is bent out to make room for the number. The stress diagram is shown in Fig. 206. Rolling loads are not included in this course, so that no further reference need be made to them. Fig. 207 shows an irregularly loaded girder, and in a case of this kind a separate stress diagram may be made for each load, and the algebraical sum of the stresses taken on each member for the final result, or the combined stress diagram may be produced in one operation, as in Fig. 208.

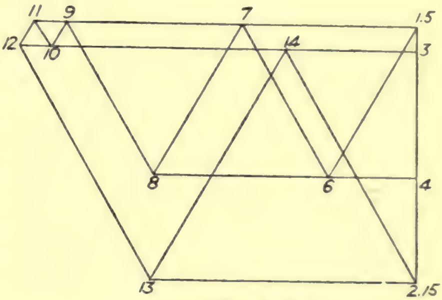
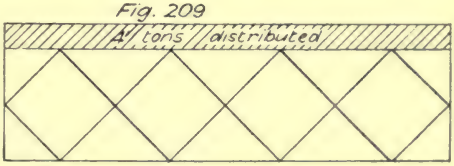


Fig. 208

Fig. 209



The simplest variety of lattice girder, after the Warren pattern, is that with two sets of bars crossing each other at 45 degrees, and some very instructive examples of stresses may be made by adopting certain proportions. Fig. 209 carries a

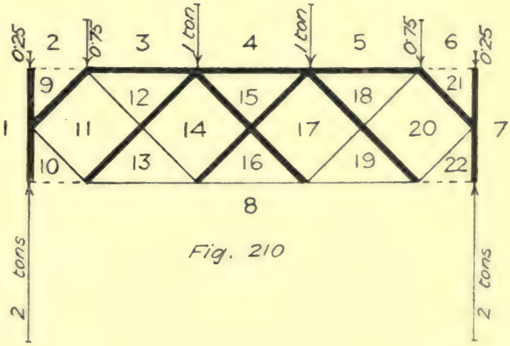


Fig. 210

uniformly distributed load on the top flange; Fig. 210 is the frame diagram for this, and Fig. 211 is the corresponding stress diagram. The frame diagram is utilised to show the nature of the stresses, thin lines for tension, thick for compression, and dotted for no stress. It will be seen that the reciprocal diagrams take account of direct stresses only, and that in such cases as the present, the very important transverse stresses produced by the bending moments due to the load between the points of support, must be dealt with separately. Strictly the top flange is in the condition of a continuous girder, and the true loading will not be quite as shown in Fig. 210. As, however, lattice girders generally have at least eight bays, no serious error will be introduced in practical work by considering the load to be divided as shown. The bending moments on the several bays of the top flange will be as shown in Fig. 212. Then the combination of the direct stress and transverse stress will be worked out by formula

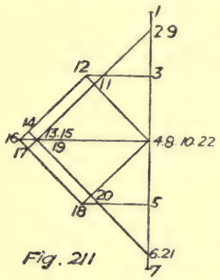


Fig. 211

$$\frac{W}{A} \pm \frac{M}{Z},$$

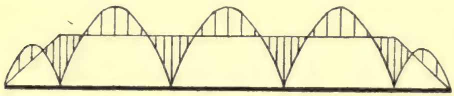


Fig. 212

where W is the direct stress, A the sectional area of the top flange, M the maximum bending moment on the bay, and Z the section modulus.

A lattice girder, as Fig. 213, with a simple concentrated load on the bottom flange, as shown, produces very remarkable results. The

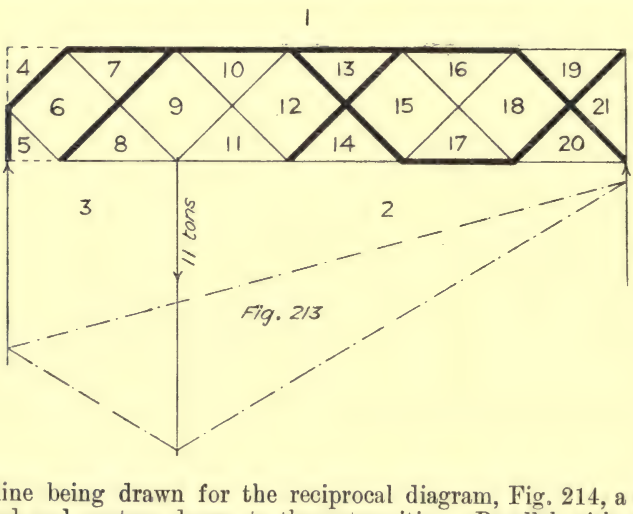


Fig. 213

load line being drawn for the reciprocal diagram, Fig. 214, a pole is selected and vectors drawn to the extremities. Parallel with these a funicular polygon is drawn on the frame diagram, Fig. 213, and the closing line gives the direction of the vector for dividing the load line

into the two reactions. Then the reciprocal diagram being completed, the nature of the stresses should be indicated on the frame diagram by thick, thin, and dotted lines as usual.

In practical work a continuous girder depends for its agreement with theory upon maintaining the level of its supports. It is evident that if any support should sink it will throw more work upon the others and alter the distribution of the stresses. This may be illustrated by taking a lattice girder continuous over two spans and drawing a series of stress diagrams from (*a*) all load on end supports, to (*b*) all load on central support, when it will be found that the minimum stresses occur when $\frac{3}{16}$ of the load is carried by each of the end supports, and $\frac{5}{8}$ of the

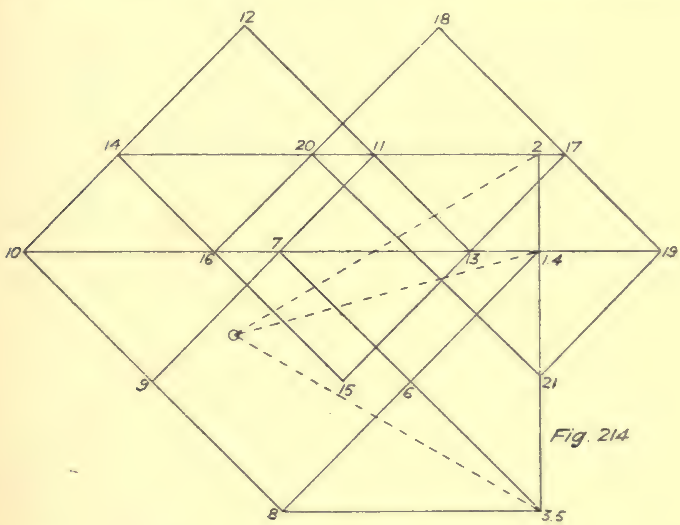


Fig. 214

load by the centre support, thus proving practically the result arrived at by the theorem of three moments.

Bent lattice girders are much used for overhead footbridges at railway stations. Fig. 215 shows the frame diagram of one from actual practice, and Fig. 216 the corresponding reciprocal stress diagram. The verticals 25–26 and 44–45 give a slight difficulty. The work is straight forward until point 25 is reached, then a new start must be made from the centre point 35. As this is the centre point of a symmetrical girder with symmetrical loading, it must come somewhere upon the horizontal line from point 1. Assume any given position for it and proceed with the diagram until point 26 is reached, as indicated by the dotted lines. This point will give the level at which to cut off 25–26, and point 26 being now in its correct position, the dotted figure may be reconstructed in its proper place. It is a mere accident that point 26 occurs on line 10–31, and no attention must be paid to that.

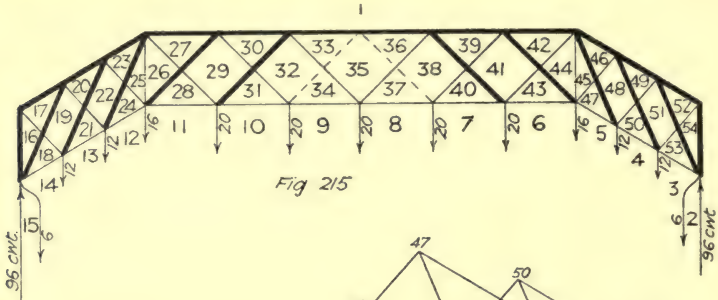


Fig 215

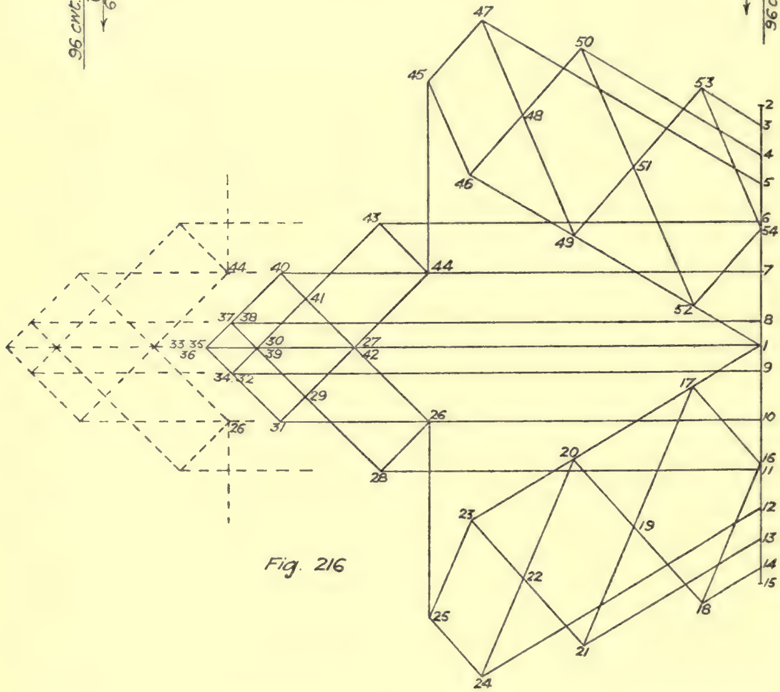


Fig. 216

EXERCISES ON LECTURE X

Q. 45. A lattice girder of double triangulation is composed of five bays with bars at 45 degrees, and loaded as in Fig. 217. Draw the frame and stress diagrams.

For answer, see Figs. 218 to 222.

(NOTE.—This extra horizontal force 11–1 required to balance the structure appears to be due to 13–14 being in tension from 3–4, while 12–14 has compression from 2–3 and 0·5 of 4–5, leaving 0·5 unbalanced. It is clear that this external force does not exist, and the thrust therefore puts a cross strain on the end vertical which produces tension in the short portions of the top and bottom members making the beginning of stress diagram, as in Fig. 221, and the end of frame diagram, as in Fig. 220. The bending moment on the vertical will be as in Fig. 222. The tension added in top and bottom flanges acts throughout the whole length of girder to modify the stresses first found.)

Q. 46. Show by bending-moment diagrams the comparison between (*a*) two girders carrying a distributed load, supported at the ends, and meeting on a central column, and (*b*) a continuous girder similarly loaded over the same two spans.

For answer, see Figs. 223 and 224.

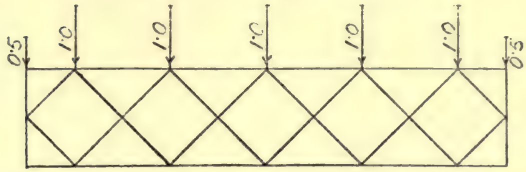


Fig 217

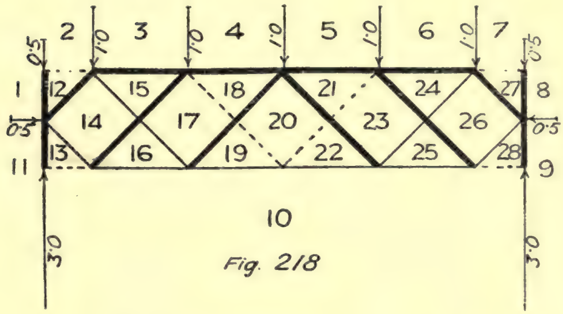


Fig. 218

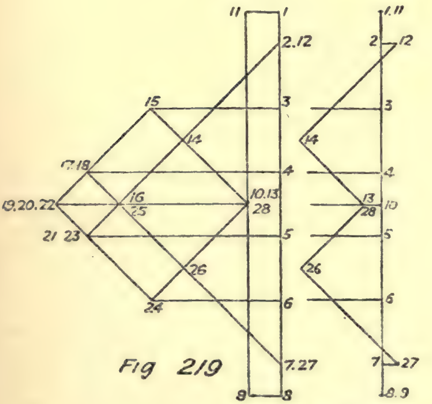


Fig 219

Fig. 221

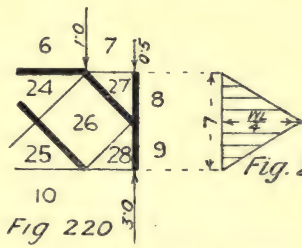


Fig 220

Fig. 222

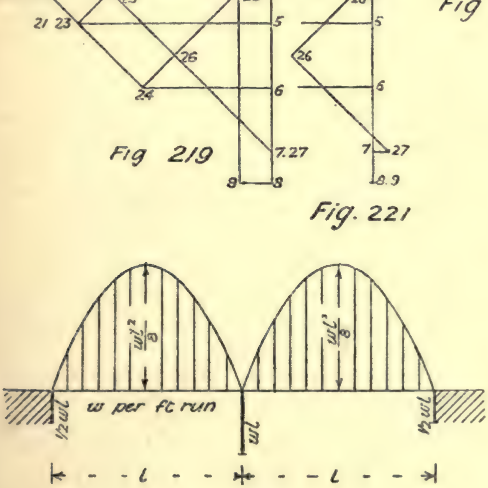


Fig. 223

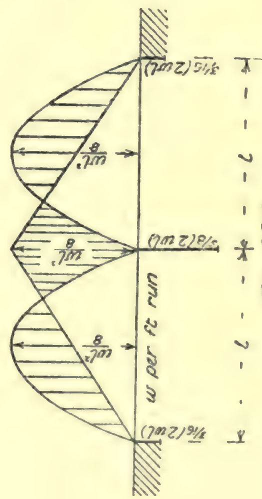


Fig. 224

LECTURE XI

N Girders—Lattice Girder with Uprights—Trellis Girder—Compound Trussed Girders—Trussed Beams—Combined Longitudinal and Transverse Stresses

LATTICE girders formed of inclined ties and vertical struts as Fig. 225 are the original form of the Pratt truss, although Prof. Jamieson calls them Linville or N trusses. With inclined struts and vertical ties as Fig. 226 they form the Howe truss, while Fig. 227 shows the

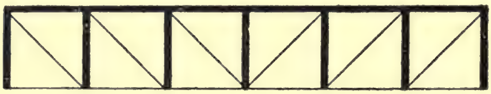


Fig. 225

Fig 226

Fig. 227

modified Pratt truss, or single quadrangular truss, very largely used all over the world for railway bridges. There is no difficulty in ascertaining the stresses under a uniformly distributed load in either of these girders, but with a rolling load and single diagonals those near the centre would be in compression and tension alternately, and to avoid this the central bays are, in practice, braced both ways, so that there will always be a diagonal to take the tension. The maximum stresses under a rolling load may be found by assuming consecutive bays covered by the load and forming a new stress diagram at each step in advance. The stresses will then be collected in a table, and the maximum provided for in the design of each member.

Lattice girders with verticals as Fig. 228 were formerly called Howe trusses, but by some writers are known as Whipple-Murphy trusses or Pratt trusses of single intersection. They are strictly indeterminate structures, as the stresses in the various members depend to some extent upon the workmanship. They may be considered as two N girders superposed, as Figs. 229 and 230, each carrying half the

load, Figs. 231 and 232 being the corresponding stress diagrams, and this consideration provides a very simple solution by the summation of the stresses, as Fig. 233. It will now be found that by assuming half

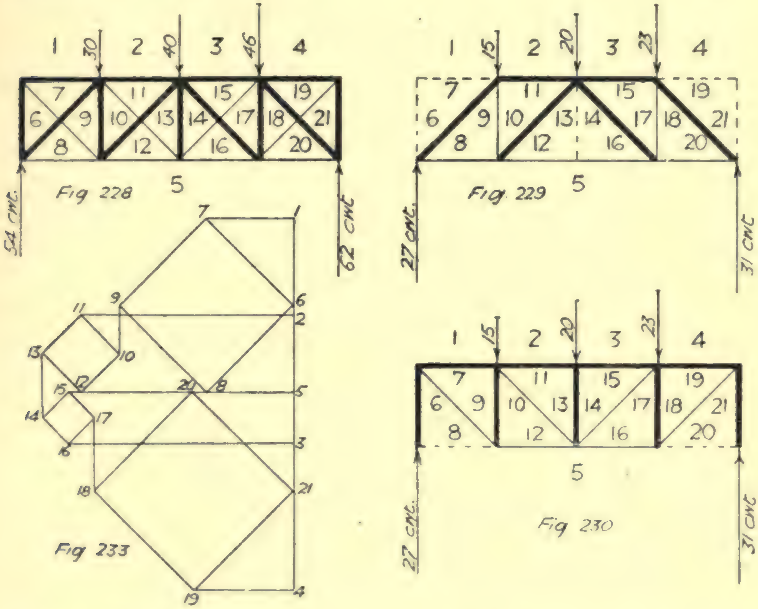


Fig 228 Fig 229 Fig 230 Fig 233

the load to pass down each vertical the final diagram could have been drawn direct, but the proof that there is more in the subject than meets the eye may be gathered from the fact that in Crehore's "Mechanics of the Girder" nearly 100 pages are taken up in elucidating the stresses in a truss of this type. Let N = number of bars in any framed structure. V = number of joints in structure, then a

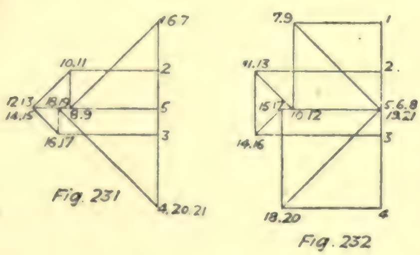


Fig. 231 Fig. 232

simple structure occurs when $N = 2V - 3$, in any other case there are redundant members and the stresses require certain assumptions.

A trellis girder is one with extra diagonals, as Fig. 234, and this of course involves a little more trouble owing to the number of bars, but there is no inherent difficulty. The stress in the end pillar may be

found by inspection, observing the amount of load passing down each part to the support. For instance, the load passing down 7—8 is equal to 3—4, being a part of 3—4 and part of the similar load at the other end of girder; and the load passing down 7—10 is that coming from 7—8 + 2—3 + 4—5. No difficulty will then be found in drawing the stress diagram Fig. 235.

Compound trussed girders, such as the Linville, Whipple-Murphy, or double quadrangular truss, are the most complicated girders to work out, particularly when they carry a rolling load. The confusion of names and types leads Fidler in his "Treatise on Bridge Con-

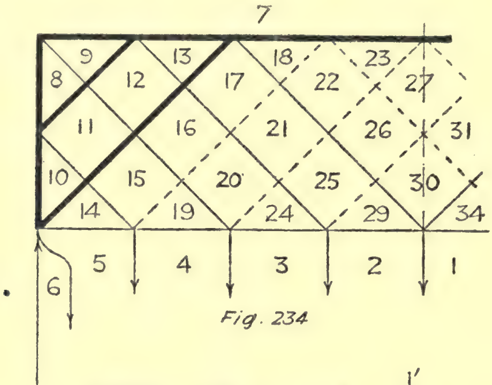


Fig. 234

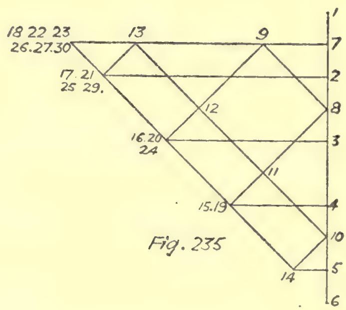


Fig. 235

struction," p. 79, to say, "Braced girders are called Linville, Pratt, Murphy and Whipple-Murphy according to variations not easily defined." An elementary case of the latter type may be taken as in Fig. 236, which is loaded on the bottom flange. This may be divided up into three component girders which are shown with their stress diagrams in Figs. 237 to 242. The stresses being marked on the frame diagram, a complete stress diagram, Fig. 243, may be formed, to prove the accuracy of the work by properly closing, but this stress diagram could not have been formed direct in the first instance.

A single trussed beam as in Fig. 244, with its stress diagram, Fig. 245, presents no difficulty, but when the beam is divided into three bays as Fig. 246 it is necessary that the load should be uniformly distributed, giving the stress diagram, Fig. 247. When used as a

support for a travelling crane on a gantry, or any similar rolling load, it is necessary to brace the rectangular bay, as in Figs. 247 and 249. If this were not done, when the load is over one strut the other would tend to rise, but when braced with tie-rods the load from one strut is

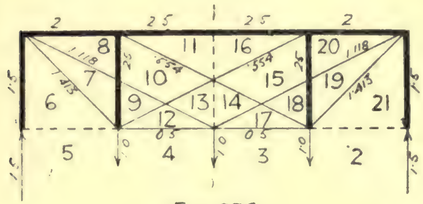


Fig. 236

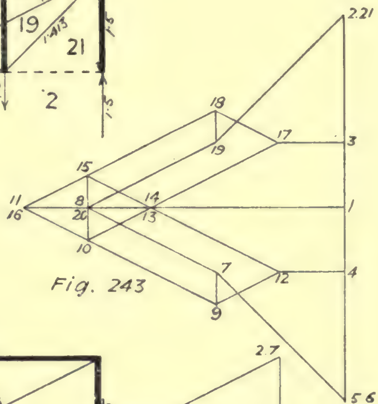
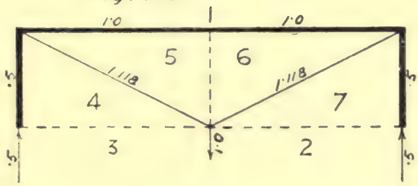


Fig. 243

Fig. 237



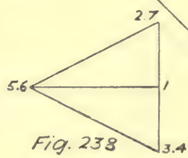


Fig. 238

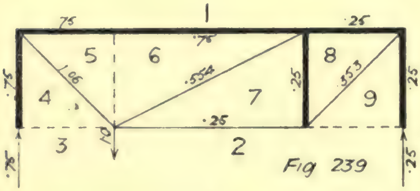


Fig 239

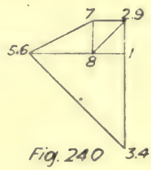


Fig. 240

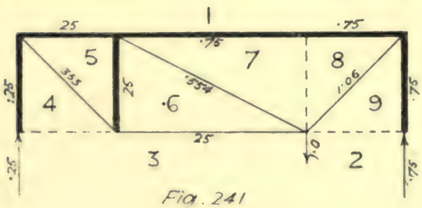


Fig. 241

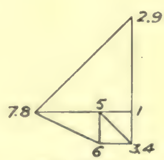


Fig. 242

transmitted to the other. To properly understand this bracing, the load must be shown over the other strut, as in Figs. 248 and 250, and the two stress diagrams, Figs. 249 and 251, compared. It would be equally correct in theory to brace with two struts, but ties are more economical than struts. In the case of trussed beams with a rolling load, when the load is midway between two points of support the beam

itself is under a longitudinal compression and a transverse stress at the same time. These may be readily combined by means of the formula

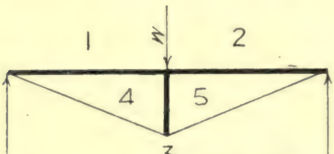


Fig 244

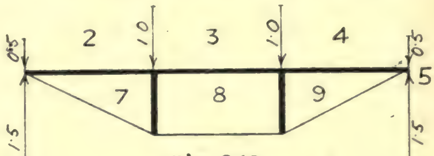


Fig. 246

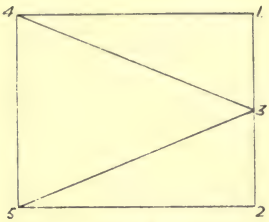


Fig 245

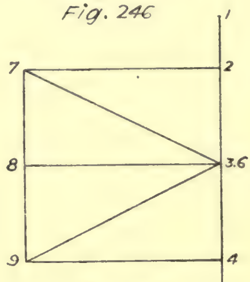


Fig. 247

$\frac{W}{A} \pm \frac{M}{Z}$, where W is the direct thrust or compression, A the sectional area of beam, M the bending moment due to the transverse load, and

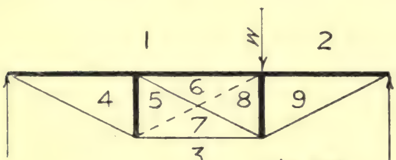


Fig. 248

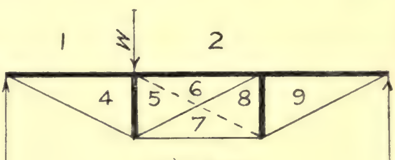


Fig. 250

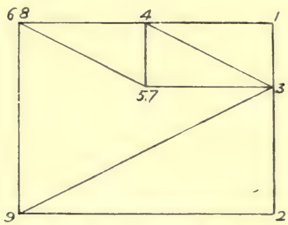


Fig 249

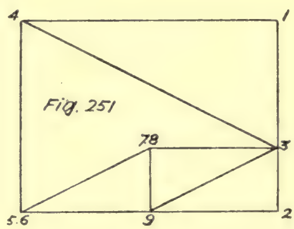


Fig. 251

Z the section modulus of the beam. The + value gives the maximum compression and the − value the maximum tension.

Span 80'0 . Depth 13'0"

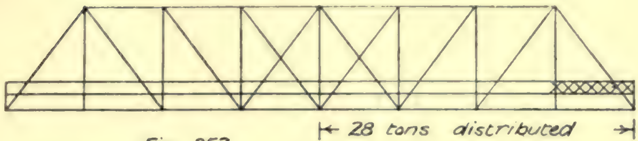


Fig. 252

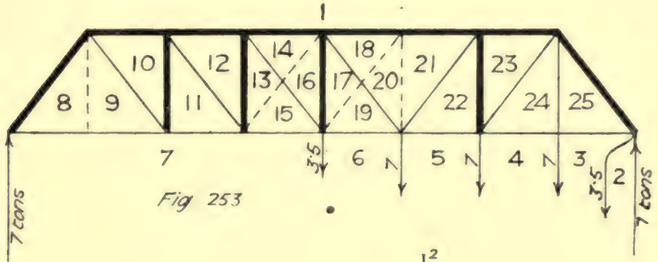


Fig 253

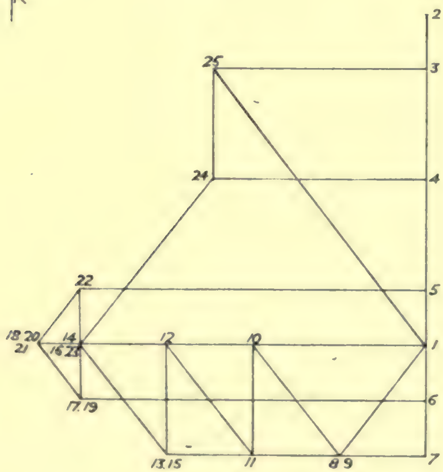


Fig 254

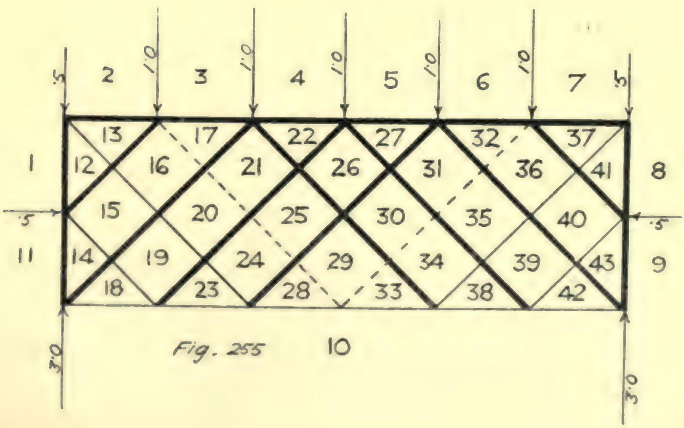


Fig. 255

EXERCISES ON LECTURE XI

Q. 47. Draw the stress diagram for the girder shown in Fig. 252.

For answer, see Figs. 253 and 254.

Q. 48. Draw a stress diagram for the trellis girder shown in Fig. 255.

For answer, see Fig. 256.

Q. 49. Draw the stress diagram for the trussed beam shown in Fig. 257, without cross bracing.

For answer, see Figs. 258 to 262.

(NOTE.—The conditions of the question are shown in Fig. 257 with the load

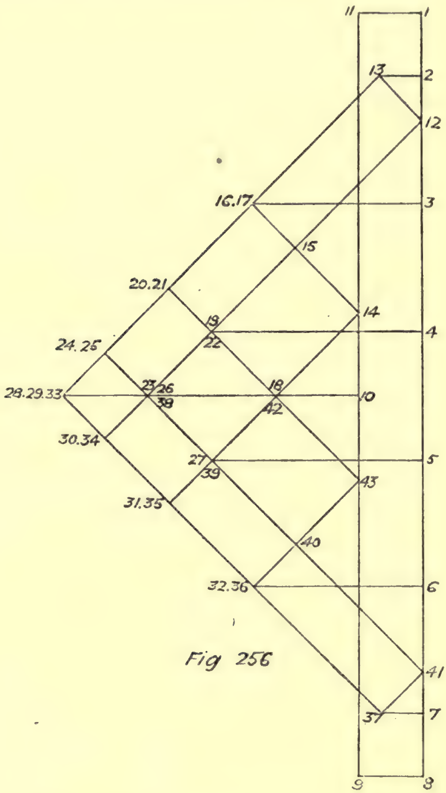


Fig 256

over one strut. As there will be a bending moment over the other strut due to the upward thrust the frame diagram must be as in Fig. 258, where the strut 4—6 and ties 1—4 and 1—6 are added as substituted forces to take the place of the bending moment on the beam and permit a stress diagram to be drawn. This stress diagram will be as in Fig. 259, and gives all the direct stresses in the original members. To ascertain precisely the value of the bending moment, take away the added members, then resolving the tension in member 3—5 gives as in Fig. 260 a horizontal component of 2 tons and a vertical component of $\frac{2}{3}$ ton. Now the 2 tons thrust against the beam will be balanced by the 2 tons compression in the beam, and as the reaction is $\frac{1}{3}$ ton acting upwards, the $\frac{2}{3}$ ton force will have a balance of $\frac{1}{3}$ ton acting downwards as in Fig. 261. At the first

strut there will be the upward thrust of $\frac{2}{3}$ ton unbalanced, but at the next strut there will be the 1 ton load acting downwards, and the $\frac{2}{3}$ ton from strut acting upwards, leaving a balance of $\frac{1}{3}$ ton acting downwards. At the right hand abut-

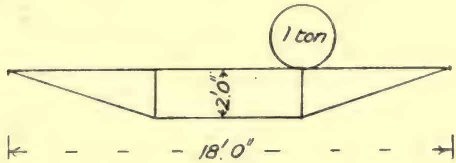


Fig. 257

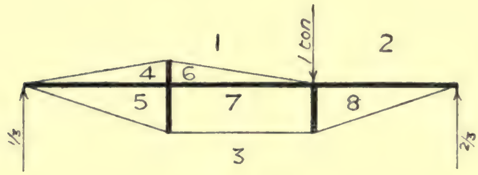


Fig 258

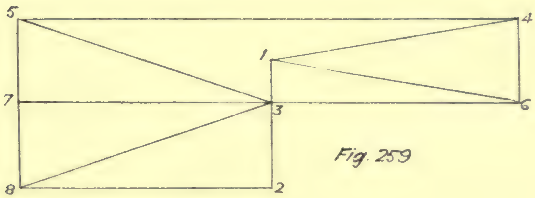


Fig. 259

Fig. 260

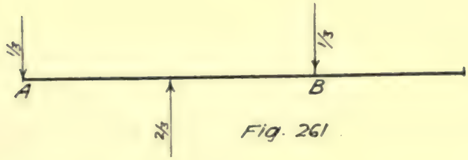


Fig. 261

ment the resolution of the tension in 3—8 will give the same results as at the left-hand abutment, but in this case the reaction is $\frac{2}{3}$ ton, so that the forces at this point will be balanced, leaving the active forces on the beam as in Fig. 261.

It will now be seen that AB is in the condition of a beam supported at both ends with a central load of $\frac{2}{3}$ ton which produces a bending moment on AB of $\frac{WL}{4} = \frac{\frac{2}{3} \times 12}{4} = 2$ ton-ft. The frame diagram with stresses and bending moments is shown in Fig. 262.)

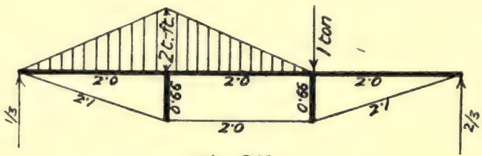


Fig 262

LECTURE XII

Struts: Timber, Iron, and Steel—Stanchions: Cast-iron, Rolled Joist Sections, Built-up Sections—Columns: Cast-iron Solid, Mild Steel Solid, Cast-iron Hollow, Stone Solid—Piers: Brick, Stone.

A STRUT is any member of a structure, or an independent piece, under compression in the direction of its length, but the term is usually given to a piece whose length is considerable compared with its cross section. A strut is often stated to be so many diameters long, taking as diameter the least width of cross section. Formulæ and tables of strength based upon the ratio of length to least diameter are simple and convenient, but are only approximately true. Upon comparing the results of calculation with actual experiments, wide differences are found when the least diameter has been taken into account, but if instead of this the formula has been based upon the least radius of gyration, the agreement of experiment and calculation is much closer. The whole subject of struts is one of great difficulty, and we must therefore proceed very cautiously. It is always useful to have an approximate rule for the safe stress per square inch, so that the time wasted in calculating the strength of unsuitable sections is reduced to a minimum. Unfortunately there is no means of arriving directly at the requisite sectional area as can be done in the case of girders, etc., we can only proceed by the system known as "trial and error." Moncrieff's formula is supposed to do it, but there are difficulties in the way of its adoption. The approximate working section of wood posts, ends flat or fixed, may be obtained by allowing $\frac{2}{10}$ ton per sq. in. on fir, and $\frac{3}{10}$ ton per sq. in. on oak, both 10 diameters long. By Gordon's formula,

$$W = \frac{FS}{1 + \dfrac{l^2}{250d^2}}$$

where W = breaking weight in tons, F = crushing force on short specimen in tons = 3·2 for oak, 2·5 for fir. S = sectional area in square inches, l = unstayed length in inches, d = diameter or least width in inches. Having chosen a probable size by the approximate rule, its sufficiency or otherwise is determined by the more complete formula. The factor of safety should not be less than 10.

Iron struts may be of cast iron or wrought iron. In cast iron the section is usually cruciform at the centre and gradually shaped down towards the ends to make the connection, but cast iron is not much used now owing to the facility with which rolled sections of wrought

iron and mild steel can be obtained. The form of section in the latter materials is usually angle (Fig. 263), tee (Fig. 264), channel (Fig. 265), sometimes rolled joist I or H section (Fig. 266), or flat bars and distance pieces (Fig. 267). The object sought in each case is to have

Fig 263 Fig. 264 Fig. 265 Fig 266

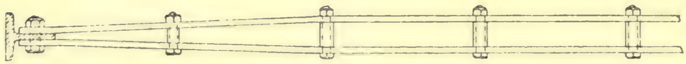


Fig 267

a form that is easily rolled, will resist bending, and at the same time be economical of material. The author has endeavoured to analyse the constants of the Gordon formula,[1] so as to make it applicable for different materials, different shapes of cross section, and different modes of fixing, as all these variations affect the result. The full statement would be as follows.

Let l = length in inches.
d = effective diameter or least cross-width in inches.
f = greatest intensity of stress in tons per square inch due to thrust and flexure when on the point of buckling,
for wrought iron = 18
,, mild steel = 26,
but by some writers f is taken = $\frac{2}{3}$ ultimate compressive strength of short specimen.
p = average thrust or compressive force in tons per square inch on section of strut, which will be the crippling stress per square inch when f is taken as above.
a = constant, varying according to different authorities.

It seems in theory to be made up as follows, $a = \dfrac{m}{nq}$,

where m = a fixing modulus,
say 1 for both ends fixed.
4 ,, ,, ,, rounded.
2·5 one fixed and one rounded.
16 ,, ,, ,, the other free.

n = a shape modulus,
say $1\frac{1}{2}$ for hollow cylindrical sections.
1 ,, solid rectangular sections.
$\frac{3}{4}$,, ,, cylindrical sections.
$\frac{1}{2}$,, X L H or T sections.
$\frac{1}{3}$ to $\frac{1}{4}$ for built-up sections.

q = a strength modulus, say tons per square inch tensile working strength × 500.

[1] The Gordon formula is really Tredgold's formula, but called Gordon's because he fixed the value of the constants from Hodgkinson's experiments.

Material.	*Stress.*	$q =$
Wood	$\frac{1}{2}$ ton	250
Cast iron	$1\frac{1}{2}$,,	750
Wrought iron	5 tons	2500
Mild steel	$6\frac{1}{2}$,,	3250

Then,
$$p = \frac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2}$$
$$\text{Factor of safety} = 4 + \cdot 05\left(\frac{l}{d}\right)$$

The Rankine-Gordon formula, taking account of the least radius of gyration instead of the least diameter, is

$$W = \frac{fA}{1 + \frac{l^2}{cr^2}}$$

where W = safe load in tons, f = maximum working stress in compression on a short specimen in tons per square inch = 6 for mild steel, A = sectional area in square inches, l = length in inches, c = constant = 36,000 for both ends flat or fixed, 22,500 for one end rounded, 9000 for both ends rounded, r = least radius of gyration, or radius of gyration in plane of compulsory bending, which is obtained from the equation $r^2 = \frac{I}{A}$. This is a good formula, but the reason it is not more used is the difficulty of obtaining the radius of gyration.

Various "straight line" formulæ have been proposed for obtaining an approximate section in a simple manner, the author's formula for mild steel struts is

$$\text{Approximate safe load lbs. sq. in.} = 8000\left(\frac{100 - \frac{l}{d}}{100}\right)$$

Cast-iron stanchions of **X H** or **E** section are still used to some extent in unimportant structures; the section may be determined by the following table:—

Up to 8	diameters	long	= 5	tons per. sq. in.
10	,,	,,	= 4	,, ,,
13	,,	,,	= 3	,, ,,
15	,,	,,	= $2\frac{1}{2}$	,, ,,
17	,,	,,	= 2	,, ,,
20	,,	,,	= $1\frac{1}{2}$	,, ,,

A rolled joist used as a stanchion may be calculated by the Rankine-Gordon formula, the radius of gyration being obtained from published tables.

Take, for example, a 6-in. by 5-in. by 25-lbs. rolled steel joist. The sectional area is 7·35 sq. in., the vertical or greatest moment of inertia

is 43·61 inch units, and the horizontal or least moment of inertia is 9·116 inch units; then,

$$r^2 = \frac{I}{A} = \frac{43·61}{7·35} = 5·93$$

and

$$\frac{9·116}{7·35} = 1·24.$$

Assume the stanchion to be 12 ft. long, then

$$W = \frac{fA}{1 + \frac{l^2}{cr^2}} = \frac{6 \times 7·35}{1 + \frac{144^2}{36,000 \times 5·93}} = 40 \text{ tons}$$

safe axial load if the stanchion is prevented from bending in the plane of either flange, or

$$\frac{6 \times 7·35}{1 + \frac{144^2}{36,000 \times 1·24}} = 30 \text{ tons}$$

if free to bend in either direction. The tabular value of ultimate strength in the manufacturers' catalogues is given as 112 tons, with a factor of safety of 4 for stationary loads and 6 for live loads, making the safe loads respectively $\frac{112}{4} = 28$ tons, and $\frac{112}{6} = 18·67$ tons, showing the agreement of the tabular load with calculation.

It must be observed that the greatest safe loads according to the direction of bending being 40 tons and 30 tons respectively, the limiting stress per square inch for a stanchion of *this section and length* will be $\frac{40}{7·35} = 5·44$ tons, and $\frac{30}{7·35} = 4·08$ tons.

There are very many cases in practice where the load is not axial, but is applied at one side of the rolled joist section, as where a stanchion is continuous through one or more floors, and the floor girders are attached by brackets or angle-pieces to the flange or web of the stanchion. In these cases, a bending moment is caused in addition to the direct stress due to the load. It is a popular error to suppose that when the girder is attached to the web of the stanchion the load is transmitted down the centre, and no bending moment results; it is contrary to fact, and although the leverage to the centre of the bearing may not exceed 2 in., a very serious additional stress is put on the stanchion, and the tendency to bend is in the direction of the least moment of inertia or radius of gyration. Taking the same rolled joist as before, if a load of 7 tons be applied at a distance of 2 in. from the centre of web, the bending moment will be $7 \times 2 = 14$ ton-inches, and the compressive stress on the edges of the flanges will amount to $\frac{W}{A} + \frac{M}{Z}$, where W is the load in tons, A the area in square inches, M the bending moment, and Z the modulus of section, or as it is wrongly called in many catalogues, "the moment of resistance in square inches."

$Z = \frac{I}{y}$, and in the present case of 6×5 stanchion loaded on the web

$Z = \frac{9{\cdot}116}{2{\cdot}5} = 3{\cdot}65.$

Then $\frac{W}{A} + \frac{M}{Z} = \frac{7}{7{\cdot}35} + \frac{14}{3{\cdot}65} = {\cdot}95 + 3{\cdot}83 = 4{\cdot}78$ tons per sq. in.

But it has already been shown that the limiting stress on this stanchion when free to bend in this direction is 4·08 tons per sq. in. so that the stress with the load of 7 tons is just over the desirable limit. On the other hand, the load may be carried from one flange of the stanchion so as to utilise the greatest moment of inertia. Then, allowing half the depth of section 3 in., and 2 in. more to centre of bearing surface, the bending moment will be $5 \times 7 = 35$ ton-ins. The modulus of section will now be

$$Z = \frac{I}{y} = \frac{43{\cdot}61}{3} = 14{\cdot}54,$$

and

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{7}{7{\cdot}35} + \frac{35}{14{\cdot}54} = {\cdot}95 + 2{\cdot}41 = 3{\cdot}36 \text{ tons per sq. in.}$$

but the allowable stress in this direction is 5·44 tons per sq. inch, so that it leaves an ample margin of safety, although the load is further from the axis of the stanchion.

When the load applied at the side of a stanchion exceeds 20 tons, it will in general be necessary to use a built-up section, and care should

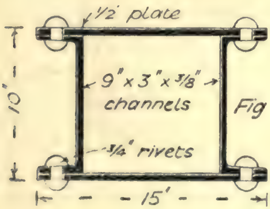


Fig 268

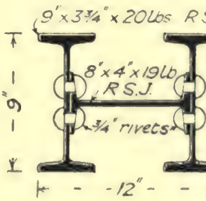


Fig 269

be taken to provide the requisite area with the least amount of riveting. It should also be noted that the material is more effective when disposed round the circumference rather than towards the centre; thus, two channels and two plates, as in Fig. 268, or three rolled joists, as Fig. 269, form economical sections. The exigencies of space, however, sometimes compel the opposite extreme to be followed, and a solid section becomes necessary.

The previous formulæ apply equally to struts and stanchions, or any compression member, but where long and important built-up stanchions have to be calculated the best formula is that known as "Fidler's Practical Formula for Columns." It is complicated and troublesome to use, but is of sufficient importance to warrant its employment in important cases. It will be found in the author's "Designing Ironwork," 2nd Series, Part II. (Spon, 1*s*. 3*d*.).

Ordinary cast-iron solid columns may be calculated by the formula

$$\text{Safe W in tons} = \frac{8S}{1 + \frac{1}{400}\left(\frac{l}{d}\right)^2}$$

Mild steel solid columns may be calculated by the formula

$$\text{Safe W in tons} = \frac{5S}{1 + \frac{1}{900}\left(\frac{l}{d}\right)^2}$$

Cast-iron hollow columns should for economy have a thickness of about $\frac{1}{12}$ of the external diameter. The section may be determined from the following table :—

Thickness = $\frac{1}{12}$ diameter,

Up to 10 diameters long	= 5 tons per sq. in.		
10 ,, 15 ,, ,,	= 4	,,	,,
15 ,, 20 ,, ,,	= 3	,,	,,
20 ,, 25 ,, ,,	= 2	,,	,,
25 ,, 30 ,, ,,	= $1\frac{1}{2}$	,,	,,
30 ,, 35 ,, ,,	= $\frac{3}{4}$	,,	,,

or by Gordon formula with $a = \frac{1}{800}$.

Stone columns may be calculated by the formula

$$\text{Safe W in tons} = \frac{8S \text{ in sq. ft.}}{1 + \frac{l^2}{400d^2}}$$

but care should be taken that the separate stones are not bedded hollow, or the outer edges will have all the load and spalling will take place. To prevent this there is sometimes a chisel draft $1\frac{1}{2}$ in. wide left round the edge of the joint and afterwards pointed up. Soft cement mortar carefully made with sand screened through a fine sieve, may be used to get a solid joint, but the interposition of a sheet of lead as is sometimes done is apt to set up "hydraulic" pressure and burst the joint.

Brick and stone piers may be calculated according to the safe load on the material, say stock brickwork in cement 6 tons per ft. sup., stock brickwork in mortar 3 tons per ft. sup.; but when the height exceeds six times the least width it is necessary to reduce the load. A suitable formula for the purpose is

$$W' = W\left(\frac{24 - r}{18}\right)$$

where W' = safe load tons per square foot on pier, W = safe load on cube of brickwork as given above, r = ratio of height to least width. When a brick pier is bonded to the end of a wall the strength may be assumed to be increased 25 per cent., and when bonded to the side of a wall, the whole thickness, including the wall, may be taken and 25 per cent. added to the strength for the adjacent bonding. These are only approximate rules to give what may be considered a fair allowance.

EXERCISES ON LECTURE XII

Q. 50. What size square post of fir 10 ft. high will be required to carry a load of 5 tons?

(NOTE.—The approximate rule of $\frac{2}{10}$ ton per sq. in. for a post 10 diameters long will not apply, as this post would then be 5 in. × 5 in. and is 10 ft. high, which would make it 24 diameters long.)

Answer. Try a 7-in. × 7-in. post, allowing a factor of safety of 10. Then

$$W = \frac{FS}{1 + \frac{l^2}{250d^2}} = \frac{2{\cdot}5 \times 49}{1 + \frac{120^2}{250 \times 7^2}} = 52 \text{ tons breaking weight,}$$

or $\frac{52}{10} = 5{\cdot}2$ tons safe load so that this size will be sufficient.

Q. 51. What is the safe thrust by Gordon's formula on a 3-in. by 3-in. by $\frac{3}{8}$-in. steel angle 6 ft. long, one end fixed and the other pivoted?

Answer. $p = \dfrac{f}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \dfrac{26}{1 + \frac{2{\cdot}5}{\frac{1}{2} \times 3250}\left(\frac{6 \times 12}{2{\cdot}25}\right)^2}$ = say 10 tons per sq. in.

on an area of 2·11 sq. in., or say 21 tons breaking weight.

$$\text{Factor of safety} = 4 + {\cdot}05\left(\frac{l}{d}\right) = 4 + 1{\cdot}6 = 5{\cdot}6,$$

giving the safe thrust as $\frac{21}{5{\cdot}6} = 3{\cdot}75$ tons.

Q. 52. What load will a 12-ft. cast-iron stanchion of cruciform section carry when the four arms are 1 in. thick and project 6 in. from the axis?

Answer. Ratio of length to least diameter $= \frac{12 \times 12}{9} = 16$ diameters long = say $2\frac{1}{2}$ tons per sq. in. on an area of 23 sq. in. $= 23 \times 2{\cdot}5 = 47{\cdot}5$ tons.

Q. 53. A detached brick pier of stock brickwork in cement 2 ft. 3 in. by 3 ft. is 20 feet high. What will be the safe load upon it?

Answer. $$W' = W\left(\frac{24 - r}{18}\right) = 6\left(\frac{24 - \frac{20}{2{\cdot}25}}{18}\right) = \text{say 5 tons per sq. ft.}$$

or $5 \times 3 \times 2{\cdot}25 = 33{\cdot}75$ tons total at base. The weight of the brickwork at 1 cwt. per cub. ft. will be $20 \times 3 \times 2{\cdot}25 = 135$ cwt. or 6·75 tons, making the safe load upon the pier $33{\cdot}75 - 6{\cdot}75 = 27$ tons.

LECTURE XIII

Finding Stresses in Roof Trusses by the "Principle of Moments," and by the "Method of Sections"—Allowance for Wind Pressure—Stresses on Roof Truss according to mode of Fixing—Stresses in Collar-Beam Truss.

THERE are so many varieties of roof trusses and elementary illustrations are so numerous in the various text books that only a few special cases need claim attention in this course.

First draw a simple truss, Fig. 270, and determine the stresses by reciprocal diagram, Fig. 271. Then take the same truss and load and find the stresses by the *principle of moments*. Take first the moments on the left about point B, Fig. 272, to find the stress in AC. The forces to consider are the reaction at A acting clockwise and the load A and

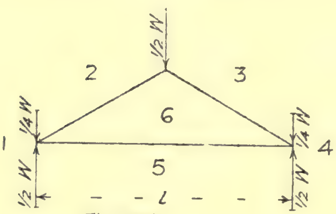


Fig 270

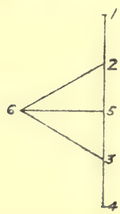


Fig. 271

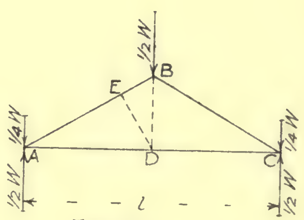


Fig. 272

tension in AC acting contra-clockwise. Then $\frac{1}{2}$W at A $\times$ $\frac{1}{2}l$ $-$ $\frac{1}{4}$W at A $\times$ $\frac{1}{2}l$ $-$ stress in AC $\times$ rise of truss $= 0$, whence

$$\frac{\frac{1}{2}W \times \frac{1}{2}l - \frac{1}{4}W \times \frac{1}{2}l}{\text{rise}} = \text{stress in AC.}$$

The stress in AB does not affect this working because it acts through the fulcrum and does not tend to turn the bars round that point. To find the stress in AB by this method, assume the fulcrum at, say,

intersection of centre line with AC, marked D. Draw a perpendicular from D to E on AB, then

$$\frac{\frac{1}{2}W \times \frac{1}{2}l - \frac{1}{4}W \times \frac{1}{2}l}{\text{leverage DE}} = \text{stress in AB.}$$

There is another method that may be used, called the *method of sections*, which is based upon the principle that for equilibrium to exist all the resolved parallel forces acting in one direction must equal all those acting in the opposite direction. In Fig. 273 draw any vertical line DE between A and B, the stress in AB may be resolved into horizontal and vertical components, but the vertical components must be equal to the loads which have to be transmitted through it. The load passing through D is $\frac{1}{4}$W from B, then DE : AD : : load passing through D : stress in AB, or stress in AB $= \frac{1}{4}W \times \frac{AD}{DE}$, which may be stated trigonometrically as,

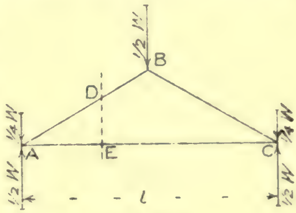


Fig 273

$$\text{stress in AB} = \frac{\frac{1}{4}W}{\sin DAE}.$$

Similarly, DE : AE : : $\frac{1}{4}$W : stress in AC, or stress in AC $= \frac{1}{4}W \times \frac{AE}{DE}$, which is, trigonometrically,

stress in AC = stress in AB × cos BAC, or stress in AC $= \frac{1}{4}W \times \cot DAE$.

An allowance of $\frac{1}{2}$ cwt. per foot super of sloping surface supported by truss, taken as a vertical load distributed over the points of support, is sufficient to include weight of truss and covering of slates, wind, and other accidental loads; generally, however, it is desirable to take the wind separately. An allowance of 21 lbs. for slated roof and 28 lbs. for tiled roof per foot super will be sufficient for the structural load, and 28 lbs. per foot super normal to the surface will provide for the wind. The actual force of the wind is a doubtful quantity. Unwin's table of wind pressures according to the angle of the roof is generally adopted, but it is based upon inadequate experiments, and a formula that leads to the impossible conclusion that the pressure is greater against a plane raised at 70 degrees than against one at 90 degrees. The author's formula for wind pressure gives a value increasing with the height of the object above the ground and decreasing with the width to be taken. It is

$$\log p = 1{\cdot}125 + 0{\cdot}32 \log h - 0{\cdot}12 \log w,$$

where p = ultimate wind pressure in lbs. per sq. ft. necessary to be allowed for against a plane surface normal to the wind, h = height of centre of gravity of surface considered, above ground level in feet, w = width in feet of part taken as one surface, and when the surface is inclined at θ degrees to the direction of the wind, the ultimate

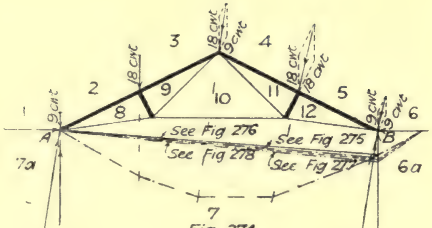


Fig. 274

Stress	Member	Stress in cwts				
		Fig 275	Fig 276	Fig 277	Fig 278	Fig 280
+	2–8	113	110	117	113·5	112
+	3–9	105	102	109	105·5	104
+	4–11	123·5	118	123	120	120
+	5–12	132	126	131	128	128
+	8–9	16·5	16·5	16·5	16·5	16·5
–	9–10	39	37	41·5	39·5	38
–	10–11	68	64	67	65	65
+	11–12	34	34	34	34	34
–	7–8	95	83	106	96·5	96
–	7–10	59·5	49	68	60·5	60
–	7–12	124	110	131	122	122

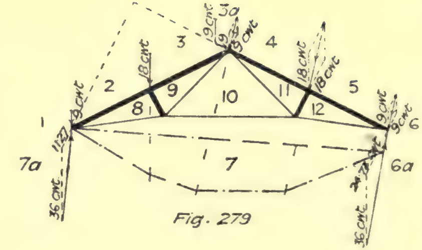
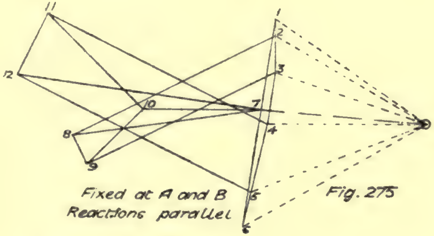


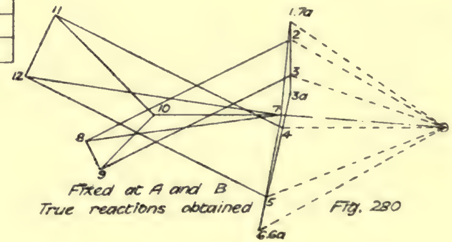
Fig. 279



Fixed at A and B
Reactions parallel

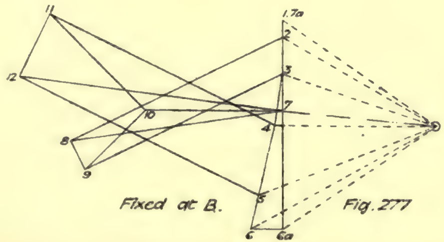
Fig. 275

Span 25′.0″
Rise 6′.3″
Camber ½″ per ft span
Trusses 10′0″ centres



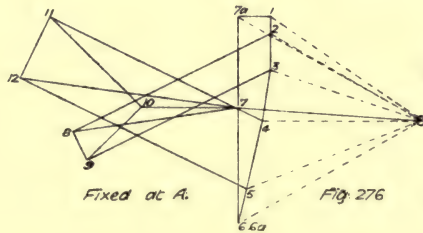
Fixed at A and B
True reactions obtained

Fig. 280



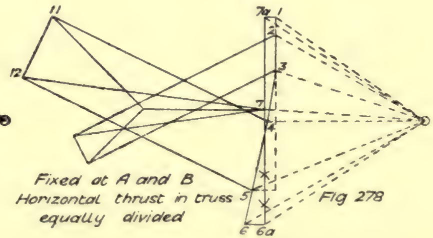
Fixed at B.

Fig. 277



Fixed at A.

Fig. 276



Fixed at A and B
Horizontal thrust in truss equally divided

Fig 278

pressure normal to the surface may be taken as $p \sin \theta$, or its effect in the same direction as the wind as $p \sin^2 \theta$. The author's practice in ordinary cases is to take 28 lbs. per ft. super as vertical load over the whole truss and 28 lbs. per ft. super normal to the surface on one side only for the wind. Taking for example a simple trussed rafter roof, Fig. 274, 25 ft. span, 6 ft. 3 in. rise, camber $\frac{1}{2}$ in. per ft. span, trusses 10 ft. centres, dead load 28 lbs. per ft. super, normal wind pressure 28 lbs. per ft. super. The ordinary method of finding the stresses will be as Fig. 275, where the reactions are taken as parallel. If the truss is fixed at A and is free to slide without friction at B, the stress diagram will be as Fig. 276. If fixed at B and free at A the stress diagram will be as Fig. 277. If fixed both at A and B, the stress diagram will be as Fig. 278. In the latter, after the load line is drawn, a vertical is dropped from point 3 and cut off by a horizontal from 5. The horizontal is bisected to give the full line 7a–6a, so that the fixing at B takes the whole of the horizontal component of force 5–6, and half the remainder equally with the fixing at A. Probably the most correct method of working is that shown in Fig. 279 and Fig. 280, where the reactions of the dead load and wind are taken separately and combined to give the total reactions, which will be found to have different inclinations. The funicular polygon can be drawn as a check upon the work, but the reactions being found independently they can be added to the load line before the stress diagram is drawn. The difference in the stresses produced by the five methods is shown in the table, and it is noteworthy that the last method gives approximately the mean of all the others.

A collar beam truss is the simplest form of truss constructively, but is a very complex one when the stresses are considered.

If the walls are taken as rigid and the loading vertical, then the frame diagram will be as Fig. 281, and the stress diagram as Fig. 282.

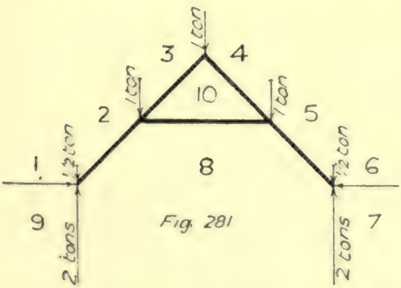


Fig. 281

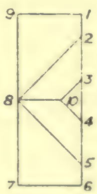


Fig. 282

The horizontal thrusts at foot of rafter being unknown, the method of working will be as follows: Set down the load line 1 to 6; then 3–10, 4–10; 10–8, 2–8, 5–8; 8–9, 1–9; and 8–7, 6–7. It will be seen that in this case all members are in compression.

If the walls are taken as yielding, the nature of the stresses will be altered and a bending moment on the rafters will be induced owing to the leverage effect of the reactions at the ends of the rafters. In order to work out a stress diagram, virtual forces at right angles to the

rafters must be introduced to counteract this effect, as shown by dotted lines in Fig. 283. To obtain force 9–1, take moments about the junction of rafter and collar. Then 9–1 will equal reaction 8a–9 multiplied by its leverage minus 1–2 multiplied by its leverage and divided by the length of rafter from foot to junction with collar. Then as the

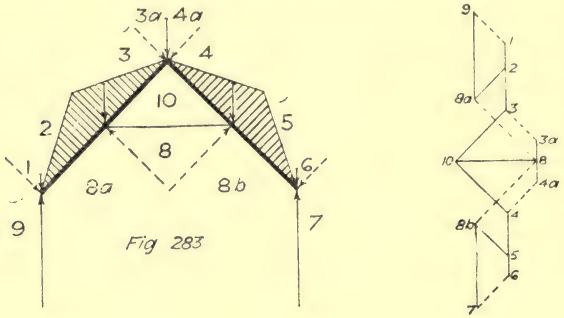


Fig 283

Fig 284

collar is half way up the rafter, the force 3–3a will be equal to 9–1, and 8–8a the equilibrant will be equal and opposite to these two forces. As the loading is symmetrical the virtual forces on the other rafter will be the same, and the stress diagram, Fig. 284, may be completed, from which it will be seen that in this case the collar is in tension. The bending moment will be found by taking moments about the junction of rafter and collar, and will equal (8a–9 × its leverage) – (1–2 × its leverage).

The next case to take will be with the supports rigid and allowing for wind pressure on one side. Assuming 2 tons distributed for the dead load and 2 tons horizontally on one side for the wind pressure, the frame diagram with loads will be as in Fig. 285. The stress diagram cannot be worked straight away, and it will be necessary to divide the truss into two portions. The upper portion of the truss is shown in Fig. 286, with the resistances obtained from the reciprocal diagram, Fig. 287. The lower portion is shown in Fig. 288, with the forces resolved graphically, and it will be seen that there is a force of 1·05 tons in the collar unbalanced. In Fig. 289 the information obtained by the other diagrams is brought into one view, including the balancing force 2–2a, which is the true equilibrant of all the other forces, and a reciprocal diagram, Fig. 290, is drawn, the funicular polygon being constructed as a check upon the work. In substitution, in Fig. 291, half the amount of 2–2a must be transferred to the supports and combined with the reactions to produce the final resultants on the supports. In making this substitution, a bending moment diagram must be added as shown, the maximum ordinate of which is equal to the horizontal force of 0·525 ton multiplied by the vertical distance to junction of collar and rafter = 4 ft. × 0·525 ton = 2·1 ton-ft.

The final case will be where the supports are not rigid and wind pressure is taken into account. Assuming the same loading as in the last case, the frame diagram with loads will be as Fig. 292. The

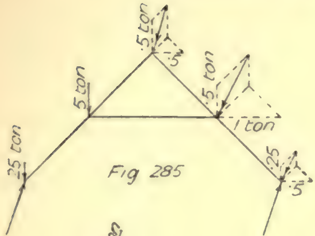


Fig 285

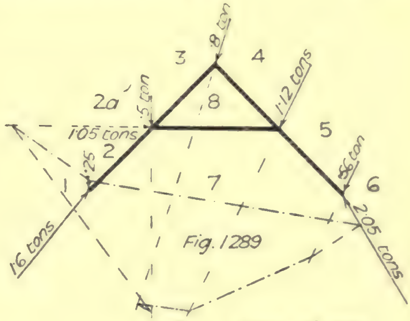
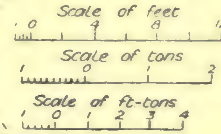


Fig. 289



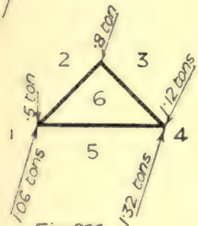


Fig 286

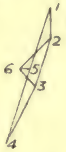


Fig. 287

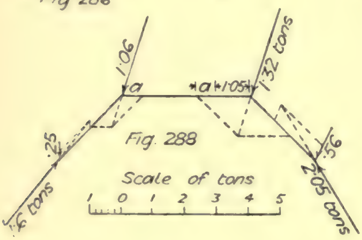


Fig. 288

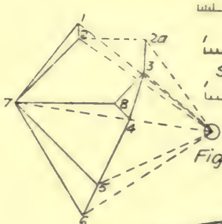


Fig. 290

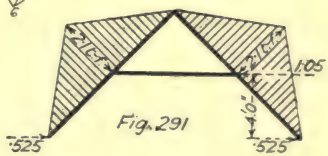


Fig. 291

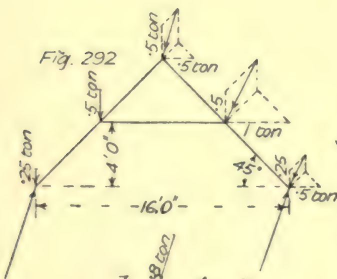


Fig. 292

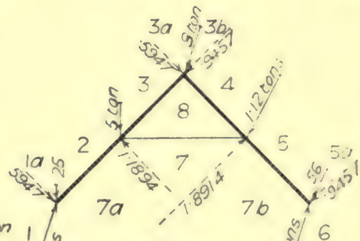
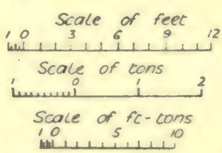


Fig 295



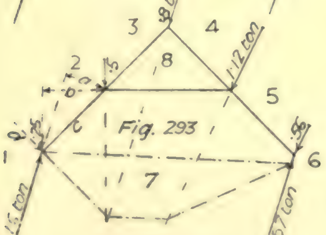


Fig. 293

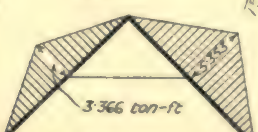


Fig 297

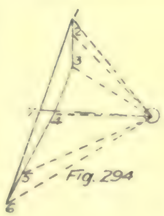


Fig. 294

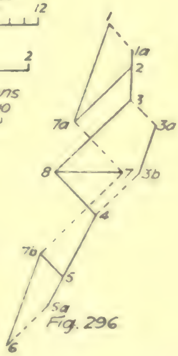


Fig. 296

combined loads at each point are shown in Fig. 293, and the reactions are obtained by vectors in Fig. 294, and funicular polygon in Fig. 293. Then the virtual force $d = \dfrac{1{\cdot}6 \times a - 0{\cdot}25 \times b}{c}$ acting at right angles to the rafter at the foot. A similar and equal force will be exerted at the head of the rafter, and at the junction of collar and rafter will be the equilibrant of the two. A similar proceeding must be adopted for the other rafter and the whole of the forces, virtual and actual, will then be as shown in Fig. 295, from which the reciprocal diagram, Fig. 296, may be obtained. The spreading tendency will cause a bending moment, as shown in Fig. 297, the maximum bending moment being equal to $1{\cdot}6 \times a - 0{\cdot}25 \times b$.

EXERCISES ON LECTURE XIII

Q. 54. Fig. 298 shows the outline of an abnormal truss for which the stresses are required, and also the bending moment and shear diagrams for the tie.

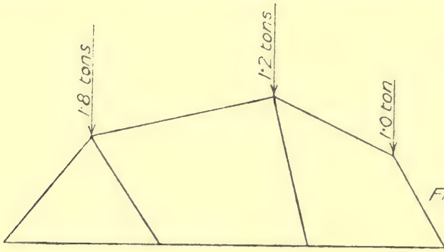


Fig. 298

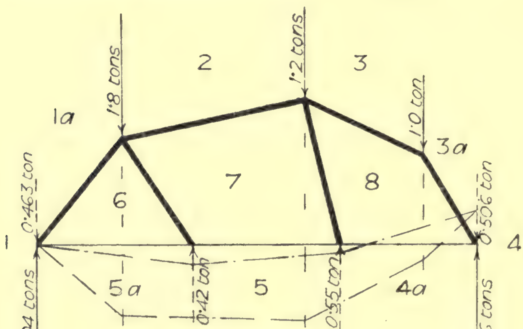
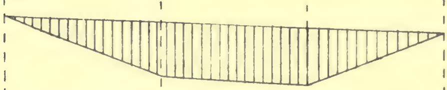
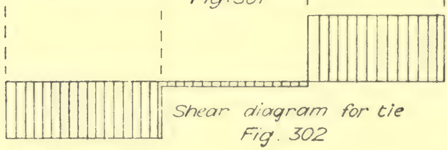


Fig. 299



Bending moment diagram for tie
Fig. 301



Shear diagram for tie
Fig. 302

For answer, see Figs. 299 to 302.

(NOTE.—Set down load line 1a to 3a. Then 3–8, 3a–8; 8–7, 2–7; 7–6, 1a–6; and 6–5a, 7–5, 8–4a, giving assumed forces 4a–5 and 5–5a. Calculate 4a–4 and 5a–1 and set down on load line, thus giving the assumed forces 3a–4 and 1–1a.

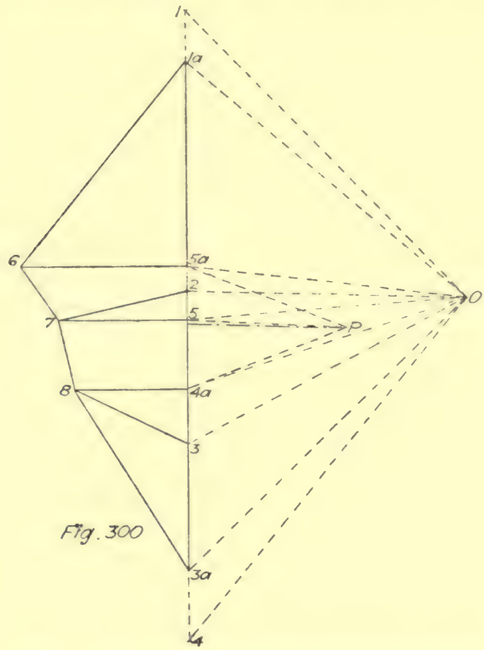


Fig. 300

Complete the diagram and test by taking pole 0 and drawing funicular polygon. For bending moment diagram take pole P and draw funicular polygon, the closing line of which happens to nearly coincide with point 5. Shear diagram drawn from reactions to virtual forces in the usual way.)

Q. 55. Fig. 303 shows a pair of rafters inclined at 39 degrees from the hori-

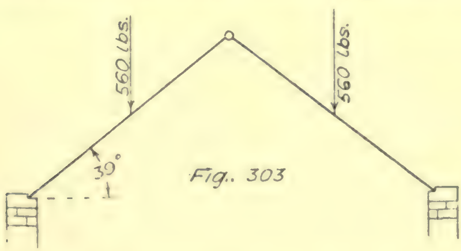


Fig. 303

zontal, with loads applied at their centres, and the feet supported as shown; (*a*) find and measure the horizontal thrust on the walls; (*b*) find and measure the thrusts in the upper and lower halves of each rafter.

For answer, see Fig. 304.

(NOTE.—The load *ab* in Fig. 304 may be resolved into a transverse force *ac*, which causes a bending moment, and a thrust *ad* in the lower half of the rafter. The force *ac* may then be resolved into the two equivalent forces *ef* and *gh*, acting

at the points of support. Then at point *e* from the other side there will be a similar force *ej*. Combine *ej* and *ef* to give the resultant *ek*, which is then resolved into the two thrusts *el* and *en*, which will be the thrust in the top portion of beam. This thrust will also be carried through to the lower portion, in addition

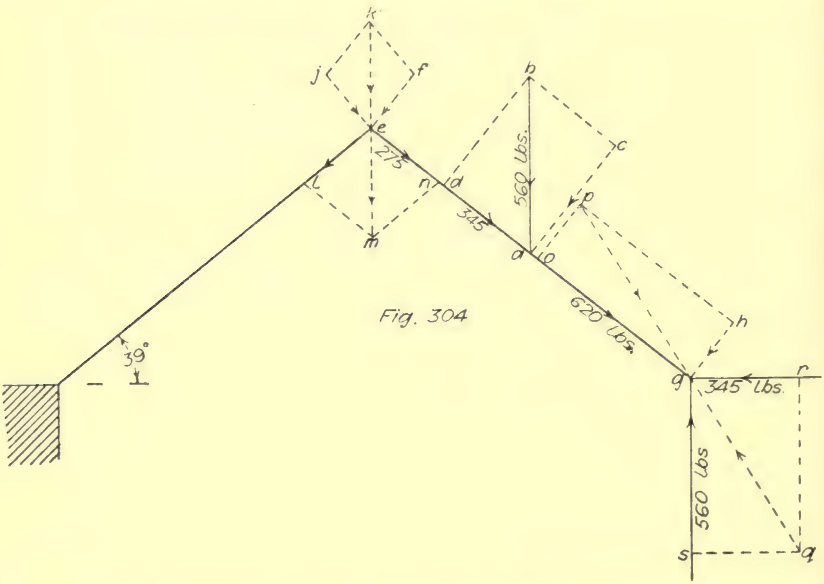


Fig. 304

to the thrust *ad* already found. Add these two thrusts together to give *go*, and combine with the force *gh*, giving a resultant *gp*. This last resultant will have to be resisted by an equilibrant *qg*, which may be resolved into the horizontal thrust *gr* and the reaction *sg* equal to the total load.)

LECTURE XIV

Stresses in Queen-post Truss—Effect of Wind—Bending Moment on Tie-beam—Lantern Lights—Substituted Members.

A QUEEN-POST truss with a rectangular space in the centre is a deformable structure as indicated in Fig. 305, the stiffness of the tie-beam alone resisting the bending moments produced by the irregular loading due to the wind on one side. To enable a stress diagram to be drawn

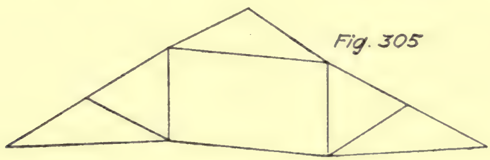


Fig. 305

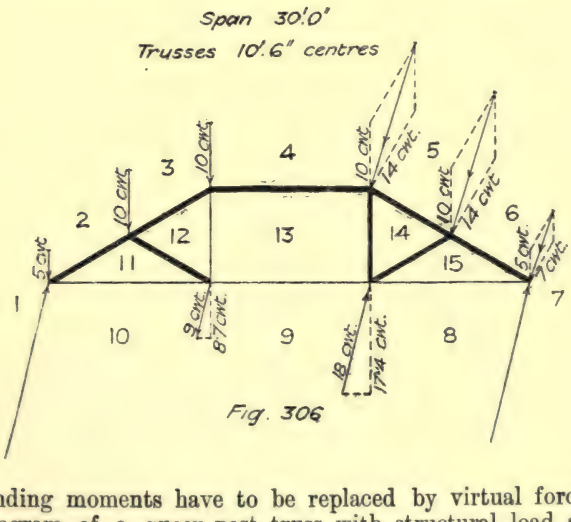


Fig. 306

these bending moments have to be replaced by virtual forces. The frame diagram of a queen-post truss with structural load and wind pressure is shown in Fig. 306. To work out the stress diagram, Fig. 307, proceed as follows:—Set down the load line 1 to 7, and join 1—7, then 2—15, 6—15; 15—8 horizontal; 15—14, 5—14; 14—13, 4—13, also giving by intersection point 9 on line 1—7; 13—12, 3—12; 12—11, 2—11; 11—10 horizontal; thus giving the virtual forces 8—9

and 9—10, which counteract the downward and upward thrusts of the queen-posts. The vertical components of these two forces will produce a bending moment on the tie-beam calculated as follows. The virtual force 8—9 will produce a negative bending moment of

$$\frac{Wab}{L} = \frac{17{\cdot}4 \times 10 \times 20}{30} = 116 \text{ cwt.-ft.}$$

as shown by the triangle ABC in Fig. 308. The virtual force 9—10 will produce a positive bending moment of

$$\frac{Wab}{L} = \frac{8{\cdot}7 \times 10 \times 20}{30} = 58 \text{ cwt.-ft.}$$

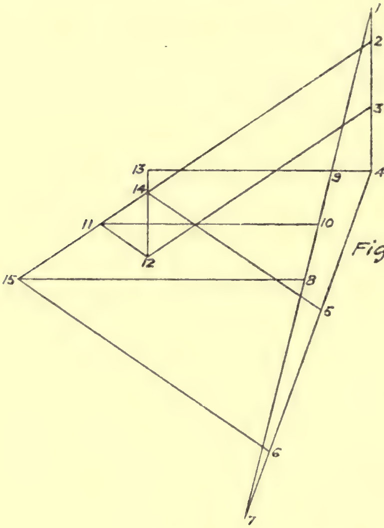


Fig. 307

as shown by the triangle ADC. Subtracting the positive from the negative bending moment leaves the triangle DBC as a negative

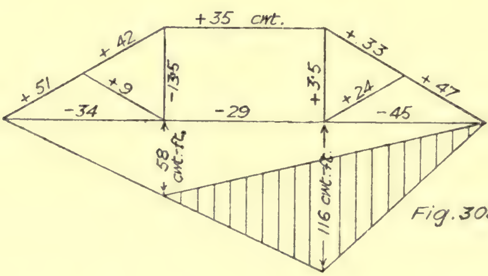


Fig. 308

bending moment on the tie-beam, *i.e.* acting downwards. The horizontal components of the virtual forces are so small that they may be neglected. In calculating the scantlings the maximum tension and

bending moment in the tie-beam must be allowed for by the formula $\frac{W}{A} \pm \frac{M}{Z}$.

When a lantern light is added to a roof the stresses on the lantern should be first considered, and then the reactions taken as external forces on the main truss. The frame diagram of a lantern light with loading is shown in Fig. 309, and the reactions are obtained by the funicular polygon in the reciprocal diagram, Fig. 310. These reactions are then transferred as loads to the main truss, Fig. 311. The lantern light is here taken as a solid structure, as only the reactions are to be made use of, but in practical designing the lantern must be duly framed or stiffened to enable it to resist the loads, the outline shown being deform-

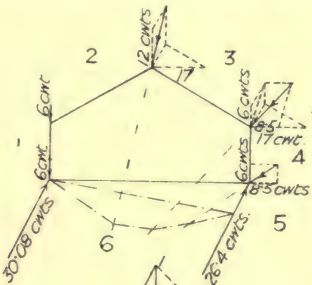


Fig 309

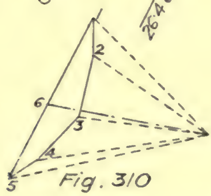


Fig. 310

able. Now set down the load line 1 to 9 in Fig. 312 and join 1—9. Select any pole 0 and draw funicular polygon in Fig. 311 across spaces 1 to 9. Then draw 2—21, 8—21 ; and 21—10 horizontal. Parallel with 10—0 draw corresponding line in funicular polygon, then draw a horizontal through point 5, cutting line 1—9 in the required point 11, and parallel with vector 11—0 draw corresponding line across space 11 in funicular polygon. Then draw a line across space 12 in Fig. 311 to complete funicular polygon, and parallel with it from pole 0 in Fig. 312 draw vector to give point 12. The stress diagram may then be completed without further difficulty. The bending moment diagram produced by the virtual loads will be calculated as described in the previous case.

In constructing stress diagrams it often happens that progress is stopped by reason of two triangles coming between two embracing spaces instead of the usual single triangle, or in other words more than two unknown stresses meet at a joint of a truss. Then a temporary rearrangement of the frame is necessary by the addition of substituted members and the ignoring of the originals. The substituted members must run direct from the points of application of the load to an external joint of the truss. The use of substituted members was first suggested by Mr. James R. Willett in a paper read before an American

Span 55' 0"

Trusses 13' 0" centres

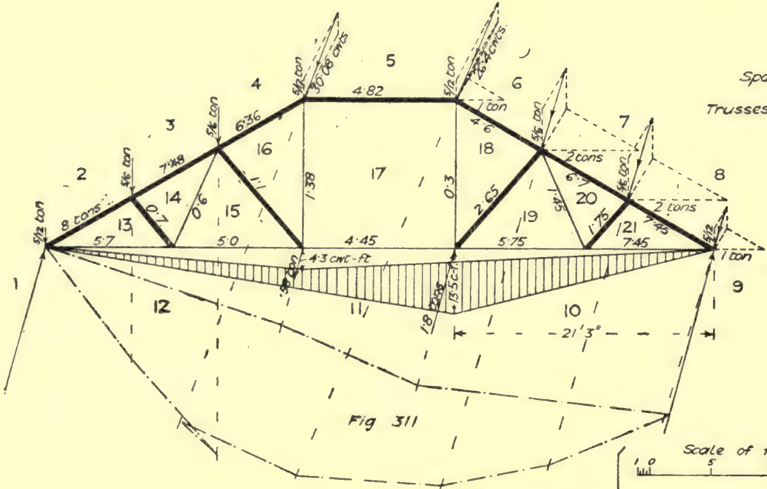


Fig 311

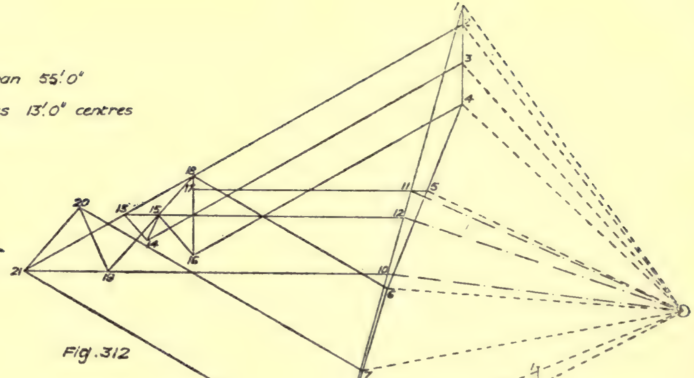
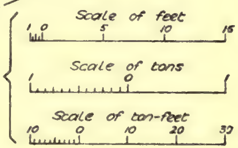


Fig. 312



Scales for Figs. 311 and 312

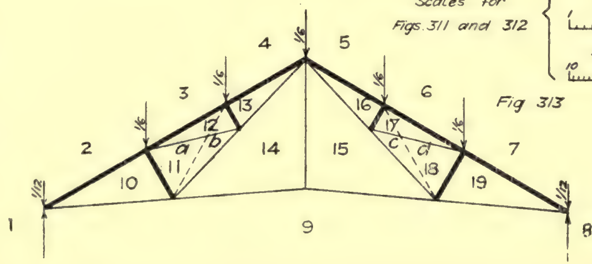


Fig 313

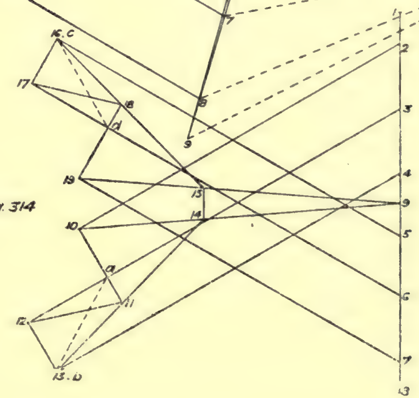


Fig. 314

Architectural Institute in 1888, a copy of which he kindly sent the author through Mr. R. H. Bow. The frame diagram of a truss to which this method is applicable is shown in Fig. 313, and in order to work out the stress diagram the substituted members *a*—*b* for 11—12, 12—13 and *c*—*d* for 16—17, 17—18 must be used. Then set down the load line 1 to 8 in Fig. 314, and proceed as follows: 2—10, 9—10; 10—*a*, 3—*a*; *a*—*b*, 4—*b*; *b*—14, 9—14; point 14 being found, points 11, 12 and 13 can now be obtained, and the same method repeated for the other side will complete the diagram.

EXERCISES ON LECTURE XIV

Q. 55 *a.* Fig. 315 shows the frame diagram of a truss 30 ft. span, slope 30 degrees, trusses 10 feet centres, structural load 28 lbs. per ft. sup., wind pressure (normal)

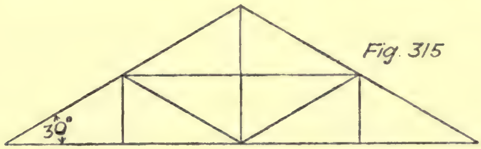


Fig. 315

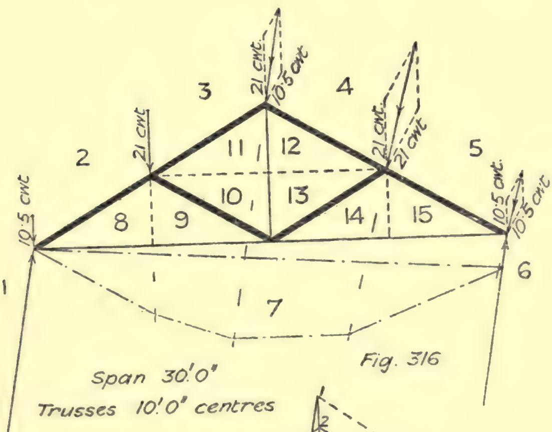


Fig. 316

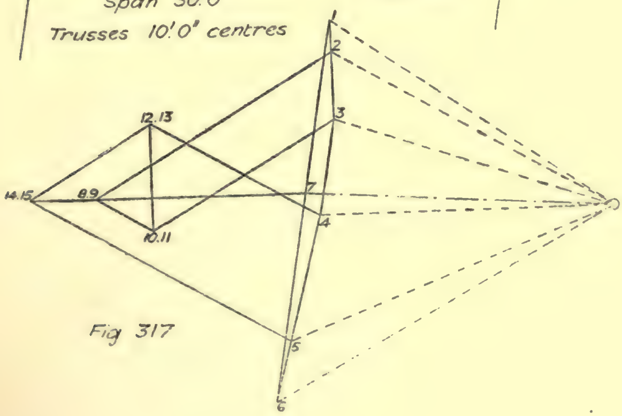


Fig. 317

28 lbs. per ft. sup. Draw the frame and stress diagrams. Scales 8 ft. to 1 in. and ¼ in. to 10 cwt.

For answer, see Figs. 316 and 317.

Q. 56. Fig. 318 shows a queen-post truss carried up to a central ridge and fixed

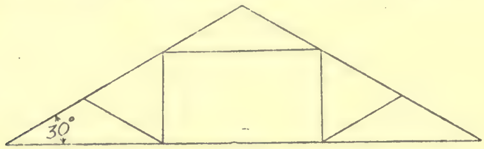


Fig. 318

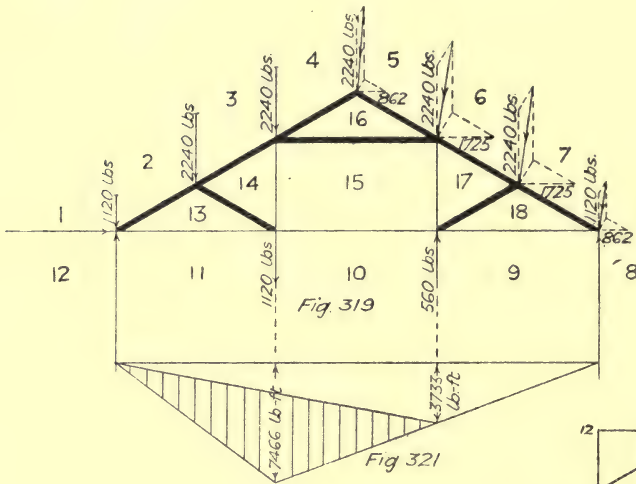


Fig. 319

Fig 321

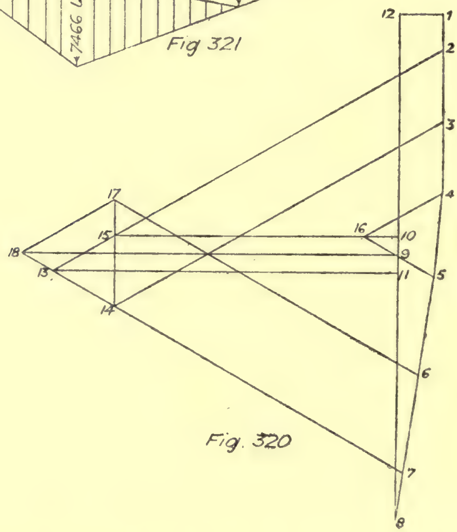


Fig. 320

at the leeward support. Span 30 feet, slope 30 degrees, trusses 10 feet centres, total structural load 6 tons, horizontal wind pressure 30 lbs. per ft. sup. Draw

frame, stress, and bending moment diagrams. Scales 8 ft. to 1 in., $\frac{1}{4}$ in. to 1000 lbs., $\frac{1}{8}$ in. to 1000 lb.-ft.

For answer, see Figs. 319, 320, and 321.

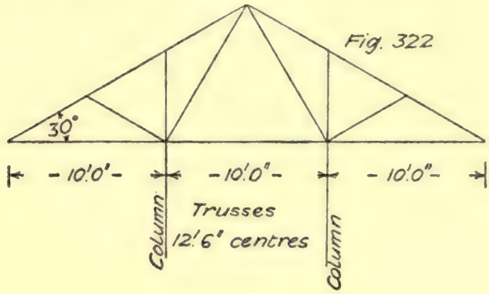


Fig. 322

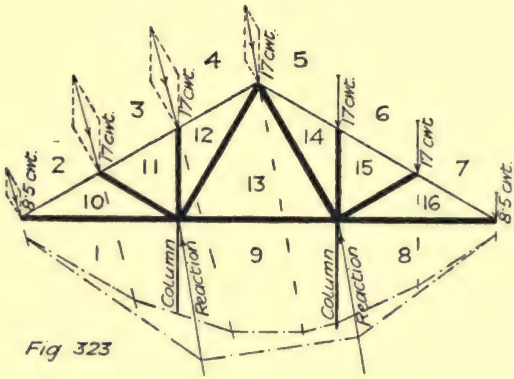


Fig 323

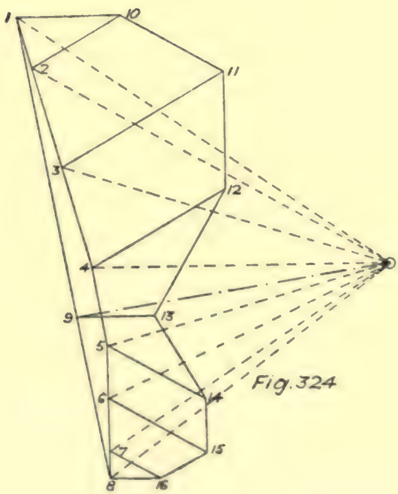


Fig. 324

(NOTE.—Order of working as follows :—Set down the load line 1 to 8, then 2—13, 7—13 ; 13—11 horizontal ; 4—16, 5—16 ; 16—10 horizontal ; 13—14, 3—14 ; 14—15, 16—15 ; 15—17, 6—17 ; and 17—18, 7—18.)

Q. 57. Fig. 322 shows roof truss for an island platform at a railway station. Span 30 feet, slope 30 degrees, trusses 12 ft. 6 in. centres, structural load 28 lbs. per ft. sup., wind pressure (normal) 28 lbs. per ft. sup. Draw frame and stress diagrams. Scales 8 ft. to 1 in., and ¼ in. to 10 cwt.

For answer, see Figs. 323 and 324.

LECTURE XV

Hammer-beam Trusses, Vertical and Inclined Loads, Rigid and Yielding Walls —Calculation of Bending Moment on Principal Rafter and Braces—Braced Collar-beam Truss.

A HAMMER-BEAM truss is a form frequently adopted for church work, and therefore of great importance, but unfortunately no entirely satisfactory stress diagram can be drawn, and the stresses are largely matter of conjecture, varying according to the nature of the support the trusses

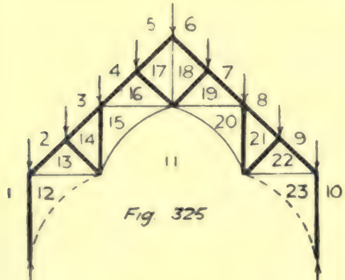


Fig. 325

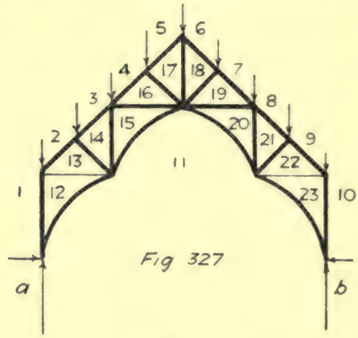


Fig. 327

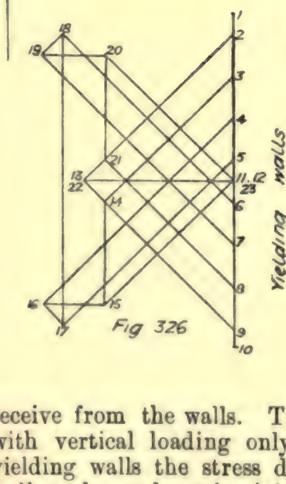


Fig. 326

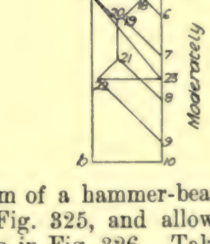
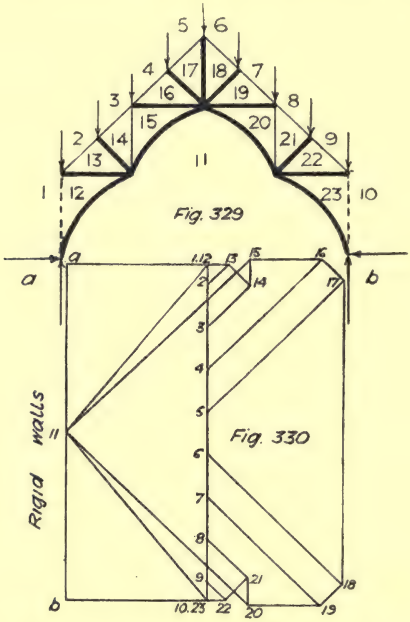


Fig. 328

receive from the walls. The frame diagram of a hammer-beam truss with vertical loading only, is shown in Fig. 325, and allowing for yielding walls the stress diagram will be as in Fig. 326. Taking the walls to be moderately rigid, the frame diagram will be as in Fig. 327, and the stress diagram as in Fig. 328, the wall piece, 1–12 and 23–10,

or wall behind it, being assumed to carry one-fourth of the total load. On the assumption that the walls are perfectly buttressed and rigid, the frame diagram will be as in Fig. 329, and the corresponding stress diagram as in Fig. 330.

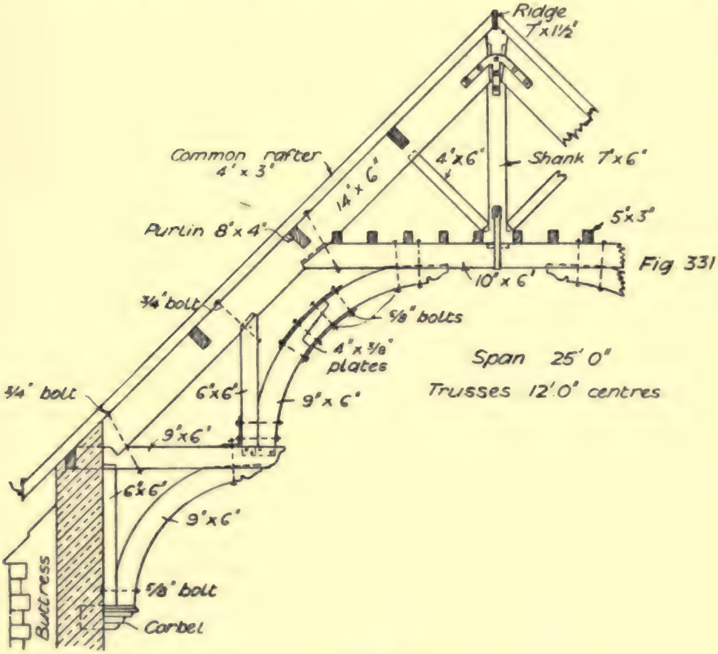
The elevation of a common type of hammer-beam truss, 25 ft. span, 12 ft. centres, is shown in Fig. 331. A slight modification of the truss must be made for the frame diagram, by introducing additional members as in Fig. 332, 15–16, 16–17, 18–19, 22–23, 24–25, 25–26, in order to

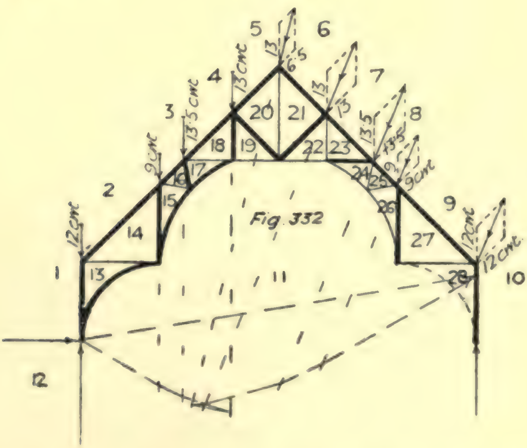


enable the stress diagram, Fig. 333, to be drawn out. It is also necessary to divide the load on the lower purlin, between the foot of rafter and junction between rafter and vertical. After the completion of the stress diagram the added members must be removed and the bending moments then produced must be ascertained and allowed for. A portion of the truss on the right-hand side where the maximum stresses occur, is shown enlarged in Fig. 334, and the stresses from the added members will be resolved into virtual forces parallel with, and perpendicular to, the various members as shown, from which the bending moments may be calculated. The diagram of bending moments on the principal rafter is shown in Fig. 335.

A braced collar-beam truss 35 ft. span, 13 ft. centres, 28 lbs. per sq. ft. vertical dead load, and 28 lbs. per sq. ft. normal for wind, is shown in elevation Fig. 336, the frame diagram with loading being taken as in Fig. 337. In order to work out the stress diagram, Fig. 338, the members 14–15 and 18–19 must be added. As these members are not actually present the effect of the stresses in them must be considered.

The stress at junction of this added member and the principal rafter may be resolved into two forces, one at right angles to and the other parallel with the principal rafter. At the junction with the collar the





stress may be resolved into a force at right angles to and one parallel with the collar, as shown dotted in Fig. 339. The bending moments produced by the forces at right angles to the rafters and collar are

shown in Fig. 340. The stresses in the various members may now be worked out and will be as follows: King post, 2 cwt. per sq. in.;

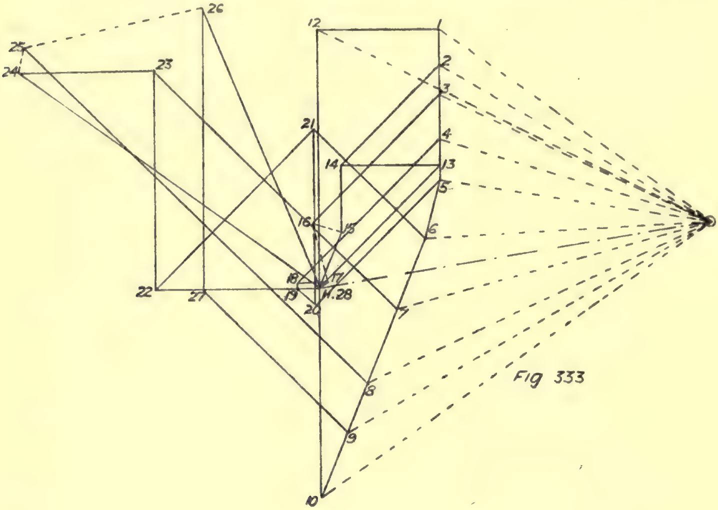


Fig 333

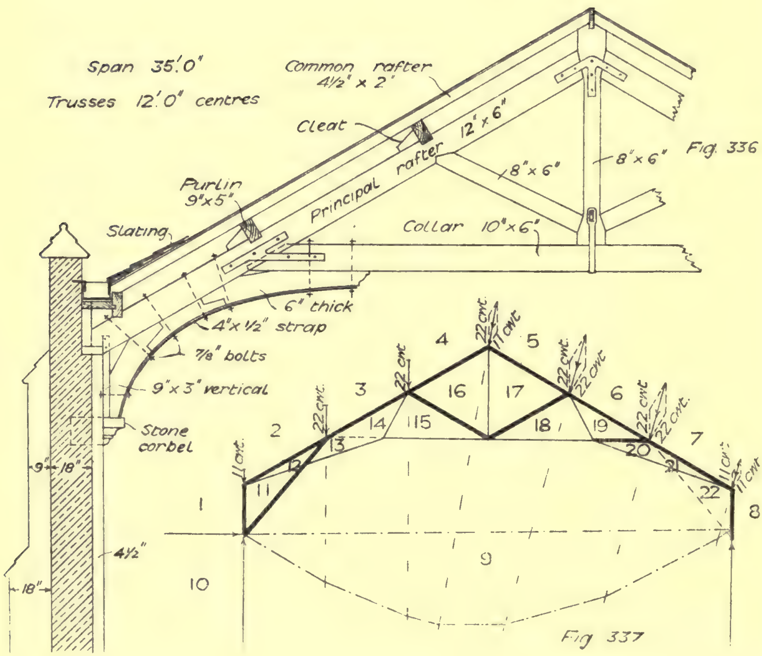


Fig. 336

Fig 337

struts, 6 cwt. per sq. in.; collar, 29 cwt. per sq. in.; principal rafter at junction with strut, 23 cwt. per sq. in.; and midway between junction with collar and foot, 7·8 cwt. per sq. in. The end bolt between curved

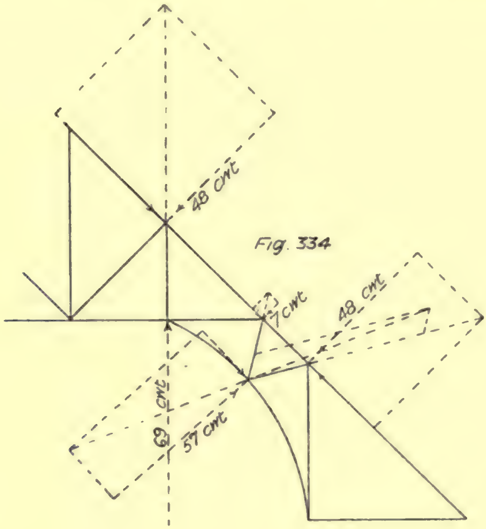


Fig. 334

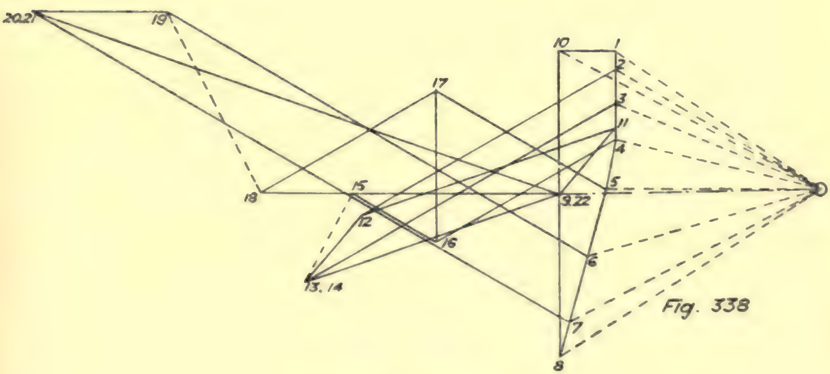


Fig. 338

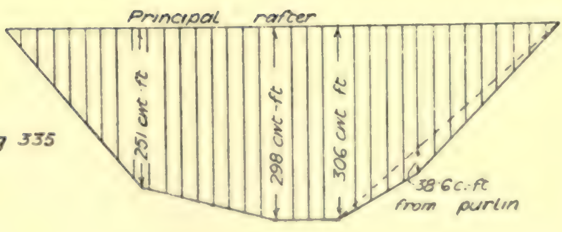


Fig 335

brace and collar will have a pull on it of 128 cwt. = 6·4 tons. The net area is 0·42 sq. in., giving a stress of say 15 tons per sq. in. It will therefore be seen that the truss will be perilously near failure if the full loading is realised.

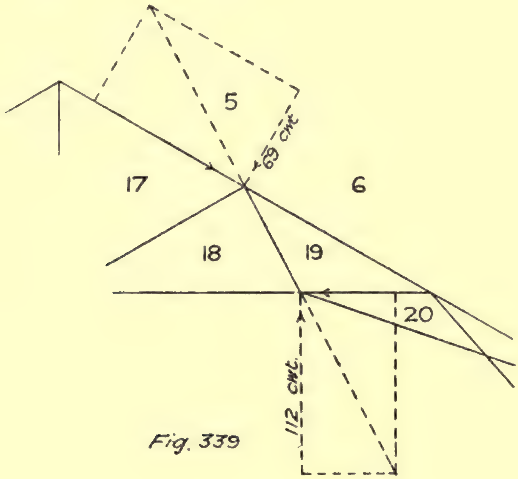


Fig. 339

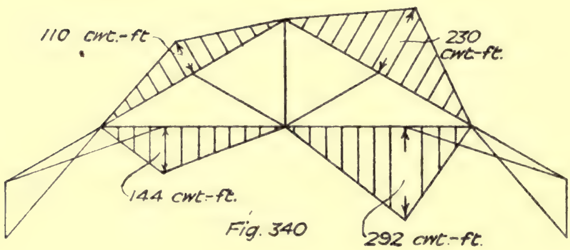


Fig. 340

EXERCISES ON LECTURE XV

Q. 57 *a.* Draw frame and stress diagrams for the modified hammer-beam truss shown in Fig. 341, allowing 28 lbs. per sq. ft. for wind pressure normal to slope in addition to structural load. Calculate the stress in the members.

For answer, see Figs. 342 to 347.

Calculation as follows :—

$$\text{King post } \frac{8 \text{ cwt.}}{48 \text{ sq. in.}} = 1\tfrac{2}{3} \text{ cwt. per sq. in.};$$

$$\text{struts } 88 = \frac{F \times 48}{1 + \frac{1}{125}\left(\frac{75}{6}\right)^2} = 4{\cdot}12 \text{ cwt. per sq. in.};$$

$$\text{collar } \frac{W}{A} + \frac{M}{Z} = \tfrac{72}{88} + \frac{2322}{\frac{1}{6}(8 \times 11^2)} = 0{\cdot}8 + 13{\cdot}5 = 14{\cdot}3 \text{ cwt. per sq. in.};$$

$$\text{principal at junction with strut } \frac{W}{A} + \frac{M}{Z} = \tfrac{140}{88} + \frac{2287}{\frac{1}{6}(8 \times 11^2)} = 1{\cdot}6 + 13{\cdot}3 = 14{\cdot}9 \text{ cwt. per sq. in.}$$

The maximum bending moment at the joint in the curved piece will be equal to the stress in 11-23 multiplied by its versin or leverage = 115 cwt. × 28 in. = 3320 cwt.-in. As a check upon this method, moments may be taken, as in

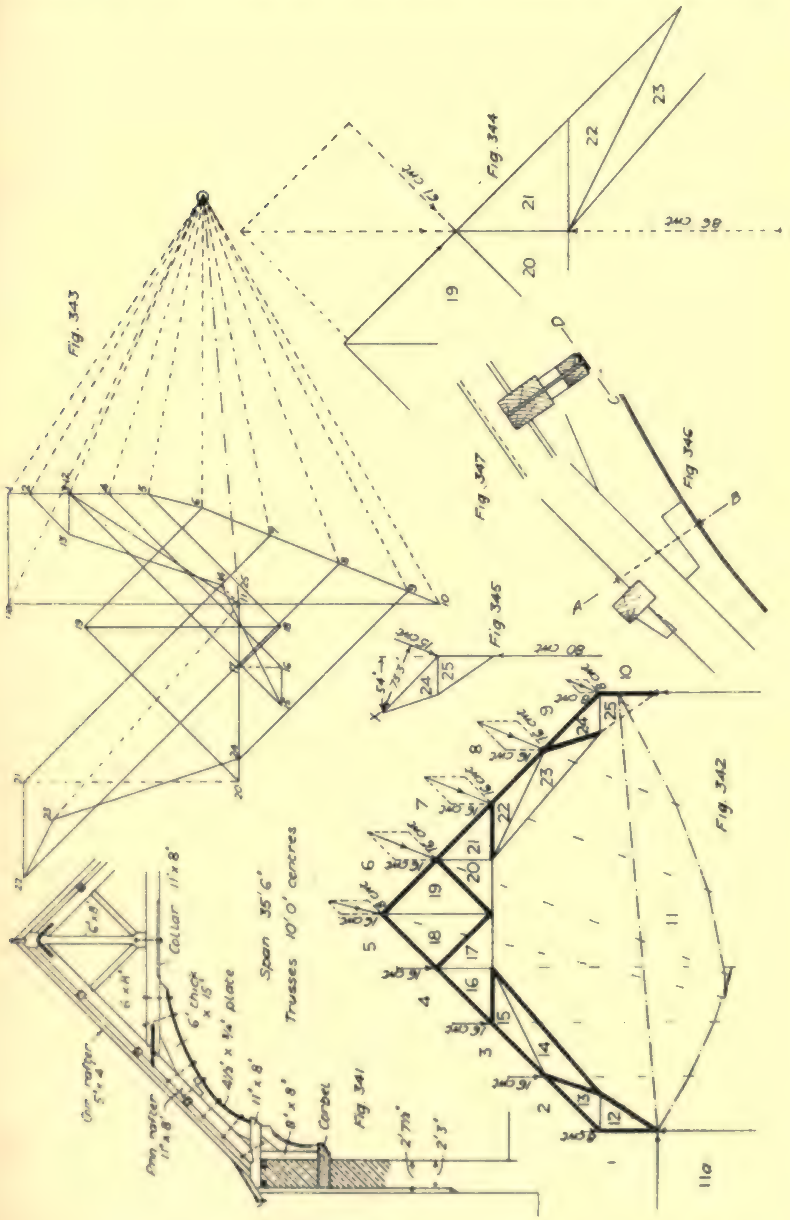


Fig. 345, about point $\times$ where the maximum bending moment will occur. Then $(80 \times 54) - (15 \times 73\cdot\dot{3}) = 3220$ cwt.-in. as before. To find the section modulus at the joint draw out an enlarged view at this point, as shown in elevation, Fig. 346, and section on AB, Fig. 347. Then to find the position of neutral axis, take moments about CD, the iron plate being taken as having a comparative sectional area of 20 times that of the timber owing to its greater strength, but the leverage remains unaltered and will still be taken about the line CD. Then

$$y = \frac{0\cdot375(3\cdot75 \times 0\cdot75 \times 20) + 5\cdot25 \times 7 \times 3\cdot875 + 7\cdot25 \times 11 \times 17\cdot875}{56\cdot25 + 36\cdot75 + 79\cdot75} = \frac{1585\cdot5}{172\cdot75}$$

$= 9\cdot2$ in. from AB, or $14\cdot55$ in. from the top of rafter, giving a virtual depth of beam of $14\cdot55 \times 2 = 29\cdot1$ in. Then

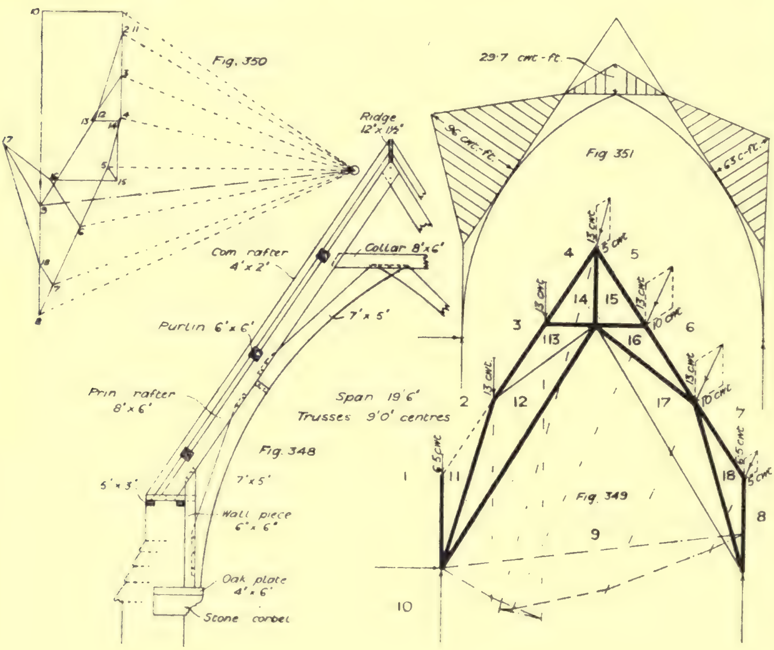
$$Z = \frac{bd^3 - h^3(b - t)}{6d} = \frac{7\cdot25 \times 29\cdot1^3 - 7\cdot1^3(7\cdot25 - 0\cdot75)}{6 \times 29\cdot1} = 1033\cdot6,$$

and the stress will be

$$\frac{W}{A} + \frac{M}{Z} = \frac{180}{152\cdot8} + \frac{3220}{1033\cdot6} = 1\cdot17 + 3\cdot11 = 4\cdot28 \text{ cwts. per sq. in.}$$

Q. 58. Fig. 348 shows the nave roof of a small church to a scale of ¼-in. to 1 ft. Draw frame, stress, and bending moment diagrams.

For answer, see Figs. 349, 350, and 351.



LECTURE XVI

Exceptional Forms of Braced Roofs, involving addition of Virtual Forces, and Bending Moments on Rafters.

THE outline of a roof truss sometimes used for sheds, but of very imperfect design, is shown in Fig. 352. Assuming the supports to be perfectly rigid the frame diagram with unity load will be as in

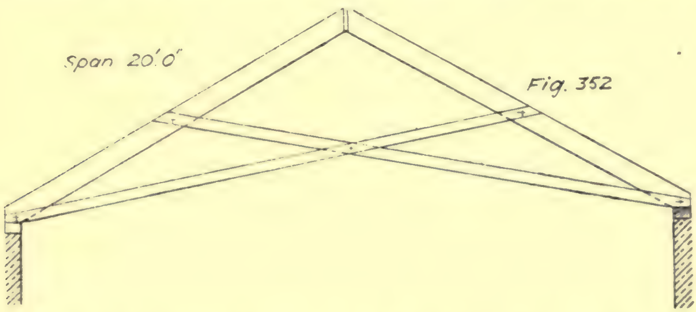


Fig. 352

Fig. 353, and the stress diagram as in Fig. 354. The following is the order of working the latter, 3—11, 4—11 ; 11—10, 2—10 ; 11—12,

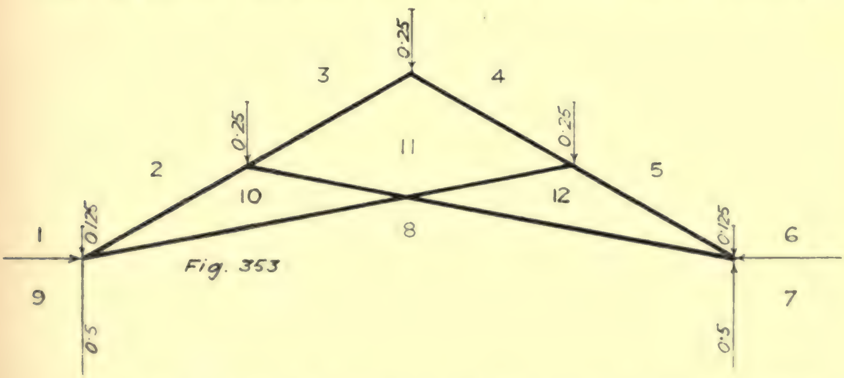


Fig. 353

5—12 ; 10—8, 12—8 ; 8—9, 1—9 ; and 8—7, 6—7 ; thus giving the horizontal thrusts at supports. With free bearings the stress diagrams cannot be directly worked as the tension in the two ties will produce

bending moments on the rafters. In order to find these bending moments an additional temporary member 9—10 must be added to the frame diagram as in Fig. 355, and the stress diagram for this will be as in Fig. 356. As the member 9—10 is non-existent, it must be replaced by virtual forces as in the frame diagram, Fig. 357, each force being equal to the amount of stress in the temporary member. The corresponding stress diagram will then be as in Fig. 358. Now in order to obtain the measure of the bending moments on the rafter, this virtual horizontal force must be resolved into two forces, one at right angles to the rafter and the other parallel with the rafter, as in Fig. 359, and the final stress diagram, Fig. 360, may be readily drawn out. The virtual forces 3—4 and 7—8 in Fig. 359 will then give the measure of the bending moments, the rafters being taken as beams

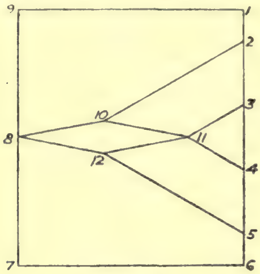


Fig. 354

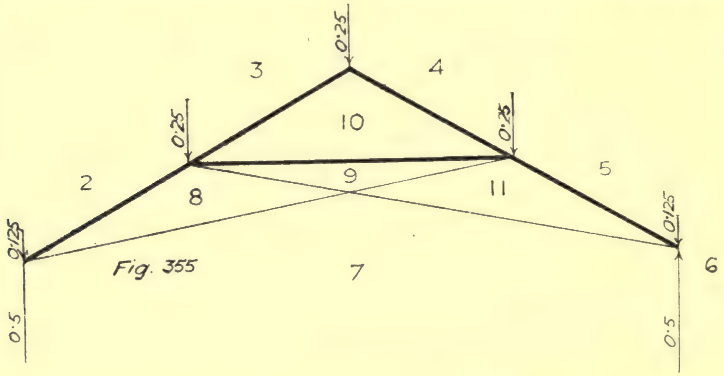


Fig. 355

supported at both ends with a concentrated load in the centre. In this case with a distributed vertical load of only 14 lbs. per sq. ft. of

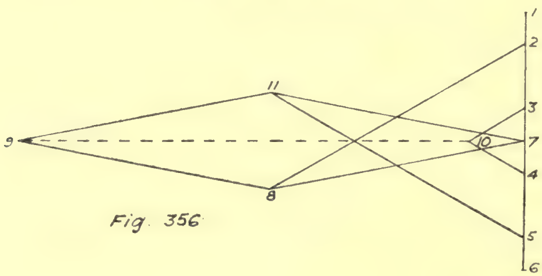


Fig. 356

sloping surface to allow for truss and covering, exclusive of wind, the cross bracing will require to be 4 in. by 2 in., and the principal rafters

10 in. by 6 in. A slight modification of this truss is shown in Fig. 361, with a king rod inserted, and the corresponding stress diagram will

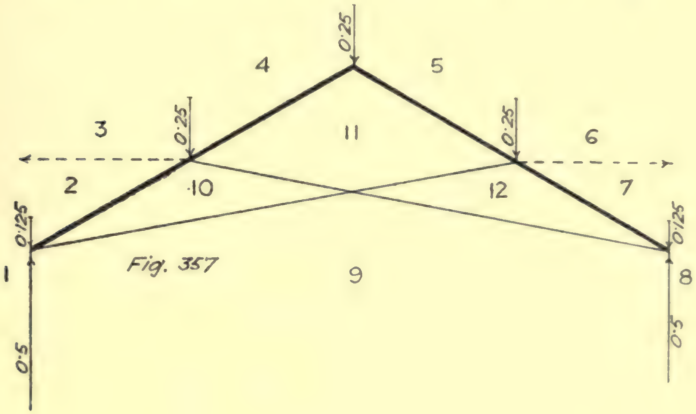


Fig. 357

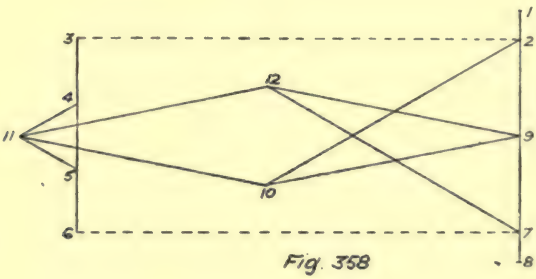


Fig. 358

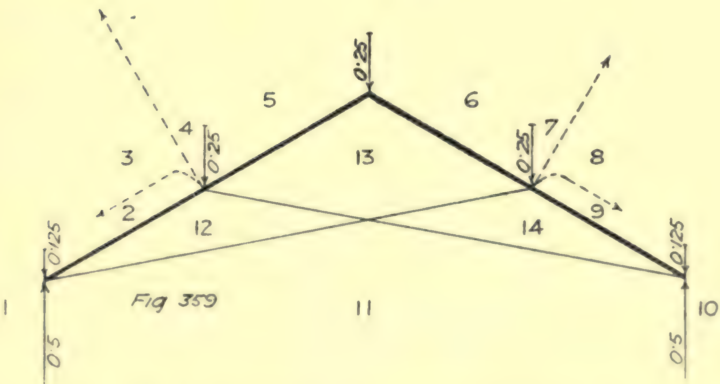


Fig 359

be as in Fig. 362, showing how an unsatisfactory form can be easily converted into a satisfactory one. A modified form of this truss, which has been used for church roofs, is shown in elevation, Fig. 363.

The frame diagram with vertical loading and rigid walls will then be as in Fig. 364, the additional member, 10*a*—11, being equivalent to the horizontal thrust to be taken by the supports. The stress diagram

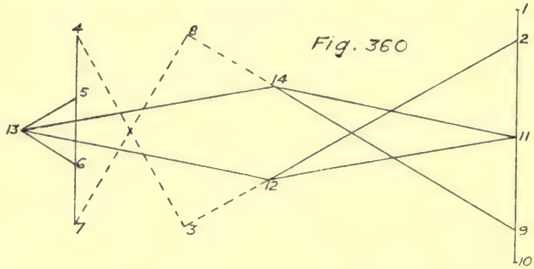


Fig. 360

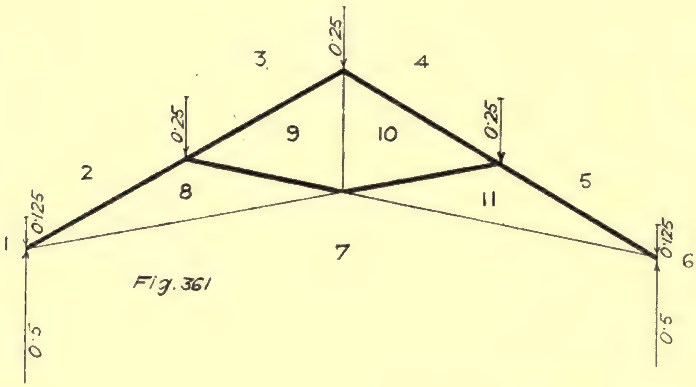


Fig. 361

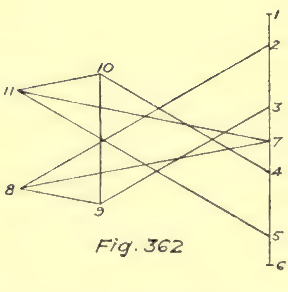


Fig. 362

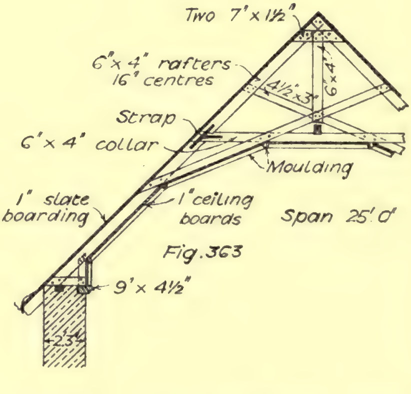


Fig. 363

presents no difficulty, and may be drawn out as in Fig. 365. If the supports are yielding, then bending moments will be produced on the rafters, and the stresses will be greatly increased. The frame diagram

will be as in Fig. 366, and in order to work out the stress diagram the bending moments must be replaced by virtual forces at right angles to

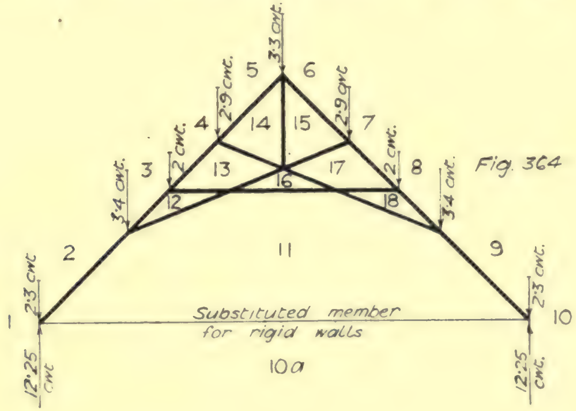


Fig. 364

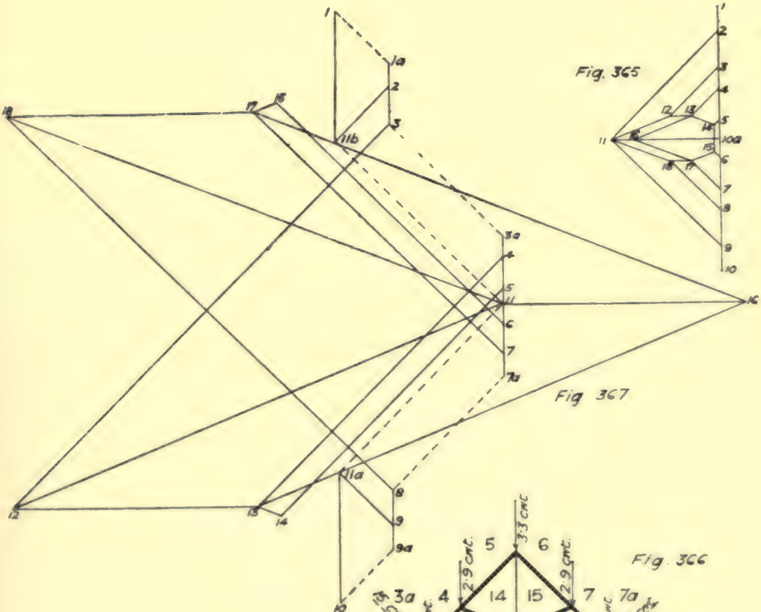


Fig. 365

Fig 367

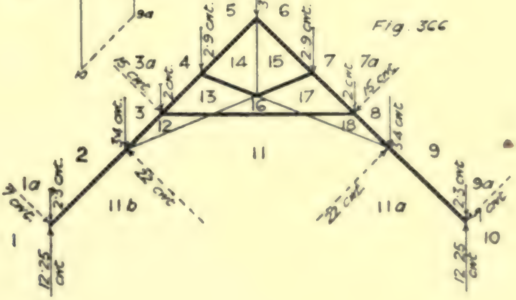


Fig. 366

the rafters. In order to find the value of these virtual forces take moments about the junction of the tie with the rafter, then virtual force $1\text{—}1a = \dfrac{(11b - 1 \times \text{its leverage}) - (1a - 2 \times \text{its leverage})}{\text{leverage of } 1\text{—}1a}$.

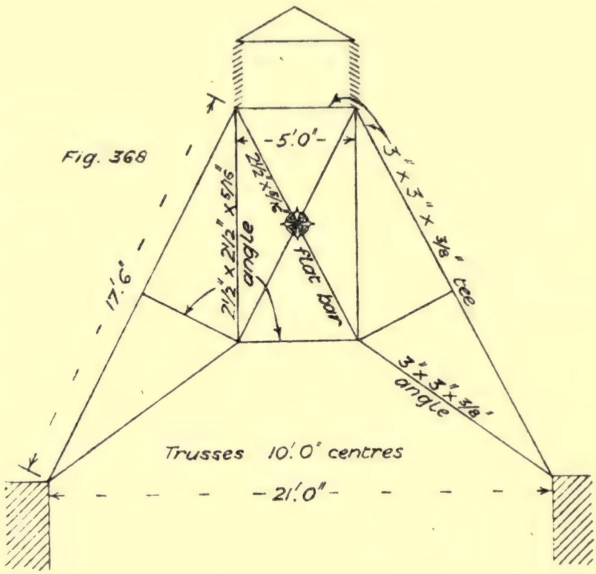


Fig. 368

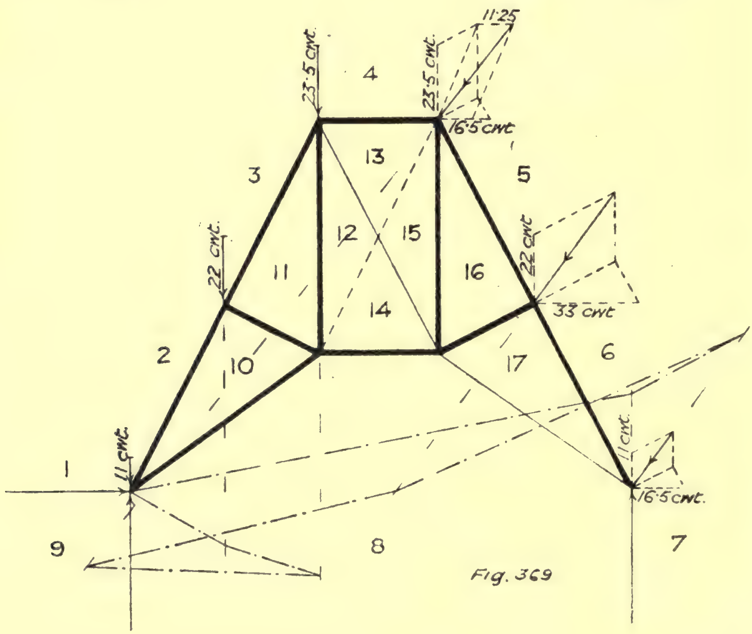
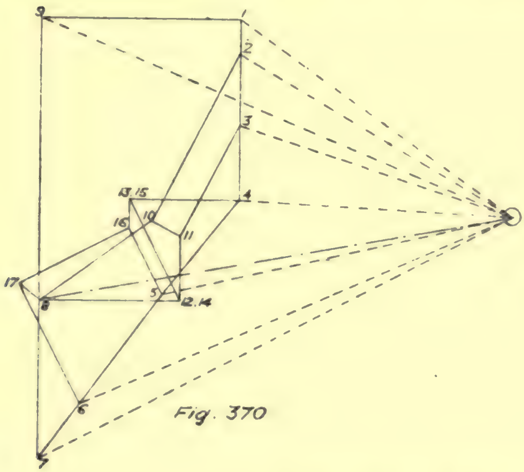


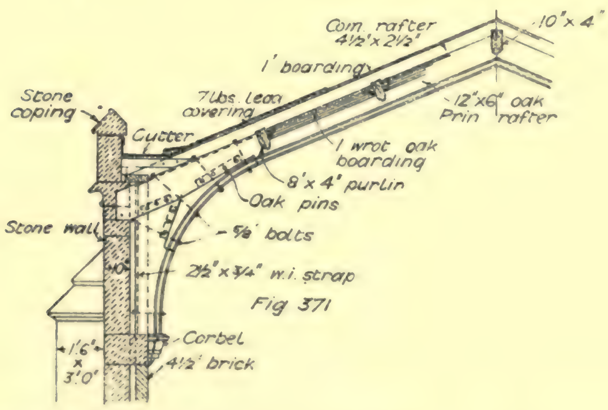
Fig. 369

Having obtained this force, 3—3*a* and 11—11*b* may be readily found, and the operations may be repeated for the other side ; in this case as the loading is symmetrical the virtual forces will also be symmetrical. The frame diagram, Fig. 367, may now be drawn out, and it will be seen that there is a large increase in stress in the majority of the members in addition to the bending moments produced by the virtual forces.

EXERCISES ON LECTURE XVI

Q. 59. Fig. 368 shows a skeleton outline of a roof truss for part of a brewery; draw frame and stress diagrams, allowing $2\frac{1}{2}$ cwt. per ft. run for the weight of the ventilator.

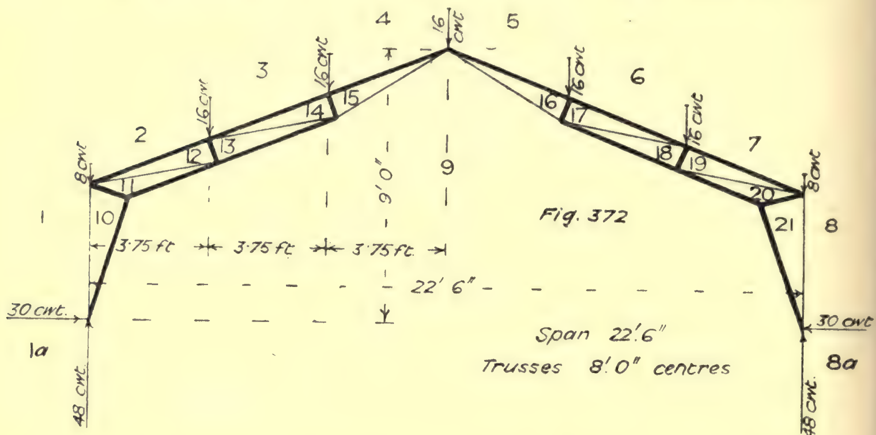




For answer, see Figs. 369, 370.

Q. 60. Fig. 371 shows oak ribs and purlins for roofing a small church; test the sufficiency of the scantlings.

For answer, see Fig. 372, frame diagram with substituted bracing; Fig. 373, stress diagram; Fig. 374, thrust in rafter; and Fig. 375, bending moment diagram. (From the frame diagram it will be seen that the roof divides itself into two portions, which may be assumed to be pivoted at the apex. Therefore the loads and reactions form two couples acting in opposite directions. The loads 1—2, 2—3, 3—4, and one-half of 4—5 are equal to and balanced by reaction 9—1a of 48 cwts., and the moment of this couple may be obtained by multiplying 48 cwts.



by 5·625 ft., which is the mean arm of the couple, giving $48 \times 5{\cdot}625 = 270$ cwt.-ft. This is balanced by the couple consisting of the force 1—1a, and the horizontal thrust at crown which will be the same value, acting with a leverage arm of

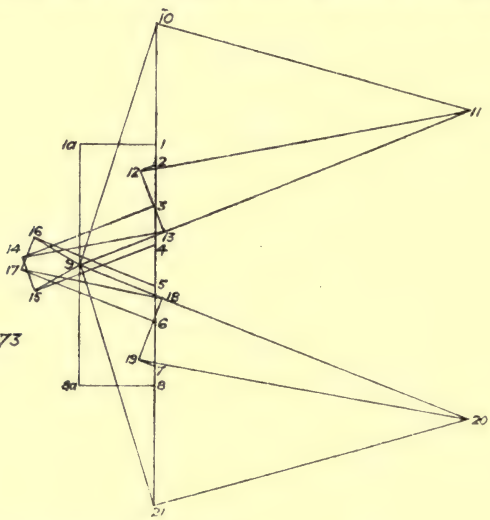


Fig. 373

9 ft. Therefore force $1—1a = \dfrac{270 \text{ cwt.-ft.}}{9 \text{ ft.}} = 30$ cwt. The load line may then be set down and the stress diagram completed without difficulty. In this case, however, the stress diagram appears to be of no assistance in calculating the

sufficiency of the scantlings. The thrust in the rafter may be obtained as shown in Fig. 374. The bending moment diagram for the rafter is next obtained as in Fig. 375, and the maximum is found to be 720 cwt.-in. Then by the formula $\frac{W}{A} + \frac{M}{Z}$ the maximum stress will be $\frac{32 \cdot 4}{54} + \frac{720}{81} = 0 \cdot 6 + 8 \cdot 9 = 9 \cdot 5$ cwt. per sq. in.

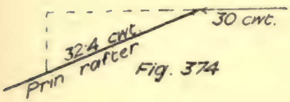


Fig. 374

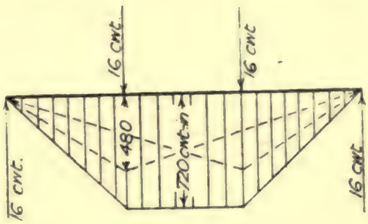


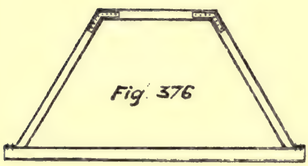
Fig. 375

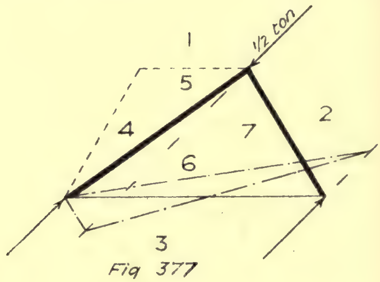
compression; and taking oak as crushing with 3·2 tons per sq. in. (as given by Hurst), $\frac{3 \cdot 2 \times 20}{9 \cdot 5} = 6 \cdot 73$ factor of safety. The size of the rafter is taken as 9 in. by 6 in. instead of 12 in. by 6 in. owing to the underside being moulded.

LECTURE XVII

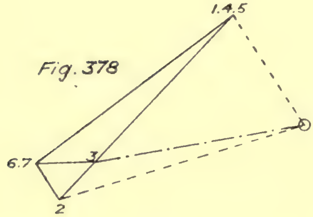
Roof Trusses formed of Bent Ribs Stiffened at Joints, with and without Tie—Calculation of Joints.

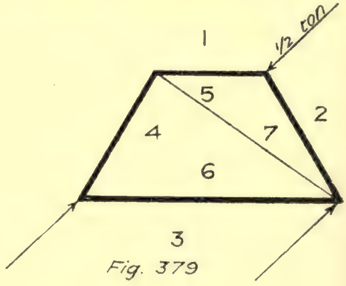
In dealing with bent-rib trusses it will be well to consider first a simple frame, as Fig. 376. It is clear that at least one additional member must be inserted to enable a stress diagram to be drawn. This would be a diagonal from one side of top to opposite side of bottom, and when





the wind blows to meet the diagonal it would put it in compression, but when it blows from the opposite side it would put it in tension. To avoid this change of stress it might be desirable to put two diagonals





both designed for tensile stress only, so that whichever way the wind blew there would be a member to take the whole diagonal stress by tension. The diagonals may, however, be arranged to each take half the duty, being in either tension or compression according to the side

from which the wind was blowing. To ascertain the stresses in that case the frame might be dealt with in two stages; first as in Fig. 377 with half the total load and the diagonal in compression, giving the stress diagram, Fig. 378; then as in Fig. 379, with the other half of the load and the other diagonal in tension, giving the stress diagram, Fig. 380. The combination of these two would give the frame diagram, Fig. 381, and stress diagram, Fig. 382, but it must be understood that if the diagonals are designed to resist tension only, the frame diagram would

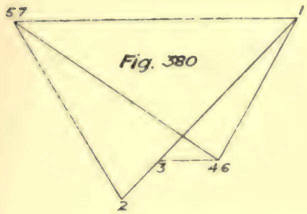


Fig. 380

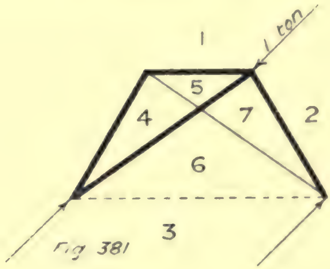


Fig 381

be as Fig. 379 and the stress diagram as Fig. 380, with all the stresses and reactions doubled.

The calculations of the bending moments on the joints had better be left for consideration with the next example.

A convenient form of roof truss for factories, involving some special considerations of stress, is shown in part elevation in Fig. 383. It

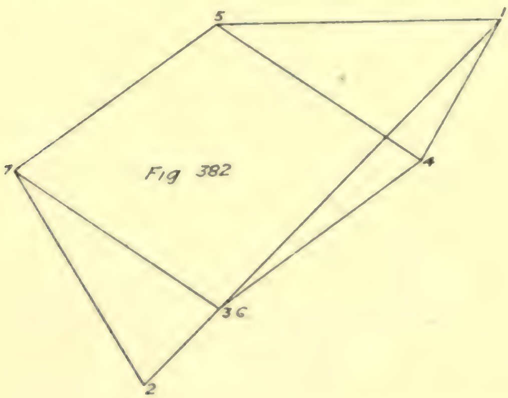


Fig 382

consists of a rolled-joist tie forming floor girder, and double channel iron rafters with gusset plates at the angles. To obtain a stress diagram additional members must be assumed temporarily as in the frame diagram, Fig. 384, for which Fig. 385 is the corresponding stress diagram, but this diagram cannot be worked out straight away; the frame diagram must be divided up into three parts, as in Figs. 386, 387,

and 388, taking one-third of the total load on each. The corresponding stress diagrams will then be as in Figs. 389, 390, and 391, from which

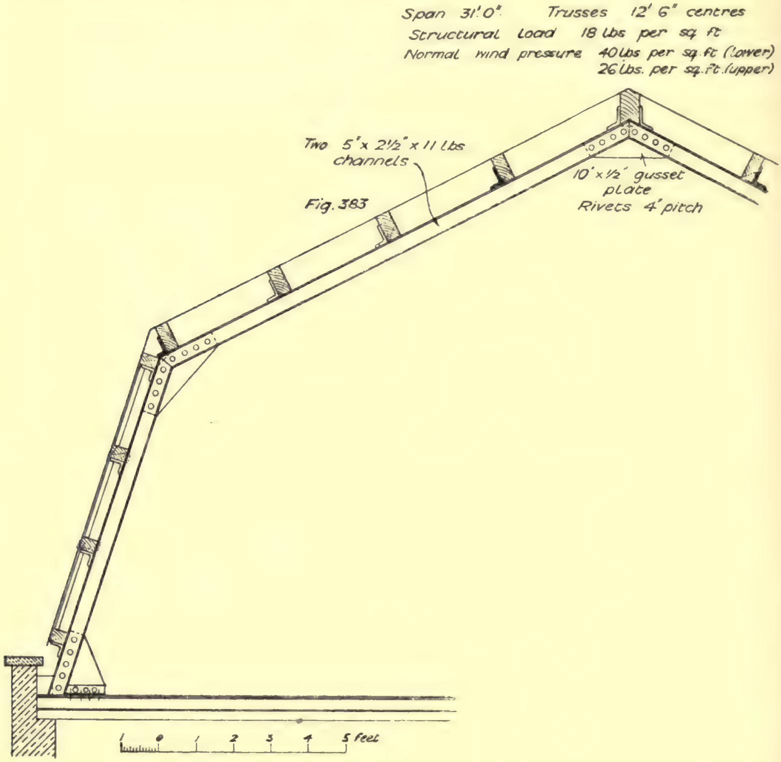


Fig. 383

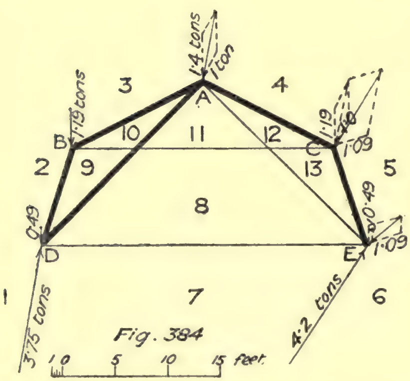


Fig. 384

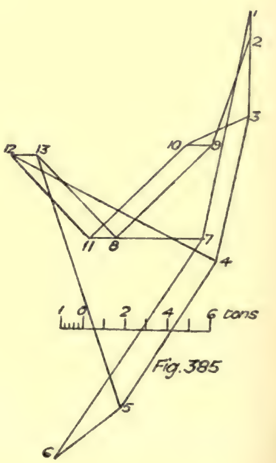


Fig. 385

the complete stress diagram, Fig. 385, to one-third the scale of the others may be constructed. The effect of the additional members will be to avoid bending moments on the joints A, B and C, the bending moments when they are removed being equal to the stress in the member

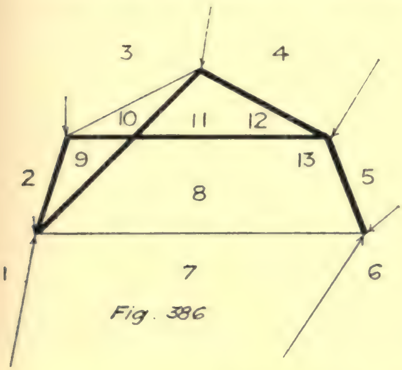


Fig. 386

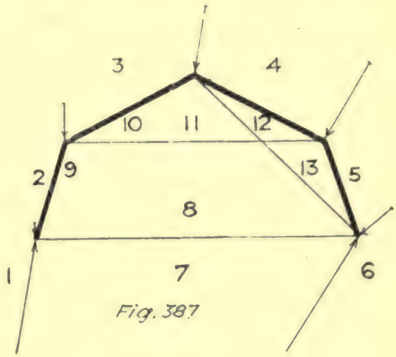


Fig. 387

multiplied by the perpendicular distance to the neutral axis of the joint, and so far it is assumed that no stress is taken by the joints D and E. Next, assuming that the joints A, D, and E take the stress, the frame diagram will have to be as in Fig. 392, and the corresponding stress

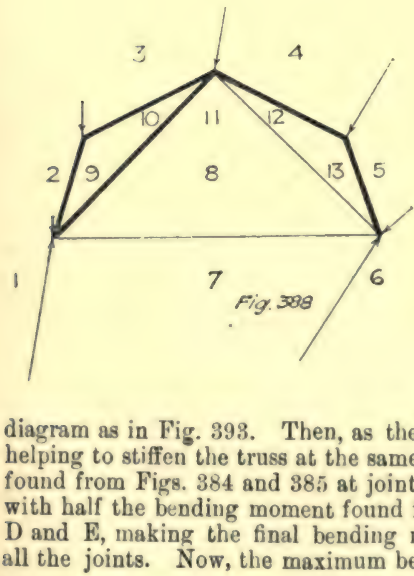


Fig. 388

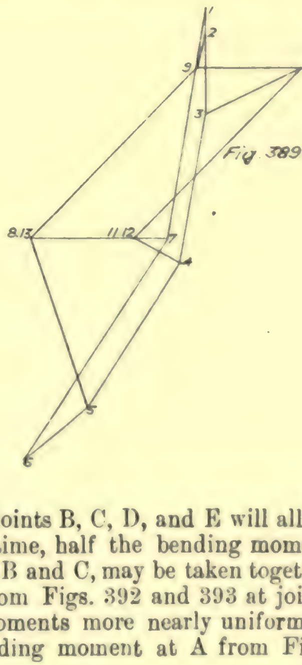


Fig. 389

diagram as in Fig. 393. Then, as the joints B, C, D, and E will all be helping to stiffen the truss at the same time, half the bending moment found from Figs. 384 and 385 at joints B and C, may be taken together with half the bending moment found from Figs. 392 and 393 at joints D and E, making the final bending moments more nearly uniform at all the joints. Now, the maximum bending moment at A from Figs.

384 and 385 will be $0{\cdot}4\dot{3}$ ton × 72 ins. = 31·2 ton-ins., at joints B and C, 2·1 tons × 52 ins. = 109·2 ton-ins., and taking half this = $\frac{109{\cdot}2}{2} = 54{\cdot}6$ ton-ins. The maximum bending moment at A from Figs. 392 and 393 will be 0·57 ton × 72 ins. = 41·04 ton-ins., and at joints D and E, 1·12 ton × 108 ins. = 120·96 ton-ins., and taking half = $\frac{120{\cdot}96}{2} = 60{\cdot}48$ ton-ins.

It will be desirable in such cases as this to make the section of the rafters uniform throughout, so that after assuming a probable section the maximum stresses and moments must be allowed for by the formula $\frac{W}{A} \pm \frac{M}{Z}$, the result being kept within safe limits. Similarly it will be desirable to have the gusset plates uniform ; the maximum case may,

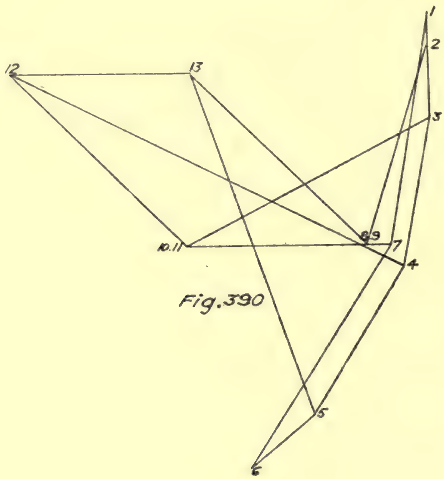


Fig. 390

therefore, be taken and calculated as follows. In principle the gusset is as Fig. 394, the stress being a maximum at outer edge and nil at apex of joint. Taking a cross section through A—B the variation of stress will be as ordinates to a triangle (Fig. 395), the centre of effort being at two-thirds the breadth of gusset, which will therefore be the lever arm. Then if the maximum stress in compression be fixed at 5 tons per sq. in. and the thickness of gusset as $\frac{1}{2}$ in., the maximum resistance per inch wide will be $5 \times \frac{1}{2} = 2\frac{1}{2}$ tons. Then $2\frac{1}{2}$ reducing to nil gives an average of $1\frac{1}{4}$ tons per inch. Let x = breadth of gusset, then the moment of resistance will be $1\frac{1}{4} \times x \times \frac{2}{3}x = \frac{5}{6}x^2$, and the greatest bending moment being 60 ton-ins., $x = \sqrt{\frac{60 \times 6}{5}} = 8{\cdot}485$, say 9 ins., from the centre of the joint, or a total of, say, 10 ins. This truss without a tie would have the reactions in Fig. 384, combined with

the forces substituted for the tie and producing combined reactions as in Fig. 396, which may be converted to vertical and horizontal forces,

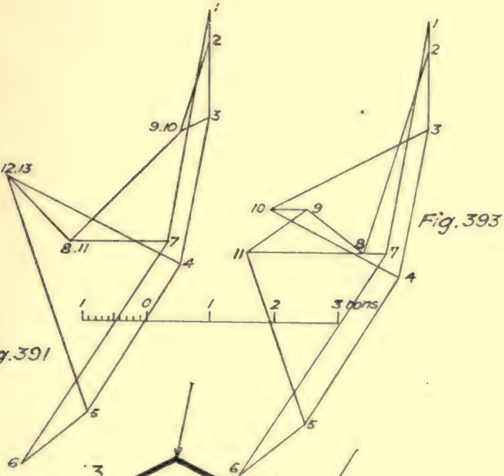


Fig.391

Fig.393

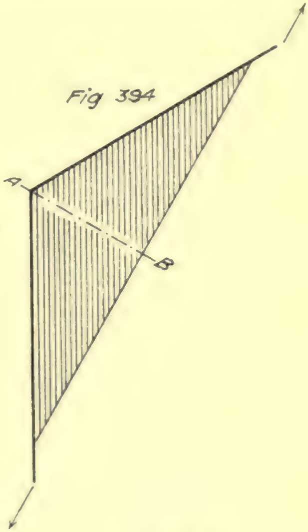


Fig 394

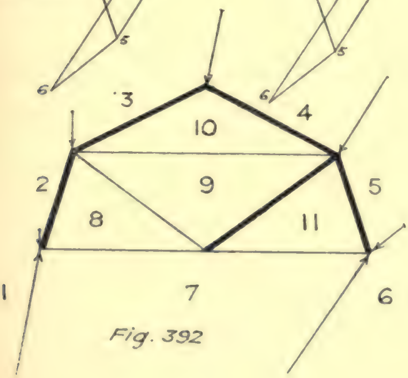


Fig. 392

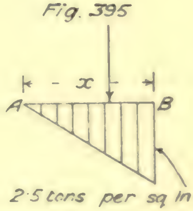


Fig. 395

Fig. 396

the former giving the direct loads and the latter the overturning force on the walls. The bending moments and other stresses would be those

due to Fig. 384, and not to Fig. 392, because the bottom gussets would be absent. If the walls are not rigid, it is the same as removing the

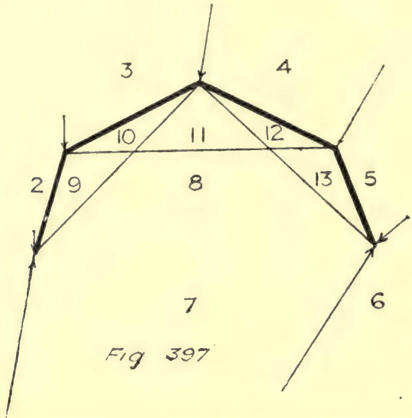
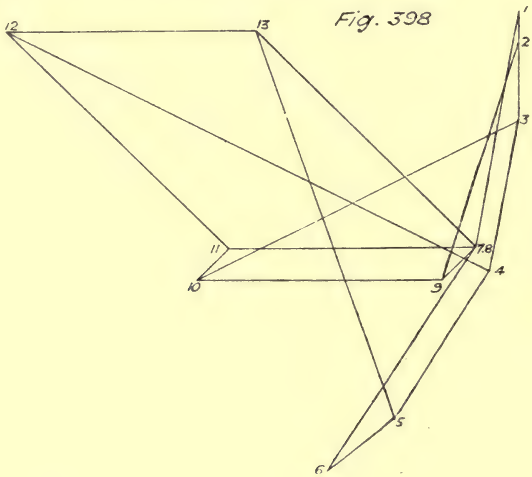


Fig 397

horizontal forces, and increasing the bending moments on the joints. Fig. 397 will give the frame diagram for this case, and Fig. 398 the stress diagram, which can be drawn direct.

Fig. 398



EXERCISES ON LECTURE XVII

Q. 61. Draw the stress diagram for the truss shown in Fig. 399. Span 25 feet, trusses 7 ft. 6 in. centres, structural load 21 lbs. per ft. sup., and horizontal wind pressure 42 lbs. per ft. sup.

For answer, see Figs. 400 to 403.

(The frame diagram with loading will be as in Fig. 400. Now, as the truss

may be considered as pivoted at the supports and at A and B, in order to obtain a stress diagram virtual forces must be introduced at the latter points. To obtain these forces divide the truss into the two parts as in Figs. 401 and 402, taking into account the amount of load 4—5 coming on to A and B. The reactions on these two parts may then be found as shown, giving the two virtual forces and the reactions on the supports which take account of the reactions from the virtual

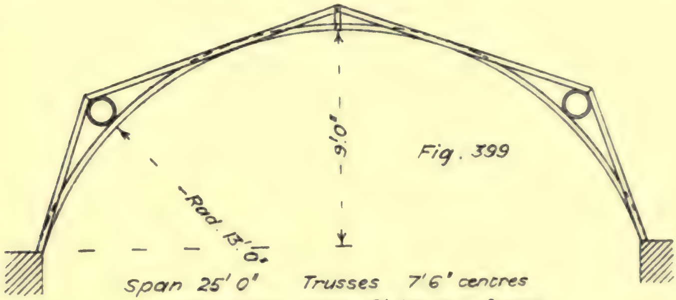


Fig. 399

Span 25' 0" Trusses 7'6" centres
Structural Load 21 Lbs. per ft sup
Horizontal wind pressure 42 Lbs per ft. sup

forces. The stress diagram, Fig. 403, may now be drawn out and completed. To find the true reactions the amounts coming on to the supports from the virtual forces must be subtracted from the reactions found by Figs. 401 and 402.

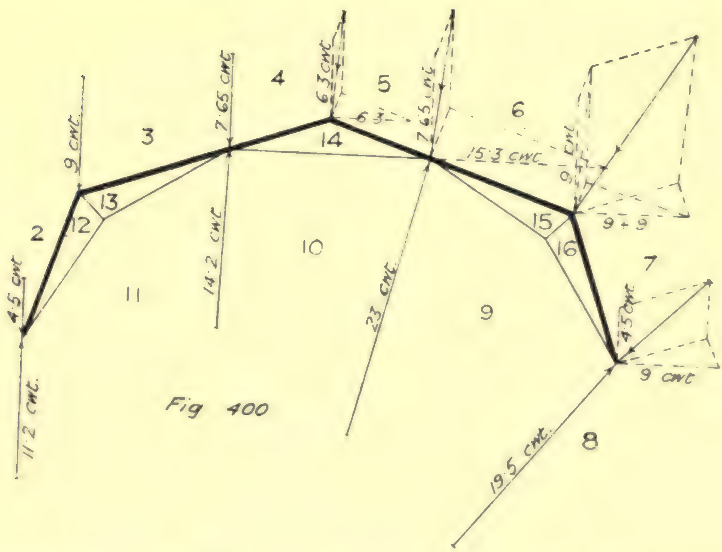


Fig 400

The method of doing this is shown in Fig. 404, and the true reactions will be as shown by the dotted lines in Fig. 403.)

Q. 62. Fig. 405 shows the line diagram of an elliptical roof truss 20 ft. span, 7 ft. 6 in. rise, and 10 ft. centres. Draw the frame and stress diagrams, allowing for a vertical load of 42 lbs. per ft. sup.

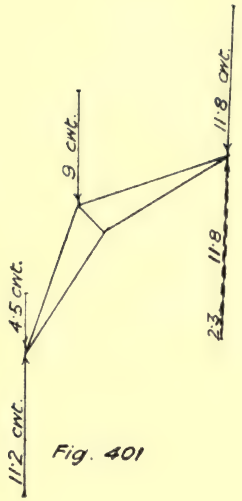


Fig. 401

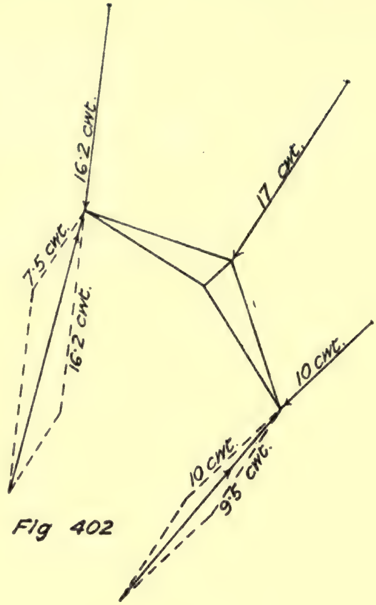


Fig 402

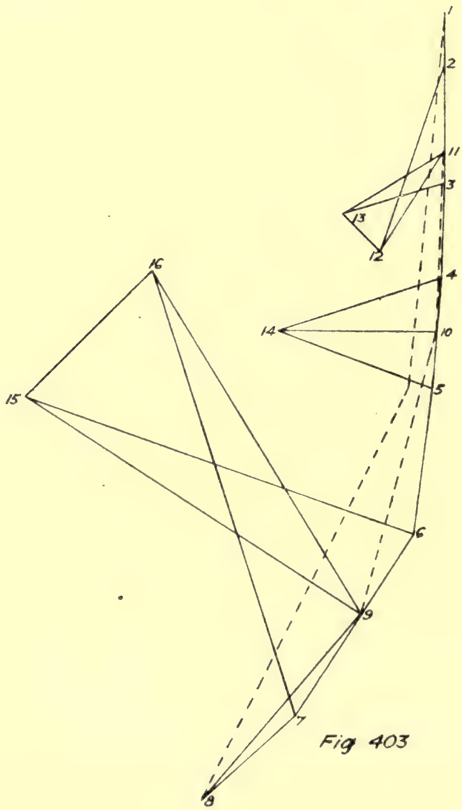
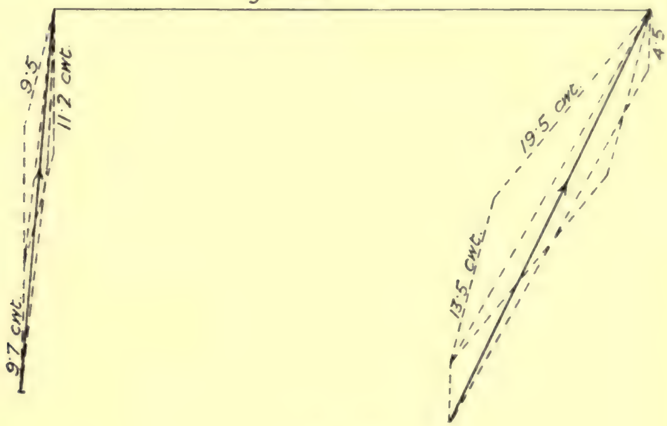


Fig 403

For answer, see Figs. 406 and 406*a*.

(NOTE.—The bending moment on the principal rafter is found by multiplying the stress in the member by the perpendicular distance to the curve, and the maximum stress is worked by the formula $\frac{W}{A} \pm \frac{M}{Z}$.)

Fig. 404



Span 20′ 0″
Trusses 10′ 0″ centres
Rise 7′ 6″

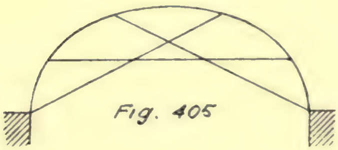


Fig. 405

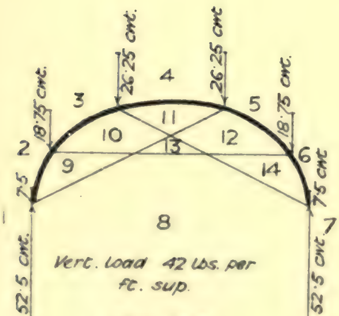


Fig. 406

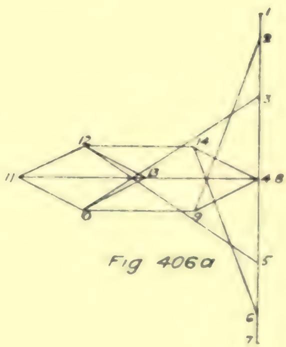


Fig 406a

LECTURE XVIII

Braced Arch Roof Trusses—Arched Rib Truss—Comparison of Stresses.

In dealing with arched ribs, or bent-rib roof trusses, it will be well to commence with a braced arch as Fig. 407. It will simplify the case if it be assumed semicircular, and the bracing should be about 45 degrees to a radial line. Owing to this example being semicircular and the

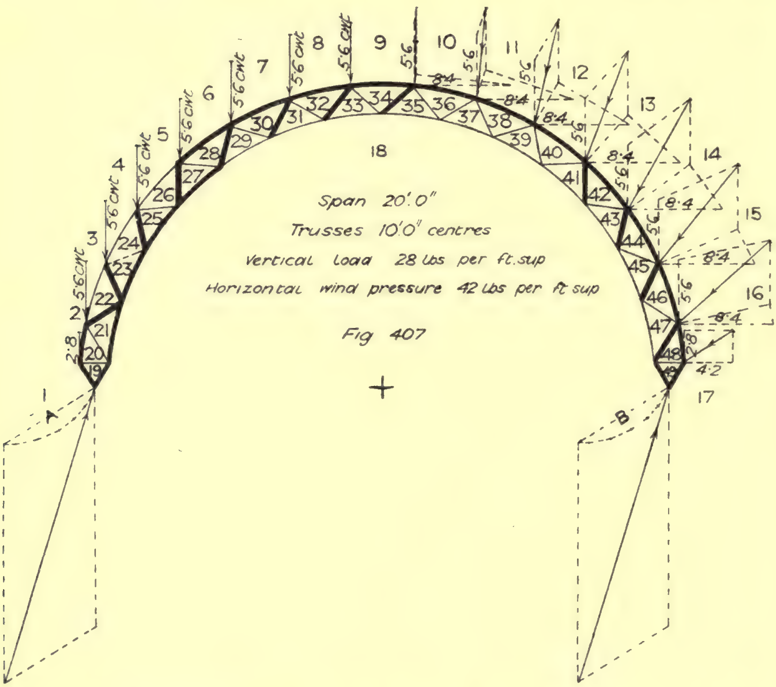


Fig 407

resultant of wind pressure on each portion being normal to the curve, all the resultants will pass through the centre and produce an equal pressure on each support; the structural load being vertical and symmetrical will also produce equal loading on the supports. The reactions may therefore be obtained by joining the extremities of the load line and bisecting, otherwise it would have been necessary to

calculate separate reactions for each differently sloping force, and the reactions would then be joined continuously to give final resultant as shown at A and B, Fig. 407. The stress diagram, Fig. 408, can be set

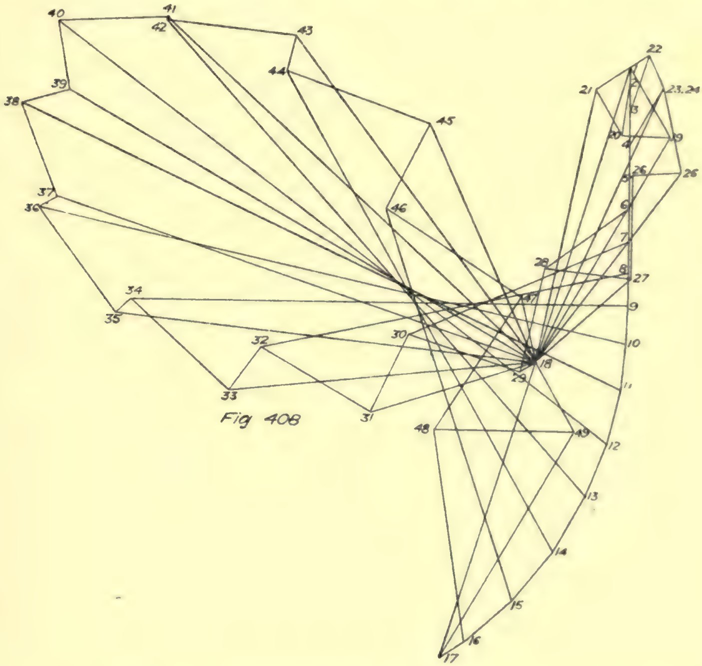


Fig 408

out in the usual way, but it should be worked from both ends, as otherwise there will probably be a difficulty in getting the figure to

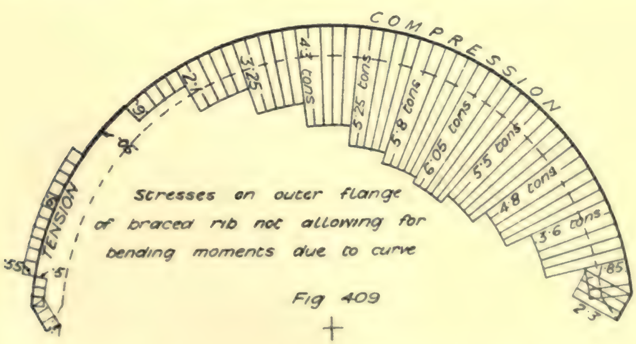


Fig 409

close, owing to the risk of small errors creeping in, due to the shortness of the lines to which parallels have to be drawn.

In order to compare the stresses with those obtained by treating the

truss as a pure arch, it will be useful to set off the stresses in the bars radially, as in Fig. 409 for outer flange, and Fig. 410 for inner flange.

Now the investigation of the case as a linear arch may be proceeded with. Draw an elevation as Fig. 411, add the resultants of the

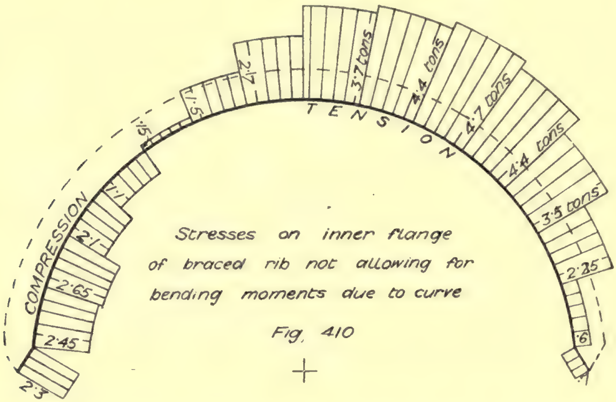


Fig. 410

structural and external loads, and set down the load line, Fig. 412. Join the extremities and bisect to give point 18, this being permissible for the reasons described above. Then draw a horizontal line both ways through point 18, and from point 2 draw a line parallel with 2–20

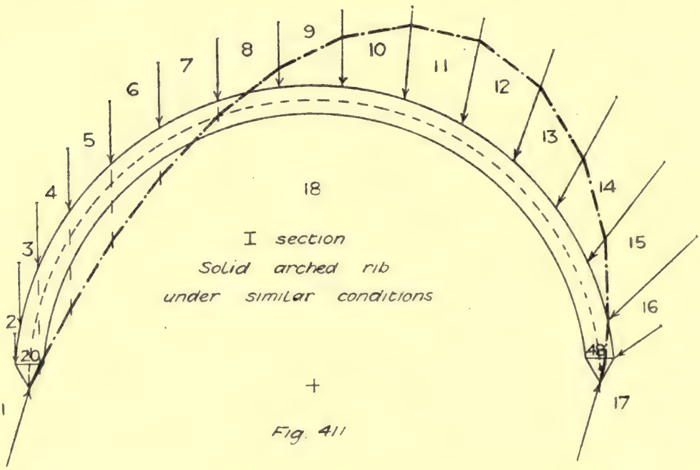


Fig. 411

(Fig. 411), and from point 16 a line parallel with 16–48 (Fig. 411), bisect 20–48 in Fig. 412, giving approximate position of pole for obtaining the best curve of thrust. Draw the vectors from pole to all the points on load line, and parallel with these draw the curve of thrust in Fig. 411. Professor Goodman says in his "Mechanics Applied to Engineering" that with irregular loading an infinite number of curves of

thrust may be drawn, but that the true curve will be the one whose ordinates give equal areas inside and outside the linear rib. Judging by the result of the braced rib this does not appear to the author to be exactly correct, and the mode of working shown in Fig. 412 is put forward as being the best working approximation.

In order to compare the maximum stresses produced in the braced

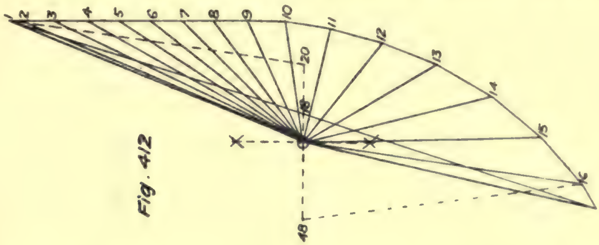


Fig. 412

rib with those given by the method now under consideration, set out on Figs. 413 and 414 the centre line of rib and radial ordinates obtained by the formula $T \pm \frac{M}{D}$, where T = thrust across each space respectively, M = bending moment given by the product of the thrust T into its perpendicular distance from the centre of the space on neutral axis of rib, and D = depth of rib between centres of gravity of flanges. Now for the stresses in the flanges, draw elevation curves as in Fig. 413 and Fig. 414, notice the position of curve of thrust in Fig. 411, and that when it passes within the line of rib the outer flange will be tension and the inner compression, and that when it passes outside the line of

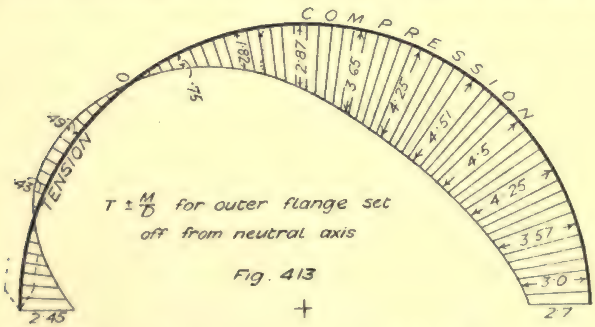


Fig. 413

rib the outer flange will be compression and the inner tension. This will indicate whether the + or − value in the formula should be given as the ordinate indicating the stress in the outer and inner flanges respectively. The regularity of the curves shows probable accuracy of calculation and plotting. Comparing Figs. 409 and 410 with Figs. 413 and 414, the similarity of stress will be apparent, although the actual amounts do not agree. It looks as if the line of thrust in Fig. 411 should be a trifle more eccentric to make the stresses in the two methods of working more nearly alike.

In comparing the maximum stresses produced in the flanges of a braced girder and a solid web girder, it will be found that while in the latter they are given by the formula $\frac{WL}{8D}$, in the former they vary according to the number of bays made by the lattice bars, being less in

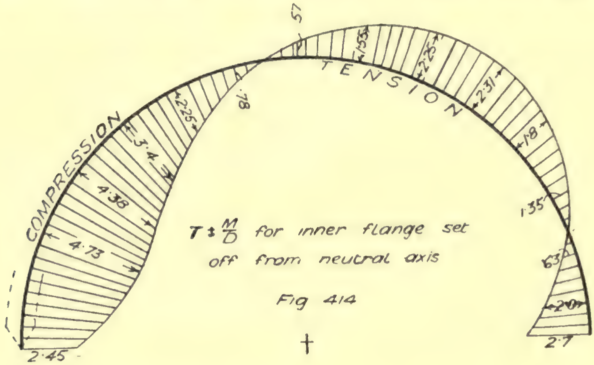


Fig 414

every case, but approaching the value of the solid web as they become more numerous. In the case of the arched rib this relation does not seem to hold good, as the flange stresses are, on the whole, shown to be greater in the braced rib.

EXERCISES ON LECTURE XVIII

Q. 63. Fig. 415 shows cast iron roof principals 36 ft. span and 10 ft. rise, placed at 15 ft. centres, over a warehouse in Manchester, and carrying the roof by

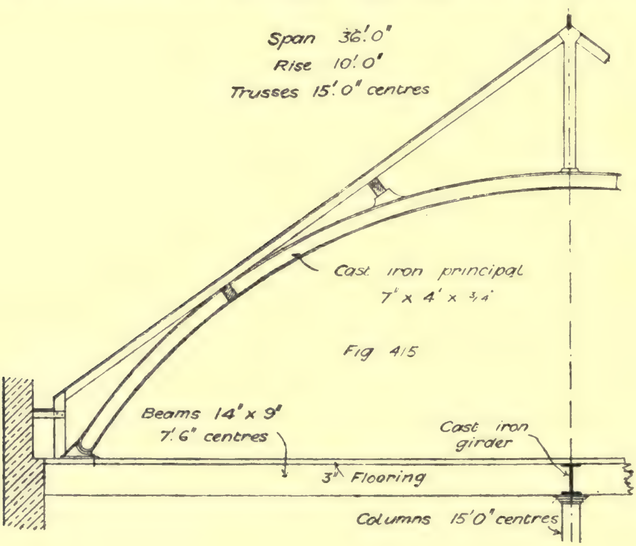


Fig 415

four symmetrically placed purlins and a strut from the ridge. Each rib is of I section, 7 in. by 4 in. by $\frac{3}{8}$ in. thick, with pivot at each end, fixed to 14 in. by 9 in. floor beam. The load from the purlins may be taken as 3 tons vertical on the windward side and $1\frac{1}{2}$ tons on the other side, the ridge transmitting the mean of these or $2\frac{1}{4}$ tons. Find graphically the reactions and curve of thrust and check the value of thrust and reactions by calculation.

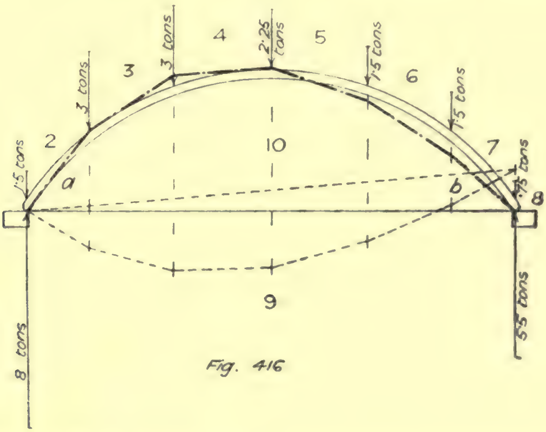


Fig. 416

For answer, see Fig. 416 and Fig. 417.

Set down the load line 1 to 8 in Fig. 417 and select any pole 0, draw vectors, and construct the funicular polygon in Fig. 416, the closing line of which will give point 9 in Fig. 417. From this point 9 draw a horizontal line, and from point 2 draw $2a$ parallel with the curve tangent $2a$ in Fig. 416, and from point 7

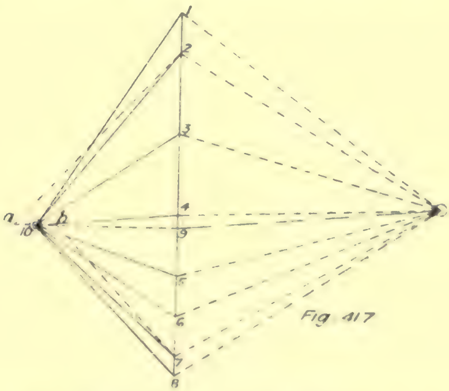


Fig 417

draw $7b$ parallel with $7b$. Bisect ab to give point 10, and draw vectors from point 10 to the various divisions on the load line, these will then give the directions for the curve of thrust crossing each space. By calculation the horizontal component of the thrust in the arch due to the central load W will be $\frac{Wl}{4v}$, where l = span and v = rise, $\frac{2{\cdot}25 \times 36}{4 \times 10} = 2{\cdot}025$. The additional horizontal thrust at

foot caused by a load W at a distance x from centre will be $\frac{W(l - 2x)}{4v}$. This will also be the increased thrust at crown.

$$\frac{3(36 - 2 \times 7{\cdot}33)}{4 \times 10} = 1{\cdot}6, \frac{3(36 - 2 \times 13{\cdot}83)}{4 \times 10} = 0{\cdot}625.$$

Then from the other side

$$\frac{1{\cdot}5(36 - 2 \times 7{\cdot}33)}{4 \times 10} = 0{\cdot}8, \frac{1{\cdot}5(36 - 2 \times 13{\cdot}83)}{4 \times 10} = 0{\cdot}313,$$

$2{\cdot}025 + 1{\cdot}6 + 0{\cdot}625 + 0{\cdot}8 + 0{\cdot}313 = 5{\cdot}363$ tons total horizontal thrust on each side. For the vertical thrust or reaction on the left we have $(3 \times 31{\cdot}83 + 3 \times 25{\cdot}33 + 2{\cdot}25 \times 18 + 1{\cdot}5 \times 10{\cdot}67 + 1{\cdot}5 \times 4{\cdot}17) \div 36 = 6{\cdot}5 + 1{\cdot}5 = 8$ tons, and on the

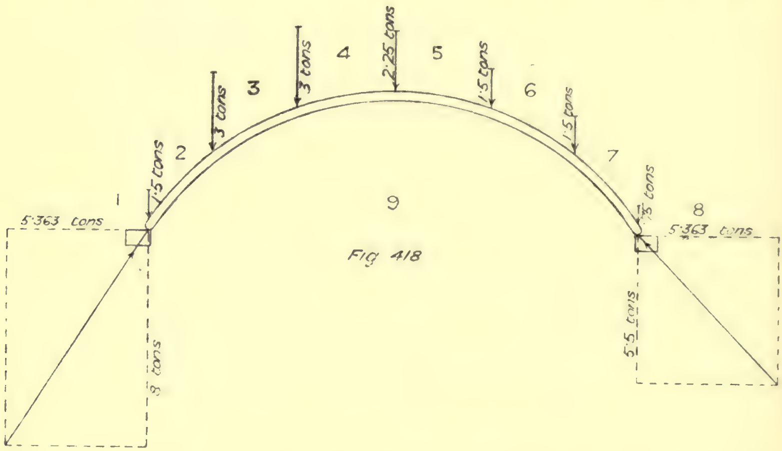


Fig 418

right $3 + 3 + 2{\cdot}25 + 1{\cdot}5 + 1{\cdot}5 - 6{\cdot}5 = 4{\cdot}75 + 0{\cdot}75 = 5{\cdot}5$ tons. Also as a check upon the horizontal thrust we have, by taking moments about the centre, $(8{\cdot}0 \times 18 - 1{\cdot}5 \times 18 - 3 \times 13{\cdot}83 - 3 \times 7{\cdot}33) \div 10 = 5{\cdot}352$ tons. Without a tie the reactions would be as Fig. 418, but the tie being substituted for the horizontal reactions leaves only vertical reactions as in Fig. 416.

LECTURE XIX

Stability of Walls—Overturning on Edge—Pressure on Base according to position of Resultant—Safe Stresses on Materials—Boundary or Fence Walls—Wall with attached Piers.

In considering the stability of walls it is usual to take a length of one foot as representative of the whole. The simplest form of calculation is that for overturning and it conveniently illustrates some of the general principles. Fig. 419 represents an ordinary 9-in. boundary wall, or fence wall. If failure takes place by wind pressure while the mortar is green it will presumably overturn at the joint just above the ground line. In Fig. 420 the upper portion of the wall is shown to a larger scale. The total force of the wind multiplied by the distance from the centre of effort to the joint under consideration is the moment of effort. The moment of the resistance is given by the weight of the wall multiplied by the leverage on which it acts, that is, half the thickness. Then if p = wind pressure lbs. per sq. ft., w = weight of wall in lbs. per cubic ft., h = height of wall in feet, t = thickness of wall in *feet*, for equilibrium,

$$p = \frac{wt^2}{h},\ h = \frac{wt^2}{p},\ t = \sqrt{\frac{ph}{w}},$$

and the parallelogram of forces will show the diagonal passing exactly through the outer edge of joint. If we consider the wind pressure which will just overturn the wall as the measure of stability, then the stability of such a wall varies as $\frac{wt^2}{h}$, or directly as the weight per cubic foot, inversely as the height, and directly as the square of the thickness.

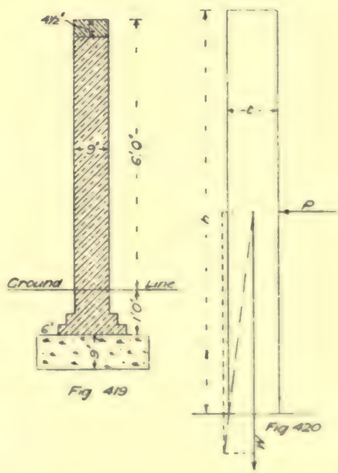


Fig 419

Fig 420

Failure may also take place by sliding when the ratio of thickness to height is equal to or less than the coefficient of friction. The coefficient of friction for fresh mortar is variously stated as 0·5 to 0·75. The shearing strength of old mortar is about ⅔ ton per sq. ft. The safe load in compression under ordinary conditions on—

Blue brick in cement	9 tons per ft. sup.
Stock „ „	6 „ „
„ „ lias mortar	5 „ „
„ „ grey lime	3 „ „
Cement concrete (6 to 1)	5 „ „
Lias lime concrete (4 to 1)	3 „ „

The safe load in tension on—

Brickwork in cement (fresh) . . .	3 tons per ft. sup.
„ 6 months old	6 „ „
„ lias mortar (fresh) . . .	$\frac{3}{4}$ „ „
„ „ 6 months old . . .	$1\frac{1}{2}$ „ „
„ grey lime mortar (fresh) .	$\frac{1}{2}$ „ „
„ „ 6 months old . . .	1 „ „

The pressure on foundations should not as a rule exceed for

Gravel and compact earth	2 tons per sq. ft.
Ordinary subsoil . . .	$1\frac{1}{2}$ „ „
Made ground (6 months)	$\frac{3}{4}$ „ „

As walls are not built to be overturned the calculation of their stability should be limited to working conditions. A plain rectangular

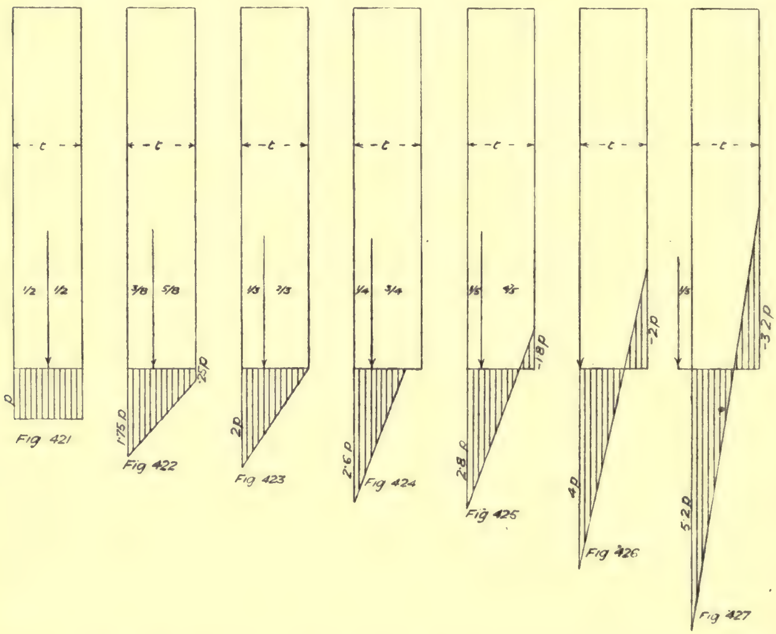
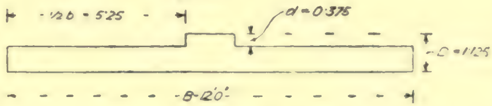


Fig 421 Fig 422 Fig 423 Fig 424 Fig 425 Fig 426 Fig 427

wall, as Fig. 421, subject only to its own weight will produce a uniform pressure on the base $= \frac{W}{A}$, as shown by the ordinates, or where the weight per cub. ft. w and the height h are given, the intensity of the pressure will be $= wh$. If, however, a horizontal force acts against

the wall at any point it will cause the resultant to deviate from the centre, increase the intensity of the pressure on the side toward which the resultant approaches and reduce it on the opposite side, as in Fig. 422. The sum of the ordinates, or the area of the figure containing them, will remain unaltered, because no change has been made in the vertical loading. When the resultant reaches the edge of the middle third of the base, as in Fig. 423, the ordinates of pressure will

Fig 428



form a triangle having twice the depth of the parallelogram, and it is generally stated that this position is the limit of safety. It is an incorrect expression to use, the limit of the middle third only signifies that there will be no tension on the inner edge, the compression on the outer edge will be according to the load, and may readily exceed the limit of safety. On the other hand, the middle third may often be passed without exceeding the limit of safety. Suppose no tensile strength or adhesion at the base of wall, and the resultant to be pushed over to $\frac{1}{4}t$ from outer edge, as in Fig. 424, then by Prof. Crofton's theory the figure containing the ordinates will be a triangle having a base of 3 times the distance from resultant to outer edge and a maximum depth equal to the load divided by $1\frac{1}{2}$ times the distance. The simplest form of the expression is $p = \frac{2}{3} . \frac{W}{d}$. Whether the resultant be shifted over by a horizontal force, or be due to unsymmetrical loading, or to a thrust at any angle, the vertical component of the resultant must be taken as W and the distance from where it cuts the base to the outer edge as d.

When the wall has tensile strength, it does not come into play until the resultant passes the middle third, as in Fig. 425, and the maximum and minimum pressures are then found by the formula $\frac{W}{A} \pm \frac{M}{Z}$, where W = the vertical load or vertical component of the resultant, A = sectional area of base, M = bending moment, or product of the load into its distance from the centre of gravity of the base, Z = section modulus of the base. For 1 foot run of a simple wall, $Z = \frac{bd^2}{6}$ becomes $\frac{1}{6}t^2$. The same units must be retained throughout, whether tons, cwts., or lbs., and feet or inches. When the resultant reaches the outer edge, as in Fig. 426, the maximum compression will be four times the mean pressure under central load and the maximum tension twice the mean pressure. The same formula applies when the resultant falls beyond the base, but the stresses are greatly increased, as in Fig. 427.

To find the wind pressure (p) per square foot on face of wall to produce any given maximum pressure (K) per square foot on base without tension on inner edge $\frac{W}{A} + \frac{M}{Z} = K$, or $K = wh + \frac{M}{\frac{1}{6}t^2}$, but when

the resultant is at edge of middle third $K = 2wh$, therefore $\frac{M}{\frac{1}{6}t^2} = wh$, or $M = \frac{1}{6}wht^2$, also $M = ph \times \frac{1}{2}h$, therefore $\frac{1}{2}ph^2 = \frac{1}{6}wht^2$, or $ph = \frac{1}{3}wt^2$, whence $p = \frac{wt^2}{3h}$. The overturning pressure will, therefore, always be three times the pressure by the middle-third rule.

When a wall is buttressed, a unit length from centre to centre of the panels must be taken as one piece in the calculations and the neutral axis and moment of inertia found as for a tee section, the line through the neutral axis being substituted for the centre line of the plain wall. A very curious fact comes to light in the course of the investigation, viz. that adding a buttress weakens the wall. The buttresses from centre to centre are usually $1\frac{1}{2}$ to $2\frac{1}{2}$ times the height of the wall or 15 to 20 times its thickness. Fig. 428 represents a unit length of an ordinary buttressed wall. The brickwork being taken as 112 lbs. per cub. ft., and the wind pressure as 28 lbs. per sq. ft. The neutral axis will be distant from face of buttress,

$$y = \frac{BD^2 - bd^2}{2(BD - bd)} = \frac{12 \times 1{\cdot}125^2 - 10{\cdot}5 \times 0{\cdot}375^2}{2(12 \times 1{\cdot}125 - 10{\cdot}5 \times 0{\cdot}375)} = 0{\cdot}717,$$

and from plain face of wall, $x = D - y = 1{\cdot}125 - 0{\cdot}717 = 0{\cdot}408$.

Then
$$I = \frac{(BD^2 - bd^2)^2 - 4BDbd(D-d)^2}{12(BD - bd)}$$

$$= \frac{(12 \times 1{\cdot}125^2 - 10{\cdot}5 \times 0{\cdot}375^2)^2 - 4 \times 12 \times 1{\cdot}125 \times 10{\cdot}5 \times 0{\cdot}375(1{\cdot}125 - 0{\cdot}375)^2}{12(12 \times 1{\cdot}125 - 10{\cdot}5 \times 0{\cdot}375)}$$

$= 0{\cdot}597$.

$$\frac{I}{y} = \frac{{\cdot}597}{{\cdot}717} = 0{\cdot}832 = Z_y, \quad \frac{I}{x} = \frac{{\cdot}597}{{\cdot}408} = 1{\cdot}463 = Z_x.$$

Then for buttress side

$$\frac{W}{A} + \frac{M}{Z_y} = \frac{6(12 \times {\cdot}75 + 1{\cdot}5 \times {\cdot}375)112}{(12 \times {\cdot}75 + 1{\cdot}5 \times 3{\cdot}75)} + \frac{12 \times 6 \times 28 \times 3}{0{\cdot}832}$$

$= 672 + 7269 = 7941$ lbs. per sq. ft. compression, and for plain side

$\frac{W}{A} - \frac{M}{Z_x} = 672 - \frac{6048}{1{\cdot}463} = 672 - 4134 = -3462$ lbs. per sq. ft. tension.

For the plain wall without buttress Z_y and Z_x will be equal, viz.

$\frac{1}{6}BD^2 = \frac{12 \times {\cdot}75^2}{6} = 1{\cdot}125$, and the stresses from wind pressure will be equal on each face of wall but of opposite sign, viz.

$\frac{W}{A} \pm \frac{M}{Z} = \frac{6 \times 9{\cdot}1875 \times 112}{9{\cdot}1875} \pm \frac{6048}{1{\cdot}125} = 672 \pm 5376 = 6048$ lbs. per sq. ft.

compression on outer edge and 4704 lbs. per ft. tension on inner edge.

Trying the effect of piers of the same width and varying projection it will be found that the resulting maximum stresses form the curves

shown in Fig. 429. Theoretically counterforts have the same effect as buttresses, but differences arise in practice. While a buttress does not tend to separate from the main wall when under thrust a counterfort does, and is only kept in connection by the bonding, and where much projection occurs the bonding requires to be strengthened with hoop iron. The popular idea is that counterforts act chiefly by their weight while buttresses act chiefly by extending the crushing edge further out from the line of the resultant, but this explanation is not supported upon analysis. The reason for putting piers on both sides is that from whichever side the wind blows there will be buttresses to resist it.

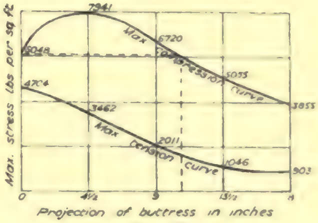


Fig 429

When a wall varies in thickness at different heights each stage should be calculated separately.

EXERCISES ON LECTURE XIX

Q. 64. What wind force per foot super would overturn a 14-in. brick wall, 10 ft. high and weighing 108 lbs. per foot cube, neglecting the strength of the mortar?

Answer. Assuming the wall to overturn without crushing

$$ph = wt^2 \therefore p = \frac{wt^2}{h} = \frac{108 \times (\frac{14}{12})^2}{10} = 14{\cdot}7 \text{ lbs. per ft. sup.}$$

Q. 65. Determine the height of a 14-in. brick wall which would just fail

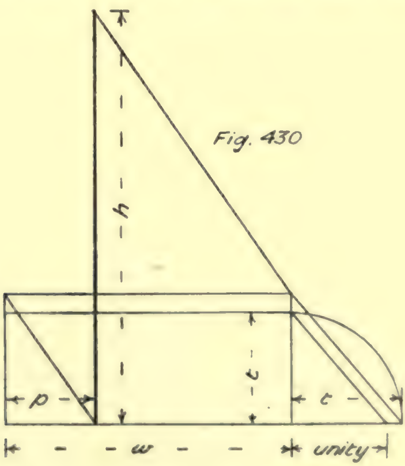


Fig. 430

under a horizontal wind pressure of 35 lbs. per foot super, neglecting the strength of the mortar.

Answer. Assuming wall to overturn without crushing. Brickwork 112 lbs. per cubic foot. Calculation as follows :—

Effort = Resistance,

$$ph \times \tfrac{1}{2}h = wht \times \tfrac{1}{2}t, \therefore h = \frac{wt^2}{p} = \frac{112 \times 1\tfrac{1}{6}^2}{35} = 4 \cdot 3\dot{5} \text{ ft.}$$

For graphic construction, see Fig. 430.

Q. 66. What will be the maximum stresses at the base of a 9-in. brick wall 7 ft. 6 in. high with 18 in. by $4\frac{1}{2}$ in. piers on both sides of the wall at 10 ft. centres? Brickwork 1 cwt. per cub. ft. Wind 21 lbs. per square foot. $Z = \dfrac{BD^3 + bd^3}{6D}$.

Answer. The stresses will be given by the formula $\dfrac{W}{A} \pm \dfrac{M}{Z}$,

$$Z = \frac{BD^3 + bd^3}{6D} = \frac{1 \cdot 5 \times 1 \cdot 5^3 + 8 \cdot 5 \times \cdot 75^3}{6 \times 1 \cdot 5} = 0 \cdot 9625,$$

$$\frac{W}{A} \pm \frac{M}{Z} = 7 \cdot 5 \times 112 \pm \frac{10 \times 7 \cdot 5 \times 21 \times 3 \cdot 75}{0 \cdot 9625} = 840 \pm 6136,$$

or 6976 lbs. = 3·11 tons per sq. ft. compression, and 5296 lbs. = 2·36 tons per sq. ft. tension.

LECTURE XX

Retaining or Revetment Walls—Earth Pressure—Rankine's Theory—Other Theories—Section of Earth Slip—Angle of Repose—Line of Rupture—Graphic Determination of Thrust on Wall—Analysis of Forces—Sloping back—Surcharged Retaining Walls—External Loads at back of Wall.

THE common theory of earth pressure against a retaining wall is attributed both to Rankine and Moseley. It assumes that the earth is in the condition of a fine granular mass whose particles possess no coherence among themselves, and are free to move over each other except for friction, the particles assuming an exterior surface slope under normal conditions, whose tangent to a horizontal line represents the coefficient of friction. This theory results in an intensity of pressure for a mass of dry earth which increases directly as the depth, similar to water pressure but at a different rate.

Mr. J. C. Meem, *Proc. Am. Soc. C.E.*, 1907, directed attention to the well-known fact, that in excavating deep trenches the lower part of the sides would stand without timbering until the moisture had dried out and the earth began to crumble. He argued from this that the thrust in sustaining earth was greatest at the top and reduced to zero at the bottom, but it is possible that the supposed greater pressure near the surface was simply due to the expansion of clay when freshly exposed, which is a familiar experience to excavators.

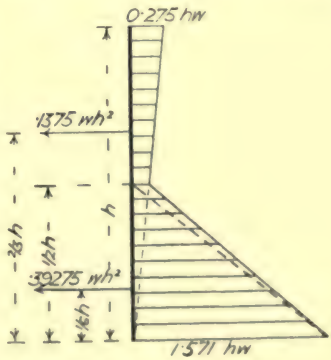


Fig 431

In the discussion of Mr. Meem's paper Mr. E. G. Haines put forward a totally different view of earth pressures; he thought the tendency to slip was in the shape of a semicircle, with the diameter vertical at back of wall, and his arguments resulted in two triangles of pressure of the proportions shown by the dotted lines in Fig. 431, giving the total pressure of 0·53025wh^2 acting at a height of 0·2963h from bottom, or say $\frac{3}{10}$ths, which is not very unlike $\frac{1}{2}wh^2$ at $\frac{1}{3}$ height. It will be observed that in this method no allowance is made for variation in the natural slope of the material, and there is nothing to recommend it over Rankine's.

The author has given some attention to earth slips in railway cuttings, and considers that the general outline, except for the collection of debris at the base, may be taken as the concave slope of a semi-parabola with the vertex at the upper level halfway to the outcrop of the natural slope; the tangent at the lowest point sloping from 2 to 1 to 4 to 1 according to the soil and representing the "natural slope" or final angle of repose, as in Fig. 432.

The angle of repose is variously given by different authorities, and no close reliance can be placed upon the figures.

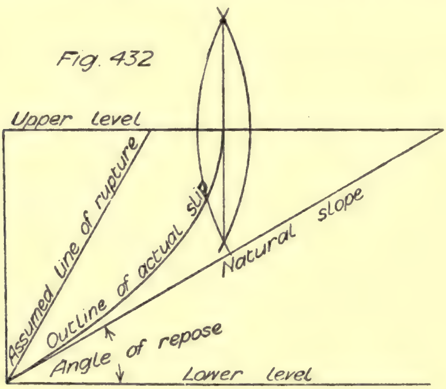
The following summary will be found sufficient for most purposes.

Angle of Repose

	$\phi =$
Wet sand, clay or vegetable earth	15
Dry sand, clay or vegetable earth	30
Loamy earth, loose shingle, clay well drained . . .	40
Firm gravel, and hard dry vegetable earth	45

The working theory in designing retaining walls is that while the

Fig. 432



natural slope is determined by the ultimate angle the soil will assume, after an unlimited period, only half the material above this slope would fall away upon the sudden failure of the wall, and therefore only half the amount requires supporting, hence the line of rupture is made to bisect the angle between the natural slope and the vertical, and the half wedge next the wall is the part considered to produce the thrust.

Fig. 433 shows the ordinary mode of finding the thrust on a retaining wall and the resulting stresses at the base. This is set off as follows. Draw the proposed section of wall ABCD, which may be already given or may be assumed approximately to be

$$\text{height} \times \frac{\text{deg. in wedge}}{90} = \text{ft. thick at base.}$$

From a horizontal line through the wall at the lower ground level set off the natural slope ϕ according to the material to be supported, bisect

the angle between this line and the vertical to give the line of rupture. Find the centre of gravity of the wedge of earth between the line of rupture and the wall, and drop a vertical to cut the line of rupture in O. Calculate the weight of the wedge of earth, and set it up to scale from O as O*a*. Draw a horizontal line through O to give the line of thrust against the wall, and from *a* parallel with the line of rupture draw *ab*, then *b*O will be the total amount of thrust from the earth,

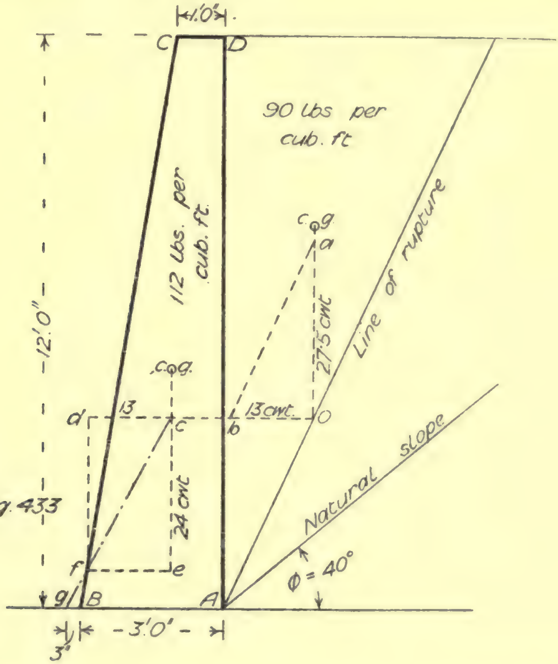


Fig. 433

collected and acting in the line O*b*. Now find the centre of gravity of the wall, and drop a vertical line cutting O*b* produced in the point *c*. From *c* horizontally set off the thrust *cd* equal O*b*, and from *c* vertically downwards set off *ce* equal to the weight of the wall in the same units as the weight of earth. Complete the parallelogram *cefd*, draw the diagonal *cf*, which will be the resultant, and produce it to meet the base line in *g*. As in this case it falls outside the actual base of wall, it is evident that the inner edge of the wall will be in tension, and care must be taken not to overstep the safe limits. The actual stresses will be given by the formula $\frac{W}{A} \pm \frac{M}{Z} = \frac{24}{3} + \frac{24(\frac{3}{2} + \cdot 25)}{\frac{1}{6}(1 \times 3^2)} = 36$ cwt.

or 1·8 tons per sq. ft. compression at outer edge, and 20 cwt. or 1 ton per sq. ft. tension at inner edge.

Among the common errors in this class of work are (1) joining the centres of gravity of the wedge and wall for the direction of thrust,

and setting off the weights in each direction from the centre of gravity of wall to form the parallelogram ; (2) calculating the bending moment from the vertical through centre of gravity of wall instead of from centre of base ; and (3) taking the length of the resultant to measure

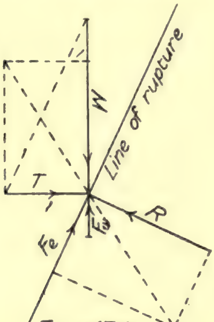


Fig. 434

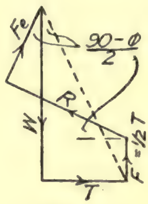


Fig. 435

the load on the base instead of its vertical component. The thrust of the earth does not increase the total load on the base, it only affects its distribution.

Now to prove this mode of working, let O be the point in the line of rupture at which a vertical through the centre of gravity of the wedge of earth would fall, then the forces at point O must be in equilibrium. In Fig. 434 let AB represent the line of rupture, and O the point at which the forces meet. Draw W vertically equal to the weight of wedge upon any suitable scale. From the upper extremity parallel with the line of rupture draw a line to intersect with a horizontal through O, giving T the direction and magnitude of the total thrust at back of wall. There will be a certain amount of friction between the wedge of earth and the back of the wall depending upon the nature of the surface, etc., which may be approximately estimated at half the thrust, equivalent to a coefficient of friction of 0·5 ; this friction will act in direct opposition to W, as shown at F_w. Deducting this from W, a parallelogram may be drawn for $W - F_w$ and T, and the diagonal or resultant produced to the opposite side of O for an equal length to form the equilibrant, which will be the resultant of the reaction of the earth behind the wedge and the friction along the line of rupture. These two directions are given, therefore the value of each force will be found by completing the parallelogram of which this equilibrant is the diagonal.

Having traced the complete forces in action at point O, a force polygon may be drawn to correspond, as in Fig. 435, which can be readily constructed without the preliminary work now that the principles are known.

It must be remembered that these forces are totals, and it will be instructive to analyse them into their details as in Fig. 436. Draw the force lines as in Fig. 434, and the line of rupture AB. The triangle ABC represents by the vertical ordinates the individual pressures on AB which go to make up the weight W. The point of application of force

W being at one-third length of AB from A the distribution of pressures will be a triangle on base AB. Join *de*, then C*f* being the perpendicular of the triangle ABC, set up O*g* = C*f*, draw *gh* parallel with *de*, and *hi* parallel with AB and cut off by the perpendicular A*i*. Join *i*B,

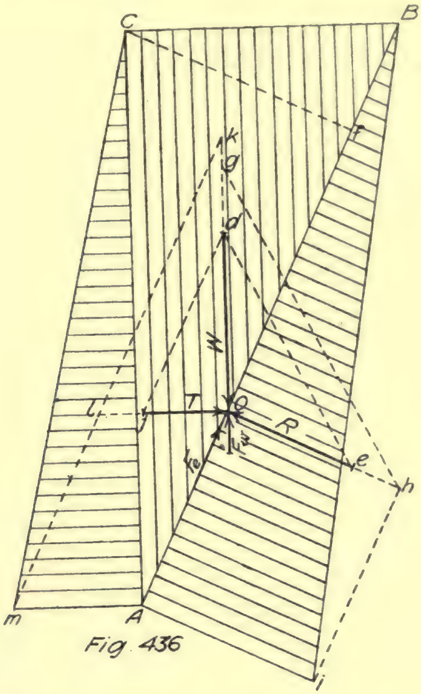


Fig. 436

then the triangle AB*i* represents the reactions on the line of rupture to the same scale as the wedge of earth. Similarly the thrust being applied at one-third the height of wall from base, the distribution of stress will be found by joining *dj*, setting up O*k* = CB, drawing *kl* parallel with *dj* to meet O*j* produced in point *l*; then *kl* produced to *m* in the horizontal line from A, and C*m* joined, gives the whole triangle of pressures AC*m*.

The graphic method shown in Fig. 433 will equally apply to a wall supporting water as in the case of a tank, reservoir, or dam, but the angle of repose or natural slope of the water will be 0°, so that the line of assumed rupture will be 45°. Numerically the thrust in any case will be. $T = \frac{1}{2} wh^2 \tan^2 \frac{90 - \phi}{2}$, or, using Rankine's form, which is a little simpler to work, but not so easy to understand, $T = \frac{1}{2} wh^2 \frac{1 - \sin \phi}{1 + \sin \phi}$.

When the material behind a retaining wall is waterlogged, through the absence of weep holes, the pressure is considerably increased beyond even

that due to water of the full depth. Sir B. Baker stated that the thrust of dry earth being taken as 1 and water $\frac{1}{2}$, the thrust of a waterlogged soil would be $1\frac{1}{2}$.

The triangle of thrust in Fig. 433 may be turned the other way round and set off as in Fig. 437. Then the thrust for walls with back sloping out or in will be perpendicular to back and at one-third the height as in

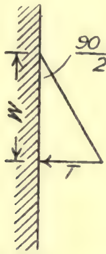


Fig 437

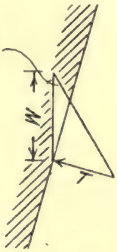


Fig 438

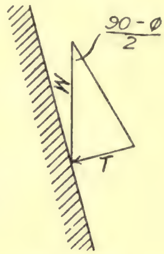
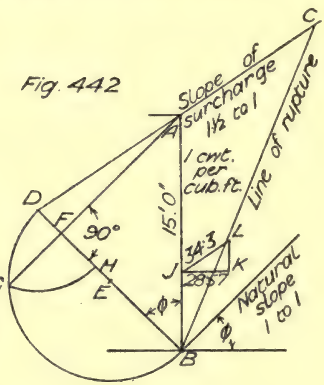


Fig 439

Figs. 438 and 439, it being remembered that in Fig. 438 the weight of wedge acting will be less, and in Fig. 439 more, than in Fig. 437, because the quantity of earth varies in the wedge between back of wall and line of rupture.

In the case of reservoir walls the triangle may be set off in the same way but the angle will be always 45°, and the vertical line will be the weight of the wedge from line of rupture as in the case of earth. Figs.

Fig. 442



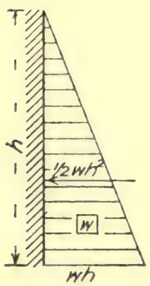


Fig 440

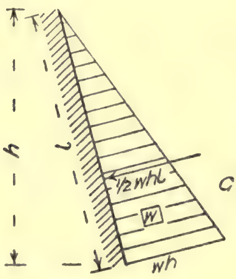


Fig 441

440 and 441 show the common methods of dealing with water pressure against walls. The pressure varies uniformly as wh from the surface to the bottom, and the whole may be represented as the ordinates to a triangle. The centre of pressure will be opposite the centre of gravity of the triangle at one-third of the height, and the total pressure assumed to be collected there will be $\frac{1}{2}wh^2$, or $\frac{1}{2}whl$, the same as found by calculating the weight of the wedge in the previous method.

A surcharged retaining wall is one supporting a sloping bank of earth rising above its top. The surcharge may be limited and definite, with a flat top, or "infinite," *i.e.* when the angle coincides with the natural slope and extends beyond the limit of the line of rupture; or it may be intermediate as when the slope of surcharge is less than the natural slope but yet extends for some distance. Rankine's graphic method applies particularly to the latter case. Let AB, Fig. 442, be the back of a wall 15 ft. high supporting a bank of earth which slopes at $1\frac{1}{2}$ to 1, while the natural slope is 1 to 1. Put in the line of rupture BC, draw the angle ABD = ϕ, and produce CA to D. Bisect BD in E, and from E as centre draw the semicircle BD. From A drop a perpendicular on to DB, cutting it in point F, and when

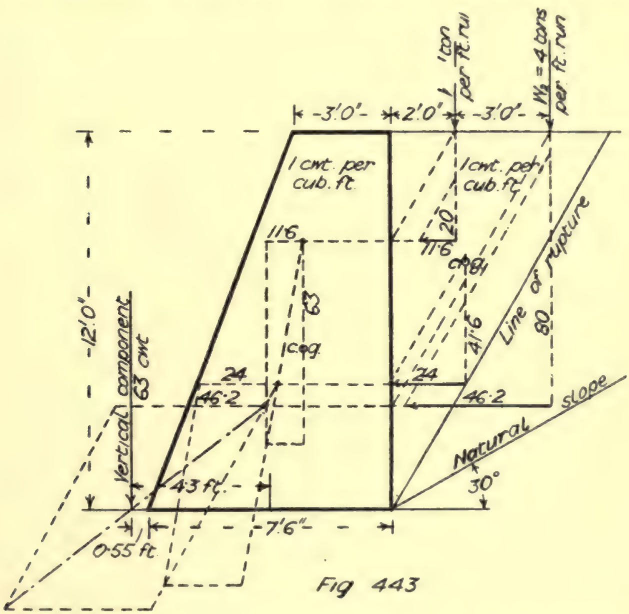


Fig 443

produced cutting the semicircle in point G. From centre D with radius DG cut DB in H. Then the horizontal thrust acting at one-third the height will be $T = \frac{1}{2}w(BH)^2$. Let the earth in this case weigh 1 cwt. per cub. ft., then the thrust JK = 28·57 cwt., which being at 5 ft. height gives an overturning moment of 142·85 cwt.-ft. From J draw JL parallel to natural slope and KL vertical, then the thrust in direction LK scales 34·3 cwt., and its perpendicular distance from B = 4·16 ft., giving an overturning moment of 142·69 cwt.-ft., showing practically the same result whichever line is taken for thrust.

From this graphic method it will be seen that when the surcharge is equal to the natural slope the horizontal thrust $T = \frac{1}{2}w(h \cos \phi)^2 = \frac{1}{2}wh^2 \cos^2 \phi$.

There is no recognised formula or means of arriving at the thrust

upon a retaining wall when the earth at the back is subject to an external load, such as a steam roller, crane, warehouse wall, stack of bricks, etc., but some such method as that shown in Fig. 443 will probably give approximate results. Find the thrust from wedge of earth as usual, then from the point of application of the external loads draw lines parallel to the line of rupture to find the point at back of wall at which thrust is to be taken. Draw triangle of weight and thrust for each and combine the thrusts in successive parallelograms.

If the front wall of a building occurs behind the retaining wall, the centre of the foundation at the underside will give the point for the line parallel to the line of rupture. Any load that lies on the earth outside the line of rupture may be ignored.

EXERCISES ON LECTURE XX

Q. 67. **It is desired to put a ½-brick wall in cement to protect a bank of earth 5 ft. high assumed to have a natural slope of 30 degrees. What should be the least batter of the wall to relieve it from all thrust?**

Answer. Natural slope $\phi = 30$ degrees, line of rupture $\frac{90 - \phi}{2} = \frac{90 - 30}{2} = 30$ degrees from vertical, which will be the batter of the wall. The wall should be in hard bricks and have rubble drains at back with weep holes at intervals as in Fig. 444.

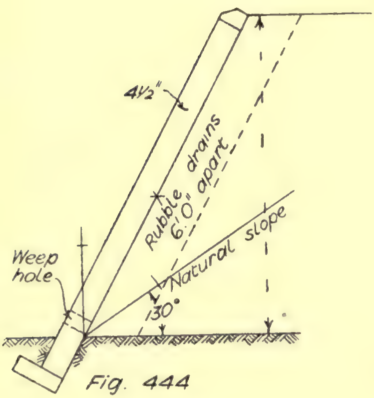


Fig. 444

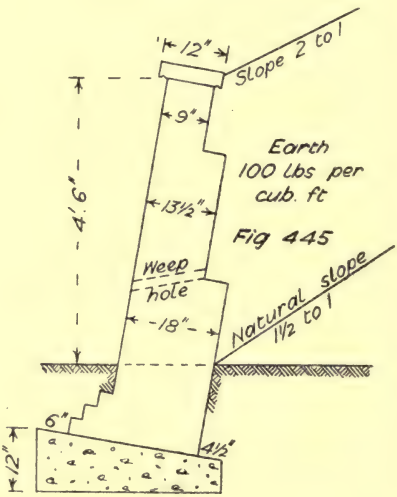


Fig 445

Q. 68. Fig. 445 shows a breast wall with an infinite surcharge. Investigate the stability.

Answer. See Fig. 446. The maximum stresses will be as follows:—

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{7}{1\cdot5} \pm \frac{7 \times 0\cdot5}{\frac{1}{6}(1 \times 1\cdot5^2)} = 4\cdot\dot{6} \pm 9\cdot\dot{3} = 14 \text{ cwt. per sq. ft.}$$

compression and $4\cdot\dot{6}$ cwt. per sq. ft. tension.

Q. 69. Fig. 447 shows a warehouse wall at the back of a brick retaining wall. Find the maximum pressure at base of wall.

Answer. See Fig. 448. The maximum pressures at base of wall will be as follows:—

$\frac{W}{A} \pm \frac{M}{Z} = \frac{84}{7} \pm \frac{84 \times 4{\cdot}9}{\frac{1}{6} \times (1 \times 7^2)} = 12 \pm 50{\cdot}4 = 62{\cdot}4$ cwt. or 3·12 tons per sq. ft. compression, and 38·4 cwt. or 1·92 tons per sq. ft. tension.

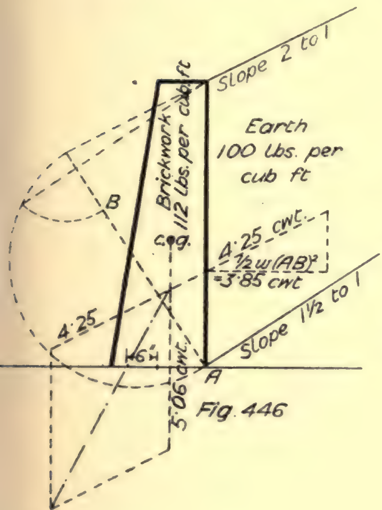


Fig. 446

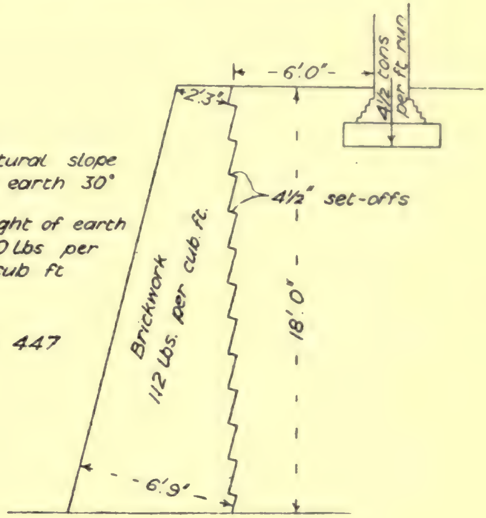


Fig 447

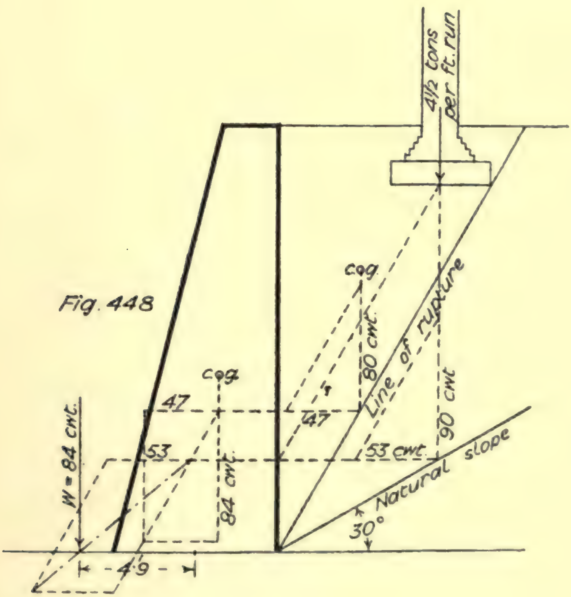


Fig. 448

LECTURE XXI

Stability of Stone Pinnacle—Pier at angle of Porch—Railway Bridge Abutment —Church Buttresses—Counterforts—Flying Buttresses.

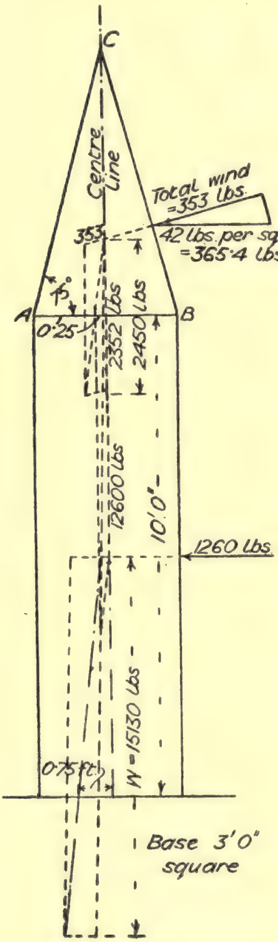


Fig 449

THERE are various interesting and special cases of stability that may now be considered. Fig. 449 shows a stone pinnacle reduced to its simplest elements. Assume it to be of stone weighing 140 lbs. per cub. ft., the shaft 3 ft. square, and the sides of apex making an angle of 75 degrees with the base. Dealing with the upper part first, the area of the side will be $\frac{AB \times BC}{2} = \frac{3 \times 5\cdot8}{2} = 8\cdot7$ sq. ft., and assuming a horizontal wind pressure of 42 lbs. per sq. ft., the total horizontal force will be $8\cdot7 \times 42 = 365\cdot4$ lbs. Part of this will slip off and the remainder will act normal to the surface, which by triangle of forces is found to be 353 lbs. Produce this force line to cut the centre of gravity line through the pyramid and set off the value, to scale, beyond the intersection. The solid contents of the pyramid are

$$\text{area of base} \times \tfrac{1}{3} \text{ vertical height}$$
$$= \frac{3 \times 3 \times 5\cdot6}{3} = 16\cdot8 \text{ cub. ft.,}$$

and the weight $16\cdot8 \times 140$
$= 2352$ lbs.

Set this off below the intersection, complete the parallelogram, and draw the diagonal for resultant = 2460 lbs. This cuts the joint at 0·25 ft. beyond the centre and, using the vertical component of the resultant = 2450 lbs., creates a bending moment of

$$2450 \times 0\cdot25 = 612\cdot5 \text{ lb.-ft.}$$

Then the stress on the two edges by formula

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{2450}{3 \times 3} \pm \frac{612 \cdot 5}{\frac{1}{6} \times 3 \times 3^2} = 408 \text{ and } 136 \text{ lbs. per sq. ft.,}$$

which is of course very little. Now the resultant force from the upper part must be transmitted through the prismatic block below, but it will be well to reduce the scale. The weight of the pier is

$$3 \times 3 \times 10 \times 140 = 12600 \text{ lbs.,}$$

set this off downwards from the centre of gravity of pyramid, complete the parallelogram and draw the diagonal for resultant. The wind pressure against this part will be $3 \times 10 \times 42 = 1260$ lbs. acting through the centroid half-way up. Combine this with the last resultant to form a parallelogram, and the resultant will then be found to be 15200 lbs., cutting the base at a distance of 0·75 ft. from the centre line. The vertical component will be 15130 lbs., then

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{15130}{3 \times 3} \pm \frac{15130 \times 0 \cdot 75}{\frac{1}{6} \times 3 \times 3^2} = 1681 \cdot 1 \pm 2521 \cdot 6 = 4202 \cdot 7 \text{ lbs.,}$$

or 1·875 tons per sq. ft. compression at outer edge and 840·5 lbs. or 0·375 ton per sq. ft. compression at inner edge.

A porch consisting of four piers with arches between is not uncommon in connection with mansions and public buildings, as in Fig. 450, elevation, and Fig. 451, plan. The thrust from each arch will be as shown in Fig. 452. The method of finding the thrust will be explained subsequently when arches are under consideration, and the accuracy of the thrust 71 cwt. must for the present be assumed. Now let Fig. 453 be the elevation of pier and thrust from front arch, Fig. 454 plan of pier and thrust from both arches, Fig. 455 diagonal elevation of pier. The next step is to find the resultant of the two thrusts in their own plane and to show this on the diagonal elevation. The order of working is shown by the small letters *g*, *h*, Fig. 456, projected from plan, $hi = ca$; $kl = ef$; *lm*, *km*, the two thrusts in the plane of the paper, *mn* resultant. This resultant carried back to *go* gives the diagonal thrust on Fig. 455, giving finally the vertical component $W = 296$ cwt. acting at a distance of 0·6 ft. from outer point on the area 3 ft. square placed diagonally. The bending moment will be

$$296\left(\frac{3}{\sqrt{2}} - 0 \cdot 6\right) = 296 \times 1 \cdot 52 = 450 \text{ cwt.-ft.}$$

The section modulus $= \dfrac{d^3}{6\sqrt{2}} = 0 \cdot 118 d^3 = 0 \cdot 118 \times 3^3 = 3 \cdot 186.$

Then the maximum stresses will be

$$\frac{W}{A} \pm \frac{M}{Z} = \tfrac{296}{9} \pm \frac{450}{3 \cdot 186} = 33 \pm 141 = 174 \text{ cwt. (say 8·7 tons) per sq. ft.}$$

compression, and 108 cwt. (say 5·4 tons) per sq. ft. tension. This

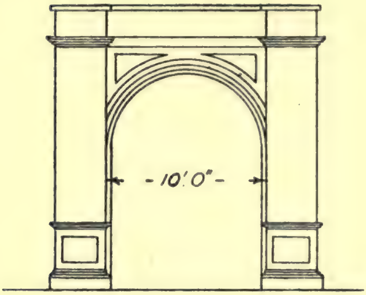
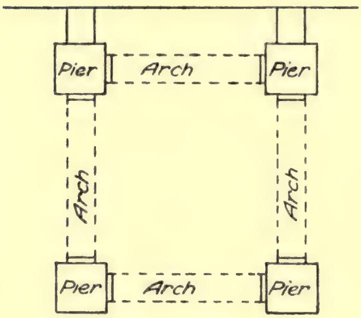


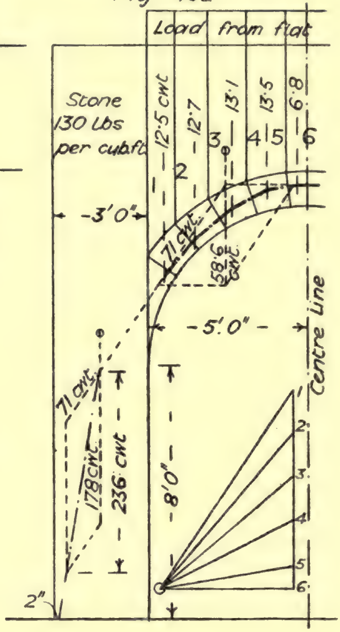
Fig 450



Porch covered with 8 Lbs lead

Fig. 451

Fig 452



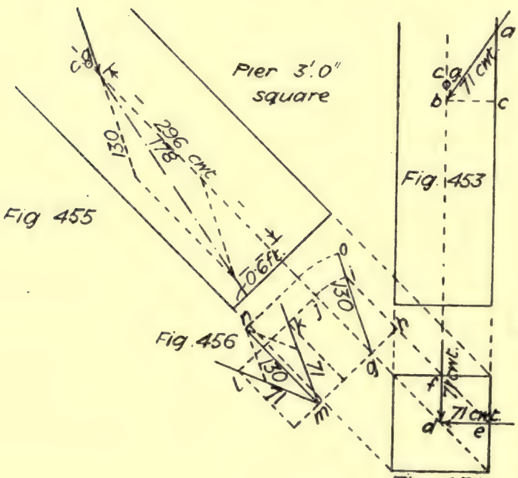


Fig 454

compression is rather high for Bath stone, but might possibly be permitted in first-class work with no flaws, to which Bath stone is rather subject. The tension is about the maximum that may be permitted with very carefully made Portland cement mortar, but four rows of hoop iron bond embedded in the masonry of the string course over the arch would remove all doubt as to the stability.

In the case of an abutment for a railway girder bridge, as in Fig. 457, there are several points to note. There will usually be wing walls to support the embankment, and although these are bonded to the abutment it is not usual to consider them as adding any strength, because the slightest disturbance of the foundations results in a crack or fissure between the two parts. When a train is standing on the embankment just clear of the bridge the weight supported over the wedge of earth will help to increase the thrust and must be allowed for; the train may, however, be on the bridge and embankment at the same time, the portion on the bridge adding weight to the abutment, and therefore increasing the vertical component of the thrust. But, again, the train may be on the bridge only, adding weight to the abutment but not increasing the thrust at the back. The maximum stresses will evidently be produced when the bridge and adjacent embankment are both loaded, and in the absence of special information it may be taken as 5 cwt. per foot super, or say 5 tons per foot run of its face on the abutment, and 5 cwt. per foot run on the wedge of earth at back. The method of working will be as shown, the final resultant cutting the ground line 4 ft. from centre of base of wall. Then

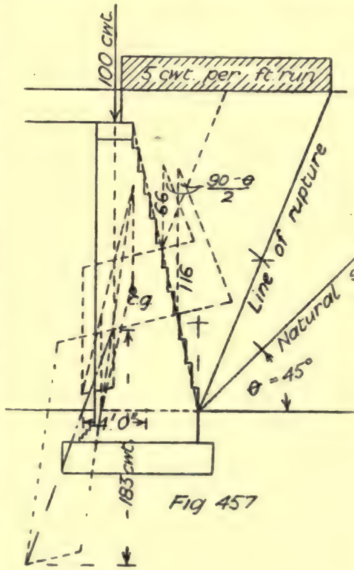


Fig 457

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{183}{6\cdot375} \pm \frac{183 \times 4}{\frac{1}{6}(1 \times 6\cdot375^2)} = 28\cdot7 \pm 103 = 131\cdot7 \text{ cwt.},$$

or 6·58 tons per sq. ft. compression and 74·3 cwt. or 3·71 tons per sq. ft. tension. When the bridge is continuous with a brick arched viaduct, the abutment must be calculated to withstand the thrust of the adjoining arch when loaded, but no difficulty will be experienced in this.

In the case of churches, school buildings, etc., with buttressed walls the buttress and portion of solid wall between the windows, forming a **T** section, may be taken as resisting the thrust from the roof. Fig. 458 shows elevation, and Fig. 459 plan of such a portion. The

vertical dead load from one-half of the roof truss, amounting to 94·5 cwt., is assumed as acting on the wall plate $4\frac{1}{2}$ in. from the inner edge of the wall, and this is combined with the whole of the wind pressure on the roof amounting to 63 cwt. acting in a direction normal to the slope of the roof surface and striking the opposite wall where shown by the arrow in Fig. 458. The result is then combined with the weight of the wall and buttress acting at their mutual centre of gravity. Having obtained the final resultant, the maximum stress per square foot will be found by the formula $\frac{W}{A} \pm \frac{M}{Z}$, where W = vertical component of resultant in cwts., A = sectional area of wall and buttress in square feet, M = bending moment in cwt.-ft., Z = section modulus in foot units. Then

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{404}{17 \cdot 15} \pm \frac{404 \times 1 \cdot 43}{9 \cdot 4} = 23 \cdot 55 \pm 61 \cdot 5 = \text{say } 85 \text{ cwt.},$$

or 4·25 tons per sq. ft. maximum compression on material of buttress, and 38 cwt. = 1·9 tons per sq. ft. tension at inner face of wall.

In many instances there are more thrusts than one on a buttress and each must be combined with the partial weight of buttress as the line of thrust is constructed downwards. The centre of gravity of each portion should be found graphically, and the mean centre of gravity of each group permanently marked by pricking in, the lines may then be rubbed out and the force parallelograms proceeded with. The force scale may be reduced at each stage so as to keep the parallelograms within reasonable dimensions, but the smaller the scale the less exact will be the results, although probably near enough. At the first working out of the curve of thrust and the stresses in a buttress it may be found desirable to modify the height or projection of the various stages, but unless very great care has been taken in ascertaining the original thrusts it is useless to spend much time over the buttress.

Piers, pilasters, or counterforts are the reverse of buttresses, being placed inside a wall instead of outside. The mode of calculating them is exactly similar to that of buttresses, but unless hoop-iron bond is inserted between wall and counterfort no tension should be allowed on inner edge owing to the facility with which separation in the bonding occurs between a wall and counterfort.

Flying buttresses are more or less detached from the wall to which they give support. By standing further out they act more as raking shores, and can therefore be somewhat reduced in bulk, while they are also capable of architectural treatment. Take a simple case, as in Fig. 460, and divide into the portions shown by lines at A, B, and C, and find the weight and centre of gravity of each part. Combine the weight of the wall and buttress down to line A with the given thrust, and then combine the resultant thus found with the weight of the portion down to line B. The section at this line will be as in Fig. 461, and the distance of the neutral axis from inner edge of wall will be $\frac{5 \cdot 5^2 \times 2 + 1 \cdot 5^2(6 - 2)}{2 \times 17} = 1 \cdot 5$ ft., which coincides with the outer edge of wall, and the section modulus will be

$$Z = \frac{(6 \times 5{\cdot}5^2 - 4 \times 4^2)^2 - 4 \times 6 \times 5{\cdot}5 \times 4 \times 4(1{\cdot}5)^2}{6(6 \times 5{\cdot}5^2 - 4 \times 4^2)} = 12{\cdot}84.$$

Then the stresses will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{31800}{17} \pm \frac{31800 \times \frac{16}{12}}{12{\cdot}84} = 1870 \pm 3302 = 5172 \text{ lbs.},$$

or 2·31 tons per sq. ft. compression, and 1432 lbs. or 0·63 tons per sq. ft.

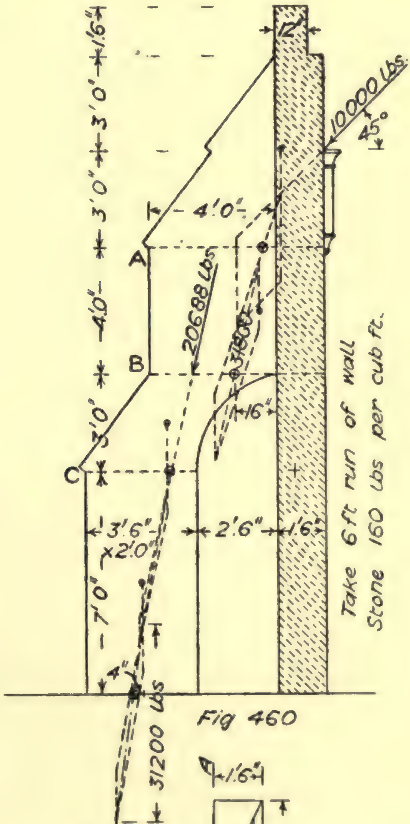


Fig 460

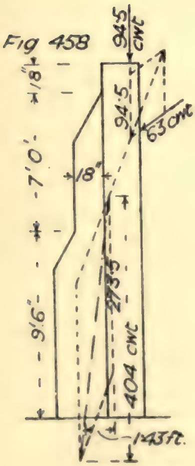


Fig 458


Fig 459

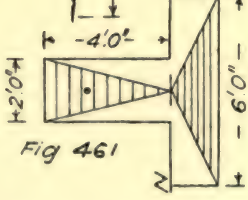


Fig 461

tension. As the neutral axis coincides with outer face of wall, the whole of the tension comes on the wall and reduces to that extent the

weight of the wall below upon the foundations, and need not be further considered. The compression which is taken by the buttress is spread over the triangle shaded by wide lines in Fig. 461, and the total amount acting at the centre of gravity of the triangle will be

$$\frac{2 \times 4}{2} \times 5172 = 20688 \text{ lbs.}$$

This force may be taken as acting parallel with the last resultant, and is then combined with the weight of buttress down to line C, and the resultant thus found is combined with the weight down to ground level, giving the final resultant. The stresses produced by this resultant will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{31200}{3{\cdot}5 \times 2} \pm \frac{31200 \times \frac{4}{12}}{\frac{1}{6}(2 \times 3{\cdot}5^2)} = 4457 \pm 2547 = 7004 \text{ lbs.},$$

or 3·12 tons per sq. ft. compression, and 1910 lbs. or 0·85 ton per sq. ft. compression, and these figures are within safe limits.

EXERCISES ON LECTURE XXI

Q. 70. Fig. 462 shows a granite obelisk weighing 160 lbs. per cub. ft. Assuming the base to be reduced 6 in. all round by weathering, what horizontal

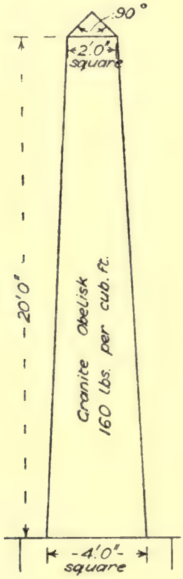


Fig 462

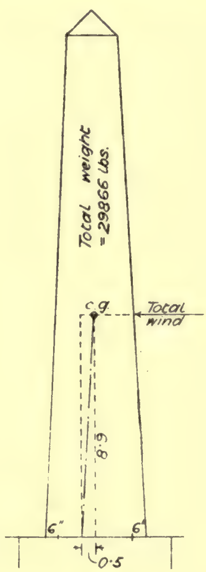


Fig. 463

wind pressure per sq. ft. would be required to produce a maximum compression with no tension?

Answer. See Fig. 463.

$$\text{Then } 8{\cdot}9 : 29866 :: 0{\cdot}5 : x, \text{ whence } x = \frac{29866 \times 0{\cdot}5}{8{\cdot}9} = 1678 \text{ lbs.}$$

$$\text{area} = \frac{4+2}{2} \times 20 = 60 \text{ sq. ft.}$$

therefore wind pressure per sq. ft. $= \frac{1678}{60} = 28$ lbs.

Q. 71. Find the centres of gravity of each portion of buttress shown in Fig. 464; trace the amount and position of combined thrust and weight at each change of section, and state the maximum stresses at the base.

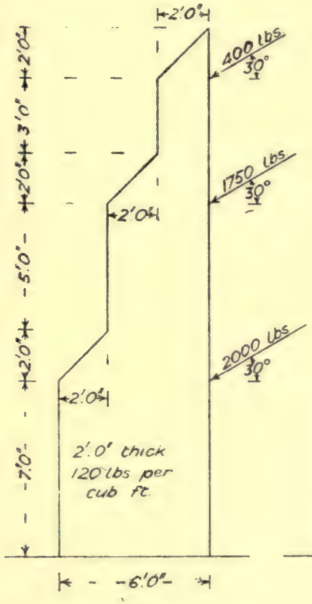


Fig. 464

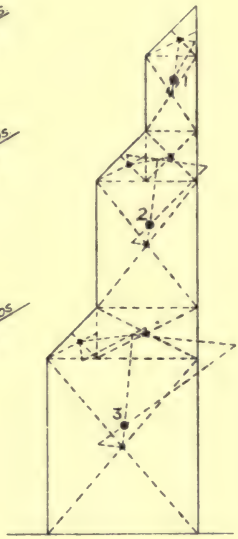


Fig. 465

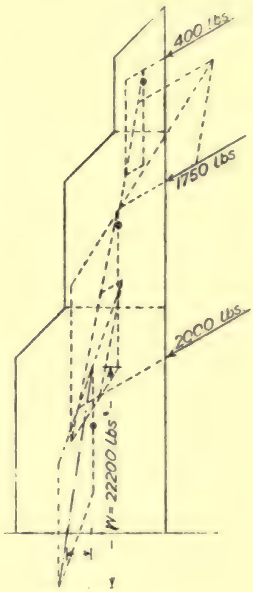


Fig. 466

Answer. See diagram, Figs. 465 and 466. The final resultant is 22200 lbs., and the resultant cuts the base at the edge of middle third, or 1 ft. from the centre of base. The stress will therefore be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{22200}{6 \times 2} \pm \frac{22200 \times 1}{\frac{1}{6}(2 \times 6^2)} = 1850 \pm 1850 = 3700 \text{ lbs.},$$

or 1·65 tons per sq. ft. compression at outer edge and nil for tension on inner edge.

LECTURE XXII

Pressure of Water—Pressure on Tank and Reservoir Walls—Stability of Masonry Dam.

THE pressure of water against a surface, at any point in it, depends only upon the depth of the point below the surface of the water. That is to say, the pressure upon a square foot at any depth will be wh, where $w =$ the weight of a cubic foot of water $= 62\frac{1}{2}$ lbs., and $h =$ height or depth in feet from surface. The pressure gradually increases

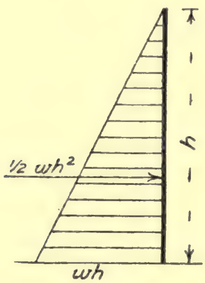


Fig. 467

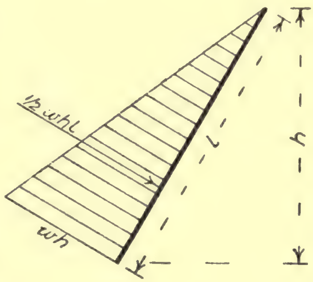


Fig. 468

therefore from the top to bottom, as ordinates to a triangle, as shown in Fig. 467. The centre of effort, or centre of pressure, will be opposite the centre of gravity of the triangle at $\frac{1}{3}$ the height, and the total amount will be $\dfrac{wh \times h}{2} = \frac{1}{2}wh^2$. The pressure at the various depths will be the same whatever the angle of the containing surface, but in the case of a sloping wall, the sloping length being greater than the vertical height, the total pressure will be greater. If h be the vertical height, and l the inclined length, the total pressure will be $\frac{1}{2}whl$, as in Fig. 468. It must be remembered that water pressure always acts perpendicularly to the surface against which it presses.

The stability of walls against water pressure may be worked graphically precisely in the same way as for earth; the natural slope of water being nil, the so-called line of rupture will be at 45°, and the wedge of water between this line and the back of wall, whether it is vertical or sloping in or out, is the weight producing the thrust. Then graphically in Fig. 469, at $\frac{1}{3}$ height, set up W = weight of wedge of water, cut off the horizontal thrust line by an angle of 45°; then

the amount of thrust will be given by the length of horizontal line. Let the wall slope away from the water as in Fig. 470, then the weight of wedge will be greater, but the thrust line will be cut off at nearly the same length as before by the angle of 45°, the difference

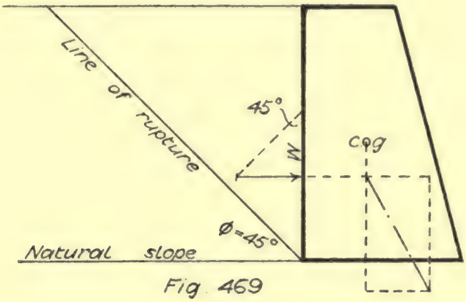


Fig 469

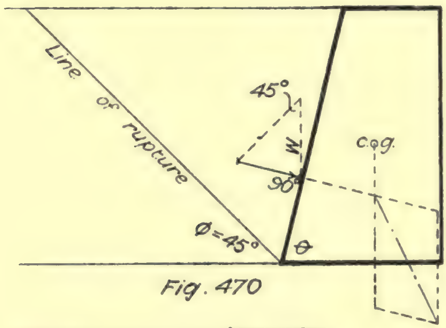


Fig. 470

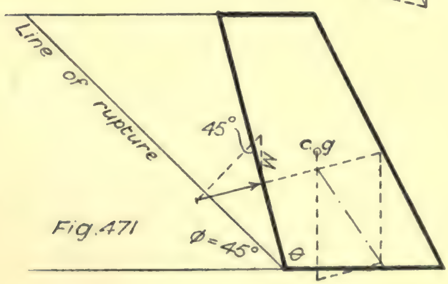
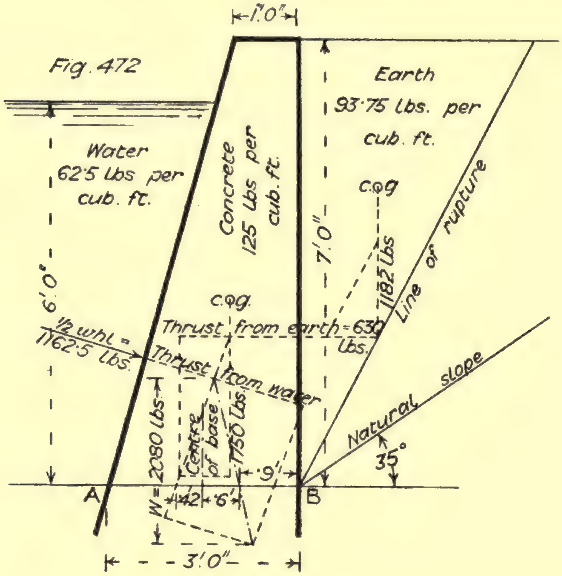


Fig. 471

being that due to the difference between $\frac{1}{2}wh^2$ and $\frac{1}{2}whl$. By formula the thrust will be $\frac{1}{2}wh^2 \operatorname{cosec} \theta$. Again, let the wall slope towards the water as in Fig. 471, then the weight of wedge will be less, but the thrust will be of the same value as before if the sloping length is the same, or $T = \frac{1}{2}wh^2 \operatorname{cosec} (180 - \theta)$.

Now let us take the case of a wall subject to earth pressure on one side and water pressure on the other, such as the wall surrounding a sunk tank or small reservoir. Assume the wall to be of concrete,

7 ft. high from bottom of reservoir, with a vertical back and battering face, 1 ft. thick at top and 3 ft. at bottom, as in Fig. 472. The earth has a natural slope of 35° and finishes level with the top of wall. The highest water-level reaches to 1 ft. from top of wall. The weight per cubic foot of the earth is $1\frac{1}{2}$ times and that of the concrete twice that of a cubic foot of water. This should be drawn to a scale of $\frac{1}{2}$ in. to 1 ft. and the stability estimated graphically as shown. Then, the figures having been scaled off, the calculation of maximum stresses at inner and outer edges of the base will be as follows: When the tank is



empty the maximum stresses from the resultant of the earth pressure and weight of the wall will be $\frac{W}{A} \pm \frac{M}{Z} = \frac{1750}{3} \pm \frac{1750 \times 0{\cdot}42}{\frac{1}{6}(1 \times 3^2)} = 583{\cdot}3 \pm 490$ $= 1073{\cdot}3$ lbs. per square foot compression at A, and $93{\cdot}3$ lbs. per square foot compression at B. Then combining this resultant with the water pressure and assuming no tension possible, the stress will be $\frac{2}{3}\frac{W}{d} = \frac{2 \times 2080}{3 \times 0{\cdot}9} = 1541$ lbs. per square foot compression at B. When tension is permissible, the stresses will be $\frac{W}{A} \pm \frac{M}{Z} = \frac{2080}{3} \pm \frac{2080 \times 0{\cdot}6}{\frac{1}{6}(1 \times 3^2)}$ $= 693{\cdot}3 \pm 832 = 1525{\cdot}3$ lbs. per square foot compression at B and $138{\cdot}7$ lbs. per square foot tension at A.

The principles of designing and calculating the main stresses of a modern masonry dam are exemplified in Fig. 473, which shows a conventional section of a dam 70 ft. high. The curve of stability when the reservoir is empty will be found by first dividing the section into any convenient number of parts, as shown by the dotted lines

A, B, C, D. Find the centre of gravity of each part, and through each centre of gravity drop a vertical line to cut the assumed base line of each part. The curve drawn through these points will be the curve of stability when the reservoir is empty. When the reservoir is full the

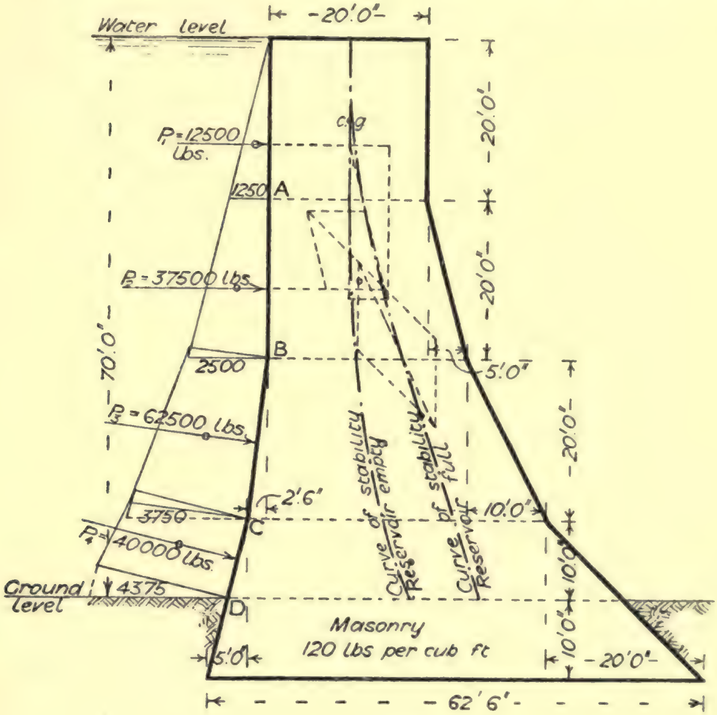


Fig. 473

method will be as follows: taking the part down to line A the pressures due to the head of water will be in the form of a triangle, with a base at A of $62{\cdot}5 \times 20 = 1250$ lbs. The total pressure will then be $\frac{20 \times 1250}{2} = 12{,}500$ lbs. acting at right angles to back of wall through the centre of gravity of the triangle. This pressure must be combined with the weight of the part considered, and where the resultant cuts the line A will be one point in the curve. The part from A to B must next be considered, and the pressures due to the head of water will now vary from 1250 lbs. at A to $62{\cdot}5 \times 40 = 2500$ lbs. at B. The total pressure acting through the centre of gravity of the quadrilateral will be $\frac{1250 + 2500}{2} \times 20 = 37{,}500$ lbs., and this amount must be combined with the resultant from the first portion. The resultant thus formed is then combined with the weight of wall from A to B, and another point in the curve will be given where the resultant of the last step cuts line

B. In a similar manner the other divisions of the wall may be worked out and other points found, through which the curve of stability when reservoir is full may be drawn.

Other calculations for shear are required in the practical designing of masonry dams, but these involve considerations rather beyond the present course.

EXERCISES ON LECTURE XXII

Q. 72. What will be the weight of water in lbs. contained in a tank 6 ft. cube when full? and what will be the total pressure upon the sides and bottom?

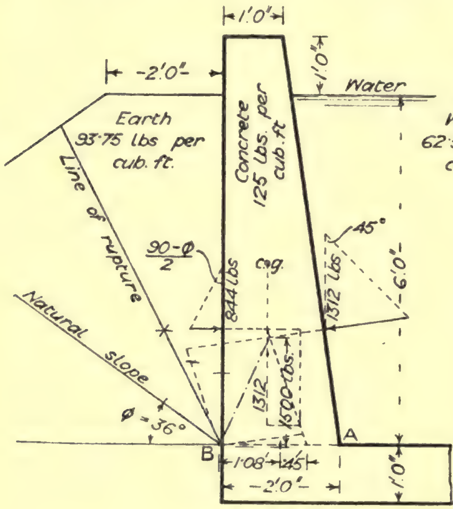


Fig 474

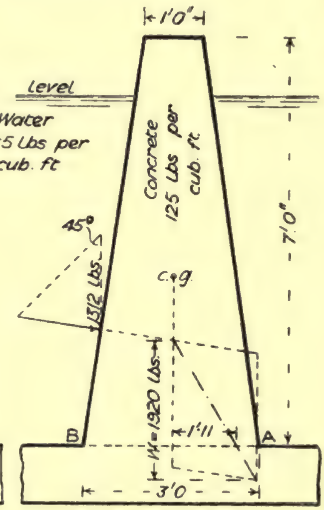


Fig 475

Answer. $6 \times 6 \times 6 \times 62{\cdot}5 = 13{,}500$ lbs. weight, $\frac{1}{2} \times 62{\cdot}5 \times 6^2 \times 6 \times 4 + 13{,}500 = 40{,}500$ lbs. pressure, or the pressure is three times the weight.

Q. 73. A storage tank is built in concrete 1 ft. below the surface of the ground. The external wall is 7 ft. high inside, 1 ft. thick at top, and 2 ft. at bottom, battered inside and vertical outside. The earth, having a natural slope of 36°, is banked up against the outside, with a 2 ft. pathway on top 1 ft. below top of wall. The water is 6 ft. deep. Find the maximum tension and compression at the base when full and empty, taking the earth $1\frac{1}{2}$ times weight of water, and concrete twice weight of water.

Answer. See Fig. 474. When empty

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{1312}{2{\cdot}0} \pm \frac{1312 \times 0{\cdot}45}{\frac{1}{6}(1 \times 2^2)} = 656 \pm 885{\cdot}6 = 1541{\cdot}6 \text{ lbs.},$$

or 0·69 ton per sq. ft. compression at A, and 229·6 lbs. or 0·1 ton per sq. ft. compression at B.

When full $\frac{W}{A} \pm \frac{M}{Z} = \frac{1500}{2{\cdot}0} \pm \frac{1500 \times 1{\cdot}08}{\frac{1}{6}(1 \times 2^2)} = 750 \pm 2431 = 3181$ lbs., or 1·42 tons per sq. ft. compression at B, and 1681 lbs. or 0·75 ton per sq. ft. tension at A.

Q. 74. A division wall in the above tank is 1 ft. thick at top and 3 ft. thick at bottom. Find the maximum tension and compression when one side is full and the other empty.

Answer. See Fig. 475. The stresses will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{1920}{\cdot 3} \pm \frac{1920 \times 1\cdot 11}{\frac{1}{6}(1 \times 3^2)} = 640 \pm 1420 = 2160 \text{ lbs.},$$

or 0·96 ton per sq. ft. compression at A, and 780 lbs., or 0·35 ton per sq. ft. tension at B.

Q. 75. Fig. 476 shows the section of a concrete dam. Find the maximum

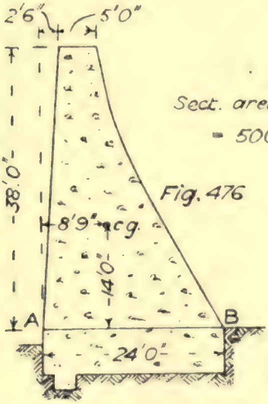


Fig. 476

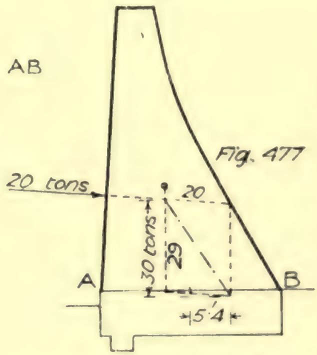


Fig. 477

stresses on the base AB when the reservoir is full. The material weighs 130 lbs. per cubic foot, and the sectional area above AB is 500 sq. ft.

Answer. See Fig. 477. The stresses will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{30}{24} \pm \frac{30 \times 5\cdot 4}{\frac{1}{6}(1 \times 24^2)} = 1\cdot 25 \pm 1\cdot 68 = 2\cdot 93 \text{ tons per sq. ft. compression at B,}$$

and 0·43 ton per sq. ft. tension at A.

LECTURE XXIII

Stability of Arches—Forces acting on a Voussoir—Curve of Thrust—Depth at Crown—Primary Thrusts—Analogy with Girders—Catenary and Parabolic Curves—Virtual Arches, Straight Arch and Lintel—Curve for Ordinary Distributed Loads—Maximum Stress—Critical Joint—Incomplete Arches—Minimum Thickness of Arch—Thrust in Semicircular Arch—Keystones not necessary.

THE first essential for the stability of an arch is that the abutments should be rigid; if they yield in the slightest degree the arch is extremely liable to fail. This is especially likely to happen at the angle of a building where there are arches on both faces, and an example of this was given in Lecture XXI. Whether an arch is made up of separate stones, called voussoirs, or is composed of half brick rings, or is a homogeneous mass such as concrete, it may be divided up into real or imaginary portions, each of which must be in equilibrium under the forces which traverse it. Fig. 478 shows a single voussoir in the haunch of an arch, subject to a vertical force due to its own weight and the superincumbent load, a thrust transmitted from the next voussoir above it, and a resisting force from the voussoir below it. The thrust and the weight combined into a parallelogram give the resultant, to which the equilibrant or resisting force must be equal and opposite. The ideal condition is when the thrust and resistance both act in the centre of and perpendicular to the joints through which they pass, but this seldom happens. The angle is usually somewhat more or less than a right angle, and the line of action is generally out of the centre, being nearer to either the intrados or extrados. The angle θ or its supplement must be less than the limiting angle of resistance, or sliding will occur, and the line of action must be within the middle third of the joint in order that the effect may be one of pure compression.

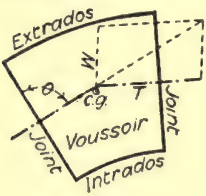


Fig. 478

The curve drawn through the points where the lines of thrust cut the joints will give the line of thrust or curve of thrust, or curve of equilibrium, which for perfect construction should be followed throughout its course by the centre line of the thickness of the arch. That is to say the curve of thrust is the theoretical shape of the arch, which requires to be clothed with material to make it practical. Other considerations of construction or æsthetics necessitate the use of a

simple curve such as a semicircle, segment of circle, ellipse, or gothic arch, without reference to the curve of thrust, but it is requisite that the curve of thrust should nowhere be so near the edge as to make the stress in excess of the safe load. In designing an arch the depth of arch ring may be assumed for trial as $D = n\sqrt{R}$, where D = depth at crown in feet, R = radius in feet, n = constant = 0·3 for block stone, 0·4 for brickwork, 0·45 for rubble stone.

In the investigation of the stability of an arch it will be found that many different curves of thrust may be drawn depending upon the points taken on the centre line and skewbacks where it is to pass through. Nature chooses that one of them which gives the minimum stresses, and we cannot be sure of selecting the right one in practice. Sometimes it is necessary to make a second trial in order to get a better curve, but usually the best result is obtained by making the curve pass through the upper edge of middle third at the centre line and lower edge of middle third at the skewbacks, except in semicircular or elliptical arches, where it is better to keep to the upper edge of middle third at the skewbacks also.

With regard to the position of the line of thrust in an arch, Prof. Goodman's "Mechanics Applied to Engineering," pp. 522 to 535, is worth careful study. He says, "An infinite number of lines of thrust may be drawn in for any given distribution of load. Which of these is the right one, is a question by no means easily answered, and whatever answer may be given, it is to a large extent a matter of opinion. For a full discussion of the question the reader should refer to I. O. Baker's 'Treatise on Masonry' (Wiley and Co., New York); and a paper by H. M. Martin, *Inst. C. E. Proceedings*, vol. xciii. p. 462."

Having determined the guiding points for the curve of thrust, the next step is to ascertain the amount of thrust at those points, or the primary thrusts. Fig. 479 shows the elevation of half an arch with its load, a length of 1 ft. being assumed as in the case of retaining walls, and the centre of gravity of the whole is found by suspension of the figure when cut out in drawing paper. A horizontal line is drawn from the point on the centre line through which the curve has to pass, to cut the vertical through the centre of gravity, and from the intersection a line is drawn through the point selected on the skewback. Then the total load is ascertained by taking the whole area of the figure with a planimeter, which load is then measured downwards from the point of intersection on centre of gravity line, and the parallelogram completed to give the horizontal thrust H, and thrust at skewback T; T will always be greater than H, and that is why some arch rings are made thicker towards the abutments.

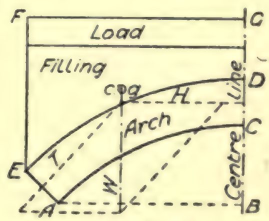


Fig. 479

The horizontal thrust in an arch under a uniformly distributed load is determined mathematically on the same principle as the compression in a girder flange, the formula $\frac{WL}{8d}$ applies equally to both, W

being the distributed load, L the span, and d the depth of girder or rise of arch. Generally speaking, the clear span of an arch between the abutments, and the rise of the arch from the springing line to the soffit at crown, will not appreciably differ from the true ratio of span and rise, which, however, ought strictly to be measured from the curve of thrust.

If the load were distributed uniformly over the ring of the arch, as in an arch ring without superincumbent weight, the curve of thrust would be a catenary such as occurs in the inverted form of a suspended chain. If it were distributed uniformly over the horizontal width of span the curve would be a parabola, but owing to the increased weight towards the abutments, the curve is not a true geometrical outline, but approximates more to the catenary.

A straight arch as in Fig. 480 is considered by many persons to be no true arch, but if the conditions be investigated it will be found that it virtually contains an arch ring half the depth of the straight arch,

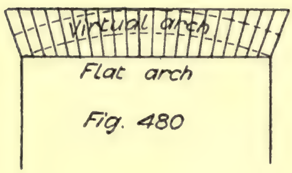


Fig. 480

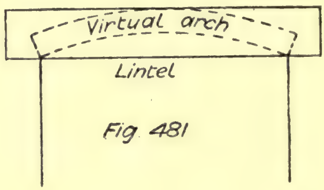
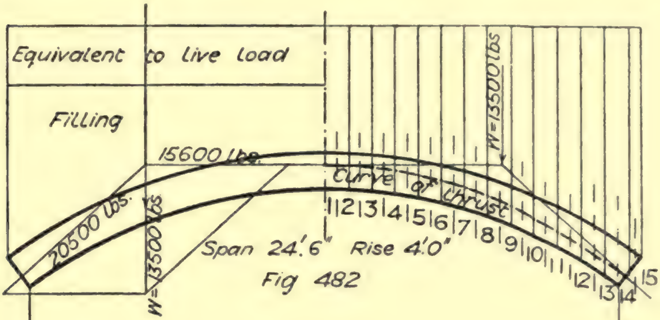


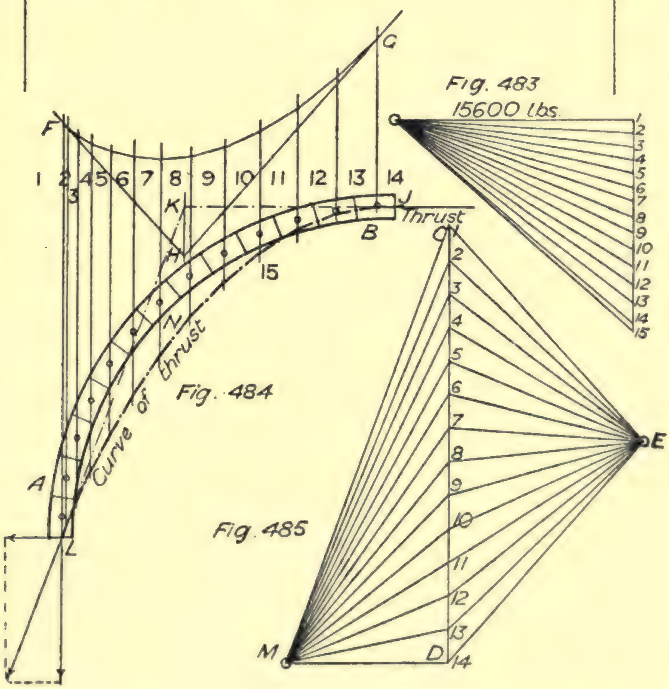
Fig. 481

and with a rise of the same amount, as shown by the dotted lines, and these lines determine the available strength of the arch. A straight lintel follows the same law, provided that the abutments are rigid, and although there is no appearance of a skewback, there is virtually one contained in it as shown in Fig. 481, being similar to an arch in one piece with the impost stones.

In the practical investigation of the stability of an arch, the outline is drawn to scale as in Fig. 482, where the part marked "filling" includes the weight of material in the superstructure, road metal, etc., for 1 ft. run, and above this is added the equivalent height of material, of the same weight, which would be equal to the external load due to the traffic, say 2 cwt. to 5 cwt. per foot super, according to circumstances. Upon one half of the diagram the centre of gravity is found as previously described, and a vertical line drawn through it. Then a horizontal through top of middle third of arch ring at crown, and from the intersection a line through the lower side of middle third at skewback, the total load on the half arch is then set off to scale and the parallelogram completed to give the primary thrusts. The next step is to divide up the other half-span into portions, which may be equal or unequal, and may consist of actual voussoirs with the superstructure resting upon each, but the simplest division is given by equidistant vertical lines, and the result is perfectly accurate as regards the production of the line of thrust. The weight of each portion and a line through the centre of gravity is next required; these are to be considered as force lines, and the spaces numbered as for ordinary reciprocal diagrams.

Then in the diagram, Fig. 483, take any pole, 0, set off the horizontal thrust, and on the load-line mark off to scale the separate loads 1–2, 2–3, 3–4, etc. Draw vectors as shown, and parallel to the vectors construct the funicular polygon upon the half elevation of arch, as





shown. The curved side of the funicular polygon gives the line of thrust, and the straight sides the direction of the two primary thrusts. Increasing the load does not alter the direction of the line of thrust, but only the intensity of the thrust. If the distribution of the load is altered, the shape of the line of thrust will be altered, bulging upwards at the part where the increased load occurs.

The stress at any point in an arch will be given by the formula $\frac{W}{A} \pm \frac{M}{Z}$, where W is the thrust passing through the given part, A the sectional area or depth of arch ring, M the bending moment or thrust multiplied by the distance from line of thrust to centre of depth of arch ring, and Z the section modulus of arch ring $\frac{1}{6}bd^2$. It is clear that the line of thrust, being within the middle third, would be no criterion of stability, as the maximum safe load might be overpassed, but at the same time it is a much more serious matter for the line of thrust to be outside the middle third, as the chances of over-pressure on the one side or tension on the other are certainly greater. The critical joint in an arch is that joint where the line of thrust most nearly approaches the intrados or extrados. It is usually situated at from 45 to 50 degrees from the crown.

If a portion of an arch be removed, and replaced by a rigid abutment giving the same bearing surface, the thrust in the remainder will be unaltered, and therefore if the curve of thrust be found for the complete side, it may be repeated as far as it applies on the incomplete side. It is a curious fact that there is a minimum depth of arch ring according to the span and rise of the arch, independent altogether of the load upon it. This arises from the necessity of keeping the curve of thrust sufficiently within the arch ring. A covering arch, 10 ft. span, formed of one ring of brickwork $4\frac{1}{2}$ ins. thick, set in cement, was built over a tank and had to carry its own weight only. Before the centering was removed, the arch bulged at the sides, about half-way between the springing and the centre, and when the centering was removed, the arch collapsed altogether. The reason will be seen by observing the position of the curve of thrust, as shown in Fig. 484, which is constructed from the reciprocal diagram, Fig. 485. The lettering in the illustrations shows the order of working. AB is the half elevation of the arch, which is divided up not into the actual bricks, but into convenient portions for the method of working. Draw a vertical line through the centre of gravity of each portion, representing the direction in which its weight acts. Number the spaces between these force lines and draw the line of loads CD. Select any pole, E, and draw vectors to CD. From any point, F, on line 1–2 of Fig. 484, and across space 2 draw a line parallel with the vector from 2 in Fig. 485. Now, in Fig. 485 draw DM parallel to JK, that is, horizontal, and CM parallel to KL. Join all points of CD with M, then these lines will represent the thrust throughout the arch. The "curve of thrust," N, is found as follows : From point L across space 2 draw a line parallel with M–2, then continue across space 3, parallel with M–3, and so on until B is reached. For the arch to be stable without tension on any part this curve should be everywhere within the middle third of the arch ring. If the arch be made to the same curve as the line of thrust, the arch will, of course, be under the best conditions of stability, provided that in finding the line of thrust all the circumstances, such as accidental load, wind, etc., have been taken into account.

It is a great mistake to suppose, as many do, that there is no outward thrust from a semicircular arch. Whatever the horizontal thrust may be at the crown, there is a similar horizontal equivalent on each

side acting outwards. An illustration of this occurs in Fig. 485, where the inclined thrust at the skewbacks may be resolved into two directions, vertical and horizontal, when it will be found that the horizontal component is equal to the horizontal thrust at the crown. It is a law of nature that the line of thrust takes the shortest possible course from the load to the support, so that if an arch ring be assumed to have no weight, the thrust from a concentrated load on the centre would pass in straight lines to the skewbacks ; and where a distributed load is carried, the horizontal thrust at crown is depressed by the load it meets as it passes each joint towards the skewback. It is also a common error to suppose that a keystone is necessary for the stability of an arch ; it is purely a matter of taste, and the fact that countless thousands of brick arches exist without a keystone ought to be a sufficient answer to the holders of the idea that it is necessary. In the fronts of buildings the arches are often finished with a keystone or similarly shaped block of gauged brickwork ; but this is for the sake of appearance only.

EXERCISES ON LECTURE XXIII

Q. 76. State the conditions of stability of a masonry arch.

Answer. The conditions of stability are—

(*a*) The line of resistance should everywhere fall within the middle third of the arch ring, so that the joints are not under tension at any part.

(*b*) The intensity of pressure should nowhere exceed the safe load upon the material.

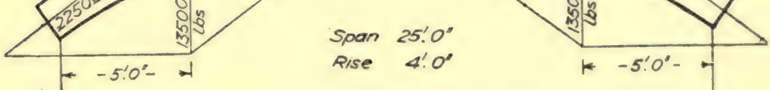


Fig. 486

(*c*) The line of thrust passing through any joint must not make an angle greater than the limiting angle of resistance of the joint to sliding.

(*d*) The abutments must be rigid to prevent the failure of the arch as a whole.

Q. 77. A brick-arched railway bridge 25 ft. span, 4 ft. rise, 4 half-brick rings, carries a total load of 27,000 lbs., the vertical through centre of gravity of each half being situated 5 ft. horizontally from the spring of the arch : determine the amount of the thrusts at crown and springing, and the maximum stresses. Scales, ¼ in. to 1 ft., 1 in. to 10,000 lbs.

Answer. See Fig. 486. Thrust at crown = 18,000 lbs., compression 5·3 tons per sq. ft. ; thrust at springing = 22,500 lbs., compression 6·7 tons per sq. ft. Had the line of thrust been through top of middle third at crown and bottom of middle third at springing, the compression would have been 9⅜ tons per sq. ft. at at crown and 12¼ tons per sq. ft. at springing.

Q. 78. A coke breeze concrete arch has a span of 8 ft., a soffit radius of 8 ft., a rise of 1 ft. ¾ in., a depth of arch ring of 1 ft. ¾ in. in centre, and radius of extrados

of 10 ft; it weighs 120 lbs. per cubic foot, and is loaded with a uniformly distributed load of 10 cwt. per foot super: what is the maximum compression in tons per square foot?

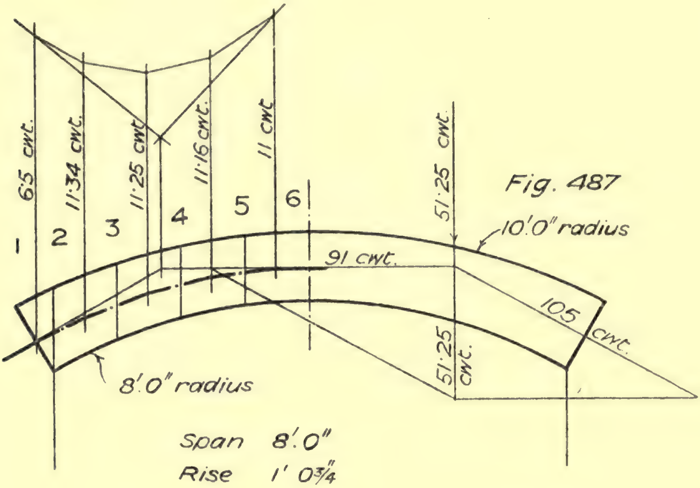
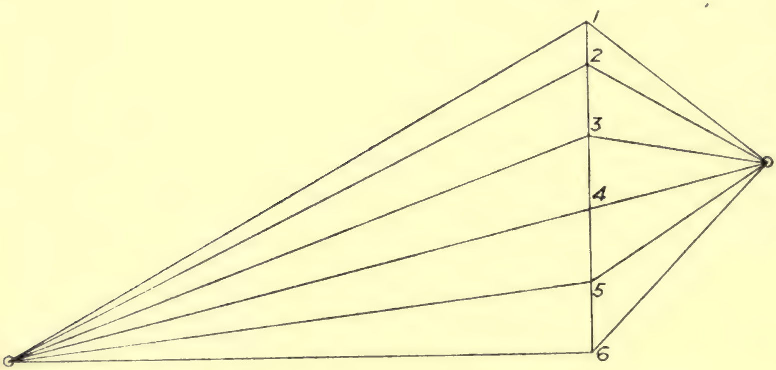


Fig. 487

Fig 488

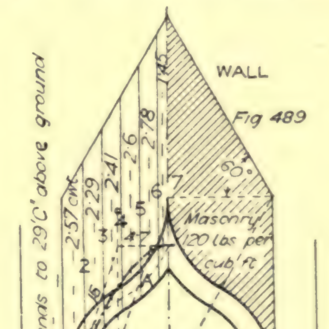


Answer. See Figs. 487 and 488. Taking the line of thrust in centre of arch ring at crown and springing, the maximum compression will be at the springing, and $= \frac{W}{A} = \frac{'105}{1\frac{1}{6}} = 90$ cwt. or 4·5 tons per sq. ft.

LECTURE XXIV

Estimation of Loads upon Arches—Load on Ogee Arch—Load on Culvert—Stability of Arch and Abutment—Arched Viaduct—Stop Abutments—Reservoir Arched Roofing—Concentrated Loads—Rolling Loads.

It is very difficult to judge the amount of load that will come upon an arch in a wall. The actual load will depend upon the nature of the material, the bonding, the strength of the mortar, the age of the work and other circumstances. When the wall extends for some distance on either side of the opening it will generally be safe to assume that the load upon the arch will be the weight of brickwork, etc., included in an equilateral triangle having the arch for its base, but when the piers at each side of the opening are narrow, say less than one-half the span of arch, the abutments will be more liable to yield, and it will be safer to allow for the whole weight of the superstructure over the span of the arch.



A curious case that arose is shown in Fig. 489. The material was a very light stone, 120 lbs. to the cubic foot, and it was assumed that the extreme load would be represented by the portion of the figure, of which one-half is shaded, and the arch itself. Then dividing up the arch and the load into vertical strips, drawing a line through the centre of gravity of each, and taking the lines to represent forces equivalent to the weight of each part, the mean centre of gravity of the load will be obtained by cutting out and suspending one-half. Then a horizontal through the centre of the crown to meet the mean centre of gravity line will give the point from which to draw the line to the centre of the skewback. The half total load being set off to scale on the vertical line through the mean centre of gravity, the parallelogram may be completed, giving the horizontal thrust at the crown as 4·7 cwt. and the inclined thrust at the skewback as 15 cwt. Then the load-line being set down in Fig. 490 and the horizontal and inclined thrust drawn in, the vectors to the other

points will give the direction of the line of thrust across the intermediate portions of the arch. The line of thrust passes outside the arch at A, and the calculations will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{8 \cdot 3}{1 \times \frac{1}{2}} \pm \frac{8 \cdot 3 \times 0 \cdot 42}{\frac{1}{6} \times 1 \times (\frac{1}{2})^2} = 16 \cdot 6 \pm 83 \cdot 6 = +100 \cdot 2 \text{ cwt.} = 5 \text{ tons per sq. ft.}$$

compression, and -67 cwt. = 3·35 tons per sq. ft. tension, which are about the extreme limits of stress for safety. At point B the calculations will be

$$\frac{\frac{2}{3}W}{d} = \frac{2 \times 12 \cdot 5}{3 \times \frac{1 \cdot 25}{12}} = 80 \text{ cwt.} = 4 \text{ tons per sq. ft. compression.}$$

An arch of this kind is not a true arch, but only an ornament that is kept in place by the adhesion of the mortar.

Where any material is interposed between the point of application of a concentrated load and the extrados of the arch, such as a bed of road material, the pressure of the load is spread in its transmission downwards, and may be considered to extend uniformly over a surface represented by the base of a cone of which the load is the apex. If the

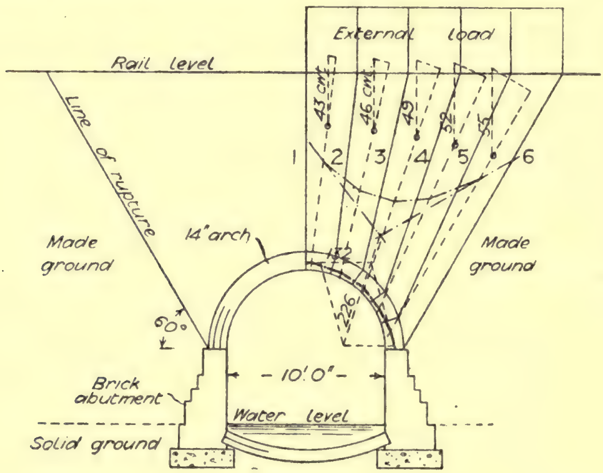


Fig. 491

interposed material is a brick wall, the pressure will extend over a surface represented by the base of a wedge of a length equal to the thickness of wall, and apex angle of 60 degrees.

As against this diffused loading may be taken the concentration that might arise in the case of a culvert in a new embankment while the filling is still in a granular condition. But this state may be aggravated by an external load acting on one-half of the arch only, as in Fig. 491. It will be seen that the line of rupture is taken as the limit of the earth pressing upon the arch, and this is divided over assumed voussoirs as

shown, and equally divided along the surface line. The centre of gravity of each portion being found, as at a, Fig. 491, the weight is set up above it to scale $= ab$, and a line drawn through a parallel with the abutment side of the portion of load under consideration, and draw the line bc at right angles to ca. Then ca is the amount and direction of the load to be taken into account in constructing the polar diagram, Fig. 492. The same being done for each portion and the vectors drawn, the funicular polygon may be constructed on Fig. 491 to give the position and direction of the mean centre of gravity line of the whole load. The next step is to draw the horizontal thrust line from centre of arch at crown to meet this line, and from the intersection drawing a line to centre of skewback, completing the parallelogram to obtain the primary thrusts. Then the curve of thrust may be drawn in as before.

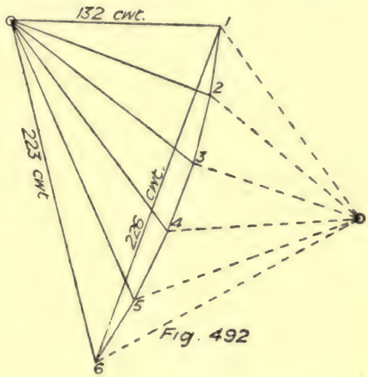


Fig. 492

To cause the line of thrust to follow exactly the outline of a semicircular arch the load would have to be increased to infinity over the ends; the vertical depth at any point $= a\frac{d^3}{h^3}$, where $a =$ depth of arch ring in centre, $h =$ height to soffit at given point, $d =$ rise of arch at centre, as shown by dotted lines on Fig. 493. Approximately the distribution of the loading can be shown by the loads required to produce equilibrium on a series of bars whose jointed ends lie in a semicircle, as in Fig. 493. The reciprocal diagram, Fig. 494, is formed by drawing vectors from point 12 and parallel to the bars, and cutting them off by a vertical line, say $\frac{1}{2}$ in. away, to give the amount of each load. Then set up these loads over the joints of the frame diagram, join the upper extremities, and the outline will be seen to be of the same character as that given by the material required above a semicircular arch to bend the line of thrust round the curve.

To find the stability of an abutment it is not necessary to construct the whole line of thrust in the arch. For example, Fig. 495 shows half an arch in a brick wall without surcharge or added weight. Find the weight of the arch and brickwork above, mark its centre of gravity and construct the parallelogram of thrusts. Find the centre of gravity of the remaining brickwork with pier, and its weight. Draw a vertical through the centre of gravity and produce the line of thrust from the arch. Complete the parallelogram and the resultant is found to cut the base 8 ins. from the outer edge, or 10 ins. from the centre. The vertical equivalent of the thrust will be $27 + 38\frac{1}{4} = 65\frac{1}{4}$ cwt. Then by the formula $\frac{W}{A} + \frac{M}{Z}$ the maximum stresses will be

$$\frac{65 \cdot 25}{3 \times 1} \pm \frac{65 \cdot 25 \times \frac{10}{12}}{\frac{1}{6} \times 1 \times 3^2} = +58 \text{ and } -14 \cdot 5,$$

that is, 58 cwt. per sq. ft. compression and 14·5 cwt. per sq. ft. tension. If no tension can be allowed the formula to be used for maximum compression will be $K = \frac{2}{3}\frac{W}{d} = \frac{2}{3} \times \frac{65 \cdot 25}{\frac{8}{12}} = 65 \cdot 25$ cwt. per sq. ft., or say $3\frac{1}{4}$ tons, which is quite safe.

An arched viaduct, or series of arches, is usually made with a stop abutment at each tenth arch. Between the other arches a thin pier is placed, just sufficient to carry the dead load and the portion of thrust

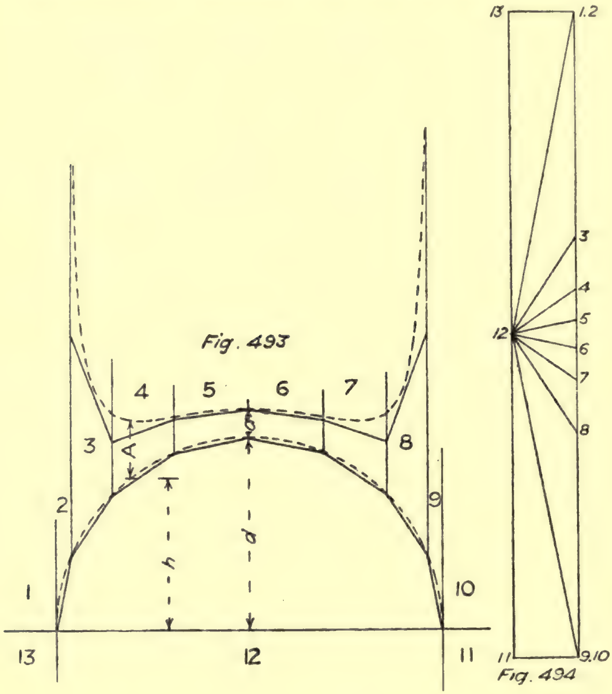
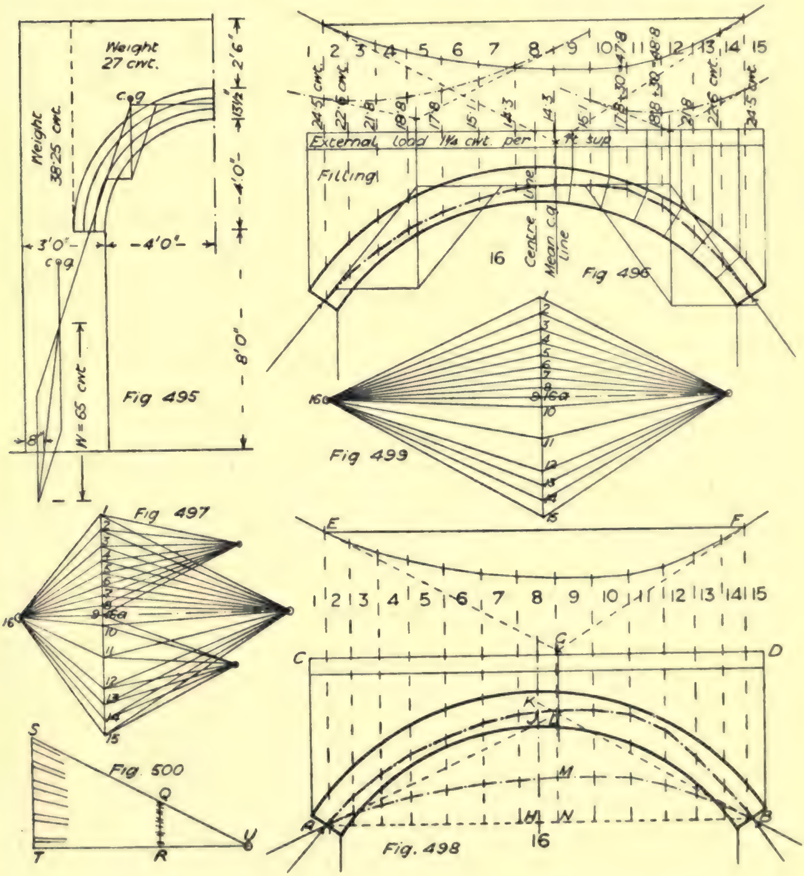


Fig. 493
Fig. 494

due to an external load on the one arch while the adjoining arch has only the structural load to carry. In the case of failure of one arch from any cause, the whole thrust from the adjoining arches would overthrow the piers, and the failure would spread the whole length of the viaduct except for the stop abutments, which are made sufficiently thick to resist the whole thrust of the full-loaded arch on one side, even when the arch on the other side has fallen. When reservoirs are covered with a series of arched roofs great care must be taken in placing any earth covering on top, as the loading of one span may cause the adjoining span to rise and collapse. The failure of an important reservoir at Madrid during construction in 1905 was probably due to this cause.

In the next diagram an example is given of irregular loading. Fig. 496 is an arch under a distributed load, and a concentrated load of 3 tons over one of the haunches, due to a rolling load coming on the

arch. The load of 3 tons may be assumed to be spread equally over two of the voussoirs, giving an addition of 30 cwt. to the load on each. It is necessary in a case of irregular loading such as this to find the mean centre of gravity of the whole load. This is done by means of the polar diagram in the centre of Fig. 497 on the right, and the corresponding large funicular polygon on the top of Fig. 496. The dotted closing lines of the funicular polygon give at their intersection the line of mean centre of gravity of the loads. The next step is to find the mean centre of gravity of the group of loads on either side, by means of the



two smaller polar diagrams in Fig. 497 and the corresponding funicular polygons in Fig. 495. Then the primary thrust parallelograms being drawn, the thrusts at abutments give the direction of the outer lines in the vector diagram on the left side of Fig. 497, the vectors of which give the parallels for the curve of thrust in the arch. Cases of irregular loading always give the greatest difficulty in finding the best position

for the curve of thrust. As it approaches the extrados and intrados at two points, the curve may be taken first to pass through the centre of arch ring on mean centre of gravity line, and through centre of skewback, but if, when the curve is drawn, a better line may be found by taking other points on mean centre of gravity line and skewback, it must be drawn over again from a new parallelogram. In the present case the curve appears to be in a very good position, and no alteration is needed, but it will be seen that the curve is not quite horizontal across the mean centre of gravity line, and it will, therefore, be well to check the work by another method of construction.

Draw out the arch and loads as before and proceed as in Fig. 498. Set down the load line 1–15 in Fig. 499 and select any pole 0; draw vectors, and parallel to these construct the funicular polygon EFG, then G will give the position of the mean centre of gravity line. From 0 draw 0–16a parallel with EF, giving the vertical reactions at the abutments 1–16a and 16a–15. Next from B, the centre of the skewback, draw a horizontal line to meet a vertical line through the centre of the arch in H. From H set up HJ = 1–16a, and HK = 16a–15. Join AJ and BK, and if produced they should intersect at L on the mean centre of gravity line. Then in Fig. 499 draw 1–16 parallel with AL, and 15–16 parallel with BL to obtain point 16. The remaining points on the load line are now drawn to point P, and the curve of thrust AMB constructed parallel with these lines. This does not pass through the arch at all, but nevertheless it is a true thrust curve, and only wants raising into its proper position by magnifying all its vertical ordinates as follows: Draw QR, Fig. 500, equal to MN, Fig. 498, and at any distance from QR set up a height ST equal to the distance from N to the centre of arch ring in Fig. 498. Join SQ and TR and produce to meet in a point U. Then the ordinates to the curve AMB being marked off on QR and lines drawn from the point V through the divisions on QR will give the corresponding magnified ordinates on ST. These being set off on the verticals of Fig. 498 will give the true curve of thrust.

EXERCISES ON LECTURE XXIV

Q. 79. Draw to a scale of ¼ in. to 1 ft. the half-elevation of a semicircular arch 10 ft. span, in three half-brick rings, with 12 ft. 6 in. of brickwork above the springing line and external load equivalent to 1 ft. 6 in. additional. On the other side of centre line, with the same springing line, draw the half elevation of an equilateral Gothic arch of the same thickness. The piers are 3 ft. wide and 6 ft. high. Obtain and draw the line of thrust for each arch and carry it down the pier, stating the maximum stresses on arches and piers.

Answer. See Figs. 501, 502, and 503. The calculations for the semicircular arch will be as follows:—

Arch $\frac{W}{A} \pm \frac{M}{Z} = \frac{60}{1{\cdot}125} \pm \frac{60 \times \frac{5}{12}}{\frac{1}{6}(1 \times 1{\cdot}125^2)} = 53{\cdot}3 \pm 118{\cdot}5 = 171{\cdot}8$ cwt., or 8·59 tons per sq. ft. compression, and 65·2 cwt. or 3·26 tons per sq. ft. tension.

Abutment $\frac{W}{A} \pm \frac{M}{Z} = \frac{111}{3} \pm \frac{111 \times 1{\cdot}25}{\frac{1}{6}(1 \times 3^2)} = 37 \pm 92{\cdot}6 = 129{\cdot}6$ cwt., or 6·48 tons per sq. ft. compression, and 55·6 cwt. or 2·78 tons per sq. ft. tension.

Fig. 501

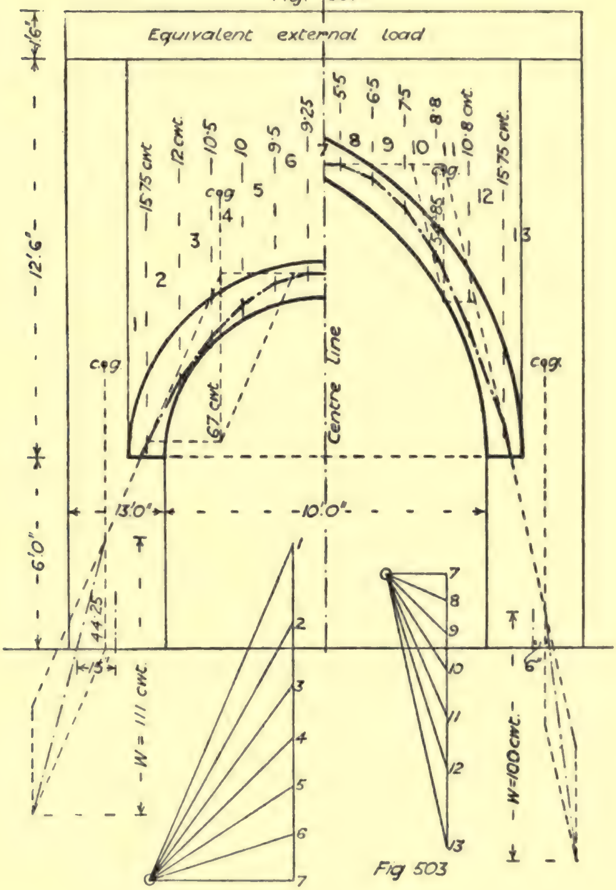


Fig 503

Fig 502

Fig. 505

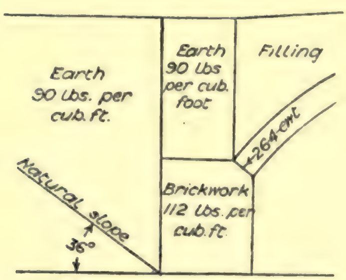
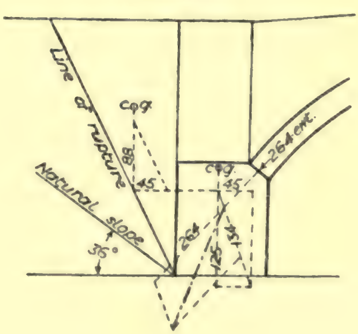


Fig. 504



The calculations for the Gothic arch will be as follows:—

Arch $\frac{W}{A} \pm \frac{M}{Z} = \frac{40}{1 \cdot 125} \pm \frac{40 \times 0 \cdot 2}{\frac{1}{6}(1 \times 1 \cdot 125^2)} = 35 \cdot 5 \pm 38 = 73 \cdot 5$ cwt., or 2·67 tons per sq. ft. compression, and 2·5 cwt. or 0·125 ton per sq. ft. tension.

Abutment $\frac{W}{A} \pm \frac{M}{Z} = \frac{100}{3} \pm \frac{100 \times 0 \cdot 5}{\frac{1}{6}(1 \times 3^2)} = 33 \cdot 3 \pm 33 \cdot 3 = 66 \cdot 6$ cwt., or 3·33 tons per sq. ft. compression and no tension.

Q. 80. Fig. 504 represents an abutment for a brick arch with a given thrust, find the final resultant allowing for the weight of abutment and filling and thrust of earth.

Answer. See Fig. 505, the final resultant being shown by stroke-and-dot line.

LECTURE XXV

Theory of the Modern Arch—Curved Struts and Ties—Arched Ribs.

An arch of brick or stone masonry is an inelastic arch ; an arch of steel or ferro-concrete is an elastic arch, or what is commonly known as an arched rib.

The following description from *The Builders' Journal* of Jan. 17, 1906, puts the case of the modern arch very clearly :—

"The 'theory' of arch design remained in an elementary state until the last century, when engineers were led to examine critically into all structural means in order to fit conditions of modern industry. They took the masonry arch and developed it a certain way, but they went little further than the greatest of the older builders, though they raised the average knowledge of the subject. Their empirical inelastic arch theory assumed the arch stones to be rigid, and required the line of resistance within the arch ring (for safety within the middle third). In the latter part of last century, however, the theory of arch design began to advance again, particularly with continental engineers. The construction of arches in metal marked the point of departure. The theory of elastic flexure was then applied to the theory of the arch, and the arched rib came into being. Instead, then, of it being a necessity for the line of resistance to be confined to the arch ring, it was easy, at small sacrifice of economy of metal, to stiffen the arch ring against flexure. This was now required to resist combined thrust and bending (with shear as a corollary). This is the elastic flexure theory of arch design. The older inelastic theory had led to the adoption of pin and similar joints at the crown and springing so as to give greater precision in design or to afford control of the line of resistance. The pin joints simplified the modern elastic flexure theory by allowing unknown quantities to be exactly determined, just as the fixing of one end of a roof truss and the freedom of the other simplifies the theoretical determination of stresses in such structural members. With the two-hinged arch (pin joints here being at the springing) or the three-hinged arch (a pin joint here being at the crown as well as at each springing), the analysis of stresses must be so conditioned as to make bending moments zero at the pin joints, whatever may be the condition of loading. This flexure theory has increased the range of the arch to immense spans. The Clifton Arch rib bridge at Niagara has a span of 800 ft. The arched rib, too, is a graceful form of construction, and is much superior in line to the truss and cantilever, and we may therefore look forward to the future

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development of engineering structures with more equanimity than formerly. The modern theory of arch design finds its architectural application to reinforced concrete, of which there are examples with and without hinges."

We have already in Lecture XI. dealt with combined longitudinal and transverse stresses, in Lecture XII. with pillars or stanchions eccentrically loaded, and in Lecture XVIII. with arched rib roof trusses, and we now proceed to consider bent struts and curved members generally. These matters are all very closely allied.

First let us consider the terms used in connection with curves and

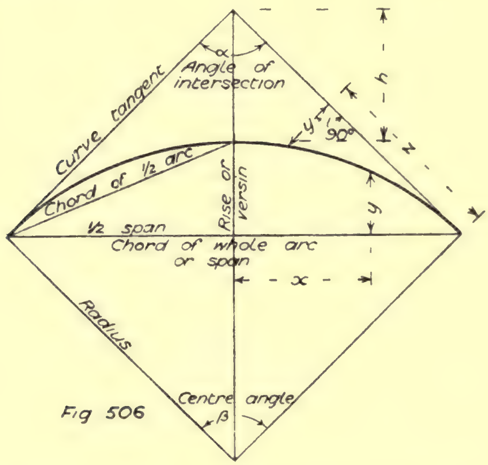


Fig 506

the formulæ for obtaining the different elements, as it may be useful in connection with other matters.

These are shown upon Fig. 506, and the formulæ will be as follows :—

$$\text{Chord of } \tfrac{1}{2} \text{ arc} = \sqrt{(\tfrac{1}{2}\text{ span})^2 + \text{rise}^2}$$

$$\text{Radius} = \tfrac{1}{2}\left[\frac{(\frac{1}{2}\text{ span})^2}{\text{rise}} + \text{rise}\right]$$

$$\cos\beta = 1 - \frac{(\text{chord of whole arc})^2}{2 \times \text{radius}^2}$$

whence β is obtained from table and $\alpha = 180 - \beta$

$$\text{Curve tangent} = \text{R}\cot\frac{\alpha}{2}$$

$$h = \sqrt{(\text{curve tan})^2 + \text{rad.}^2} - \text{radius}$$

$$\text{Rise} = \text{rad.} - \sqrt{\left(\text{rad.}^2 - \frac{\text{chord}^2}{4}\right)}$$

$$y \text{ for } x = \sqrt{\text{rad.}^2 - x} - (\text{rad.} - \text{rise})$$

$$y \text{ for } z = \text{rad.} - \sqrt{\text{rad.}^2 - z^2}$$

It may help to throw some light on the subject generally if we compare the action of a thrust on a curved member with the bending moment produced by a concentrated load on the centre.

The end thrust on a curved strut or roof member produces a bending moment = Tv, where T = thrust, v = versin of curve.

The condition of a curved strut may be illustrated by reciprocal

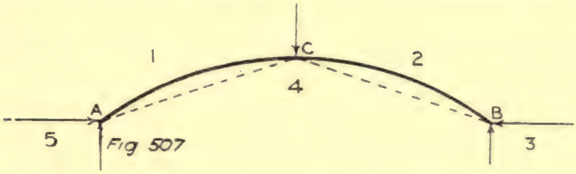


Fig 507

diagram. Let AB, Fig. 507, be a curved strut, as crane jib, upper flange of bowstring girder, principal rafter of curved roof, etc., under known end thrusts. Draw chords AC, CB, and vertical force lines as shown, and number the spaces. Then for reciprocal diagram, Fig. 508, draw 2–3 = thrust, and draw 2–4, 3–4 ; 4–1, 2–1 ; and 1–5, 4–5.

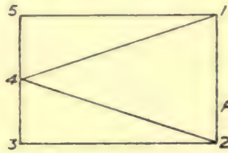


Fig 508

Then 1–2 will be the external load required to maintain equilibrium, or it will represent the virtual transverse stress on the strut resisted by its own stiffness, the bending moment on the strut being $\frac{Wl}{4}$ the same as would be produced in a straight girder of the same span under the same load.

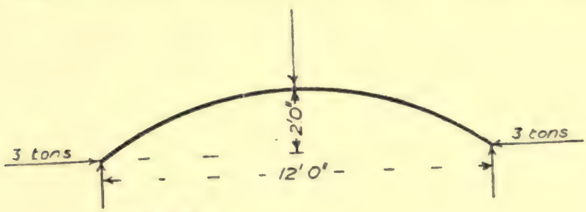


Fig 509

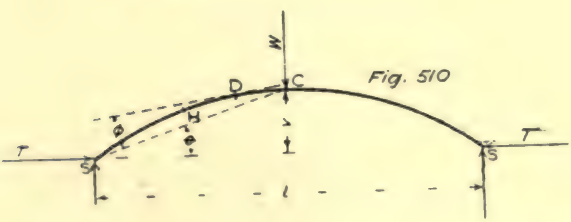


Fig. 510

Example, Fig. 509.—Bending moment by thrust = 3 tons × 2 ft. = 6 ton-ft. Bending moment by reciprocal diagram, 6 : 2 : : 3 : 1,

the vertical reaction each end = 2 tons load, $\frac{Wl}{4} = \frac{2 \times 12}{4} = 6$ ton-ft.

Again, we have in Fig. 510, $\frac{Wl}{4} = Tv$, whence $W = \frac{4Tv}{l} = 2\ T \tan \theta$,

and $T = \frac{Wl}{4v} = \frac{W}{2 \tan \theta}$

Direct thrust at C and S = T

" " " " $H = T \sec \theta = \frac{Wl}{4v} \sec \theta$

" " from H to C and S = $T \sec \theta$ decreasing to T

" " at any point $D = \frac{Wl}{4v} \sec \theta \cos \phi = T \sec \theta \cos \phi$

so that it is greater at H than anywhere else with load W because the thrust along the chord of half the arc is greater than the horizontal thrust in the ratio $\frac{\text{chord of } \frac{1}{2} \text{ arc}}{\frac{1}{2} \text{ span}}$.

If a circular arch rib be resisted by reactions tangent to the curve, the concentrated load for equilibrium would be given as in Fig. 511 and Fig. 512, where the order of working is 2–3, 3–1, then 1–2 represents

Fig. 511

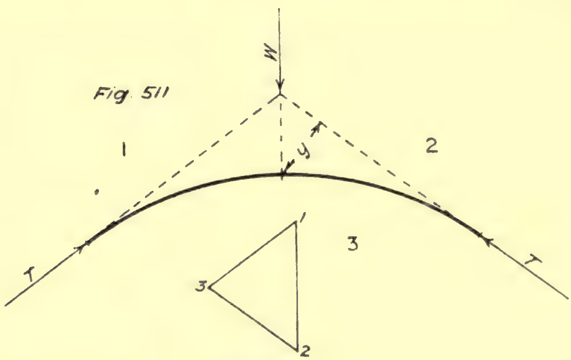


Fig. 512

the balancing force, and the maximum bending moment would appear to be T y, supposing that an actual resistance occurs at the point shown for the application of W.

Upon consideration it will be seen that the concentrated load in Fig. 507 does not represent the true condition of the balancing forces in a bent beam under thrust. The resistance to the thrust being distributed along the strut, a distributed load of double the concentrated should really be taken, but this is not capable of being dealt with completely by the parallelogram of forces or reciprocal diagram. The nearest approach will be made by taking a series of loads equally distributed along the curve and obtaining a line of thrust as for an ordinary arch, as in Fig. 513. With a sufficiently large scale we should

find that the line of thrust produced by means of Fig. 514 would be a catenary curve, almost identical with the circular arc. The diagonal resultant of the reactions, as shown on the left, would be tangent to the catenary as shown by the full line, while the tangent to the circular curve is shown by the dotted line. The bending moment at the centre by calculation will be

$$2 \times 6 - \tfrac{1}{2}(5{\cdot}38 + 3{\cdot}94 + 2{\cdot}38 + 0{\cdot}8) = 12 - 6{\cdot}25 = 5{\cdot}75 \text{ ton-ft.}$$

instead of 6 ton-ft., the difference being due to the splitting up of the

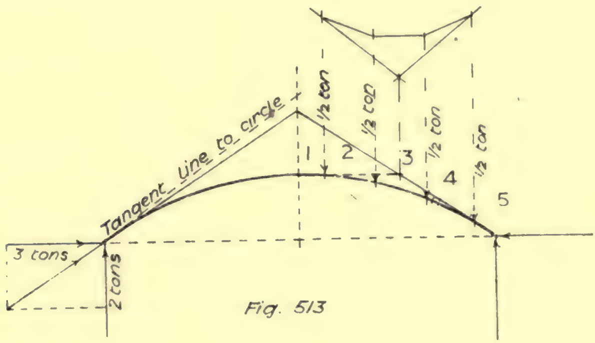


Fig. 513

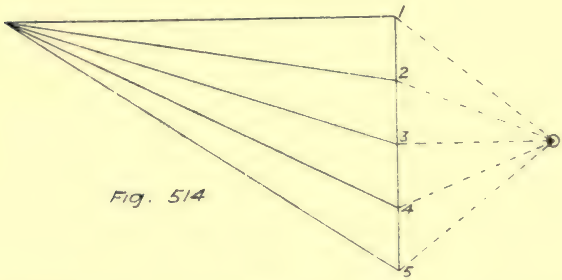


Fig. 514

load into concentrated portions. Under a fully distributed load the bending moment at any point x might be taken as $= \dfrac{4\mathrm{T}v(l-x)}{l^2}$ which would give a parabolic outline. The bending moment is, however, only part of the stress to be resisted by the curved strut, it will have in addition the direct load, the combined effect being given by the formula $\dfrac{\mathrm{W}}{\mathrm{A}} \pm \dfrac{\mathrm{M}}{\mathrm{Z}}$.

Now consider a $2\tfrac{1}{2}$-in. by $2\tfrac{1}{2}$-in. by $\tfrac{1}{4}$-in. steel tee made up of plain rectangular sections, as Fig. 515, the sectional area will be

$$\tfrac{1}{4}[2\tfrac{1}{2} + (2\tfrac{1}{2} - \tfrac{1}{4})] = 1{\cdot}1875 \text{ sq. in.}$$

The neutral axis from the edge of web will be

$$y = \frac{\mathrm{BD}^2 - bd^2}{2(\mathrm{BD} - bd)} = \frac{2{\cdot}5 \times 2{\cdot}5^2 - 2{\cdot}25 \times 2{\cdot}25^2}{2(2{\cdot}5 \times 2{\cdot}5 - 2{\cdot}25 \times 2{\cdot}25)} = 1{\cdot}783 \text{ in.,}$$

and from face of table $x = 2{\cdot}5 - 1{\cdot}783 = 0{\cdot}717$ in. The moment of inertia

$$I = \frac{(BD^2 - bd^2)^2 - 4BDbd(D-d)^2}{12(BD - bd)}$$

$$= \frac{(2{\cdot}5 \times 2{\cdot}5^2 - 2{\cdot}25 \times 2{\cdot}25^2)^2 - 4 \times 2{\cdot}5 \times 2{\cdot}5 \times 2{\cdot}25 \times 2{\cdot}25(2{\cdot}5 - 2{\cdot}25)^2}{12(2{\cdot}5 \times 2{\cdot}5 - 2{\cdot}25 \times 2{\cdot}25)}$$

$= 0{\cdot}703$ in inch units.

Then section modulus for web side $Z_y = \frac{I}{y} = \frac{0{\cdot}703}{1{\cdot}783} = 0{\cdot}394$, and for table side $Z_x = \frac{I}{x} = \frac{0{\cdot}703}{0{\cdot}717} = 0{\cdot}98$.

$$\text{Radius of gyration squared} = r^2 = \frac{I}{A} = \frac{0{\cdot}703}{1{\cdot}1875} = 0{\cdot}592.$$

It is assumed that this tee section is to be used as a bent strut or tie

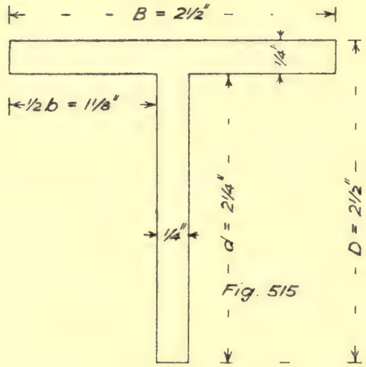


Fig. 515

across the top of the trussed sides of a footbridge, as in Fig. 516. The chord of arc may be taken as $4\frac{1}{2}$ ft. and the rise 18 in. Suppose a horizontal thrust of 2 cwt. against one end, the other being rigid, what

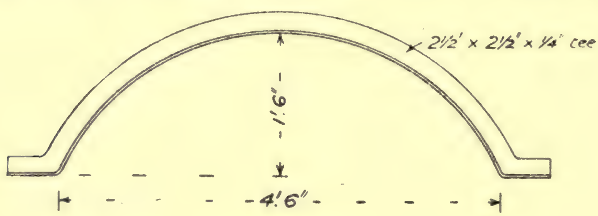


Fig. 516

stresses will be induced in the bar? By a common, although incomplete, method of calculation we have $Tv = \frac{2}{20} \times 18 = 1{\cdot}8$ ton-in. bending moment, the moment of resistance being ZC; then for web side (web

upwards) $C = \frac{1 \cdot 8}{0 \cdot 394} = 4 \cdot 57$ tons per sq. in. tension, and for table side (web upwards) $C = \frac{1 \cdot 8}{0 \cdot 98} = 1 \cdot 84$ tons per sq. in. compression. This, however, has been previously shown to be incorrect by reason of the omission of the thrust. By the formula

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{1}{1 \cdot 1875} + \frac{1 \cdot 8}{0 \cdot 98} = 0 \cdot 84 + 1 \cdot 84 = 2 \cdot 68 \text{ tons per sq. in.}$$

compression in table and $0 \cdot 84 - 4 \cdot 57 = 3 \cdot 73$ tons per sq. in. tension in web. Whether these stresses are permissible is generally considered to depend upon the crippling stress of the section as a column.

By the Rankine-Gordon formula for mild steel struts (Alexander and Thomson's "Elementary Applied Mechanics," p. 415),

$$p \text{ tons per sq. in.} = \frac{6\frac{1}{2} \text{ tons}}{1 + \frac{1}{7000}\frac{l^2}{r^2}}$$

but this is for hinged ends, and to allow for fixed ends the length l may be taken as $\frac{6}{10}$ of the actual length L. It may be open to question whether the full length round the curve should be taken, or the length of the chord. It will probably be sufficient if we take the full chord length for l.

Then $$p = \frac{6\frac{1}{2}}{1 + \frac{1}{7000}\left(\frac{54^2}{0 \cdot 5921}\right)} = \frac{6 \cdot 5}{1 + 0 \cdot 7035} = 3 \cdot 8 \text{ tons per sq. in.}$$

safe load in compression, while the actual load is $2 \cdot 68$ tons per sq. in.

For the next illustration of the stresses in bent beams take the case of a 3-in. by $1\frac{1}{2}$-in. steel channel section, as Fig. 517, bent to a semicircle 10-in. mean radius, with the flanges outwards, held at the ends and carrying a concentrated hanging load in the centre, it is required to find the ultimate crippling load W. The sectional area may be taken as $1\frac{1}{2}$ sq. in., the neutral axis 1 in. from edge of section, and the moment of inertia $0 \cdot 25$ in. units. Then R = radius = 10 in., y = distance from neutral axis to furthest edge of section = 1 in., I = moment of inertia = $0 \cdot 25$ in., A = sectional area = $1 \cdot 5$ sq. in., x = leverage of diagonal pull, or versine from line of tension to curve of neutral layer = approximately 3 in., F = ultimate tensile strength per sq. in. = 28 tons for steel, 20 tons for wrought iron. Then on the principle that $F = \frac{W}{A} + \frac{M}{Z}$, instead of W will be used the diagonal pull which in a semicircular bar will be $0 \cdot 7W$, and

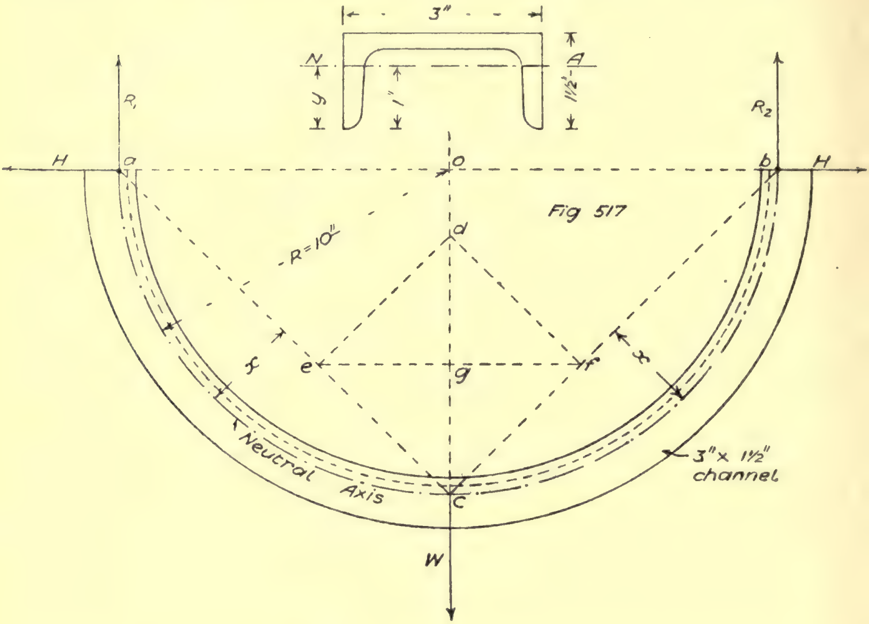
$$F = \frac{0 \cdot 7W}{A} + \frac{0 \cdot 7Wx}{I \div y}, \text{ or } 28 = \frac{0 \cdot 7W}{1 \cdot 5} + \frac{0 \cdot 7W \times 3}{\frac{0 \cdot 25}{1}}$$

or $28 = 0 \cdot 4\dot{6}W + 8 \cdot 4W = 8 \cdot 8\dot{6}W$, whence $W = \frac{28}{8 \cdot 86} = 3 \cdot 16$ tons. This

can be stated in another form which, however, does not show so readily the origin of the formula, viz.

$$W = \frac{F}{\frac{0\cdot7xy}{I} + \frac{0\cdot7}{A}} = \frac{28}{\frac{0\cdot7 \times 3 \times 1}{0\cdot25} + \frac{0\cdot7}{1\cdot5}} = \frac{28}{8\cdot4 \times 0\cdot46} = \frac{28}{8\cdot86} = 3\cdot16 \text{ tons.}$$

Graphically the forces at work can be found as follows :—Join *ca*, *cb*



and let dc = weight W. From d draw de and df parallel to bc and ac. Join ef, cutting cd in g. Then the vertical reactions $R_1R_2 = dg$ and $cg = \frac{W}{2}$. The horizontal tension $H = eg$ and $fg = \frac{W}{2}$ for semicircular beam. The diagonal force tending to straighten curved beam $= cf$ and ce.

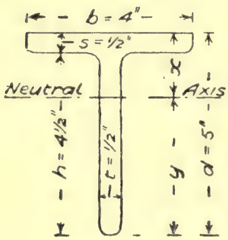
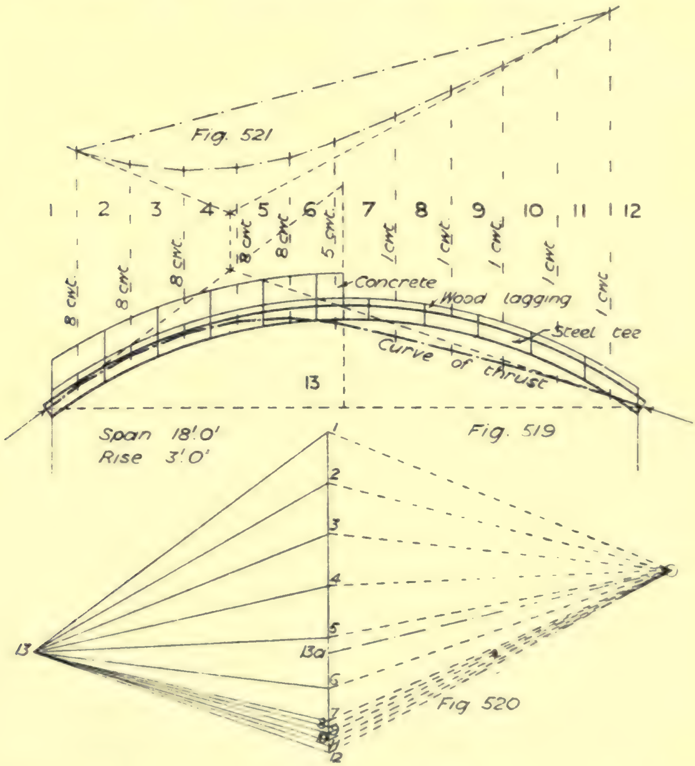


Fig. 518

Now take the case of an arched rib used in centering and consisting of bent 4-in. by 5-in. by ½-in. steel tees, as Fig. 518, 3 ft. centre to centre, used web downwards to support the lagging of an arch of 18-ft. span and 3-ft. rise. Assuming the arch to be built on half the span before the other half is commenced, the problem is to find what will be the maximum stresses produced in the tee bars. First draw the outline as in Fig. 519, divide it up into assumed voussoirs, and find the weight of each. Suppose these to be as marked on diagram, then find the centre

of gravity of all the loads by polar diagram, Fig. 520, and link polygon, Fig. 521, and obtain the vertical reactions by drawing a line $0 - 13a$ in Fig. 520, parallel with the closing line of the link polygon. From the centre of the horizontal line joining the tops of middle-thirds at abutments in Fig. 518, set up the reaction $13a - 1$ and join the top point to top of middle-third at left-hand abutment. From the intersection of this last line with the mean centre of gravity line draw a line to top of middle-third at right-hand abutment. From points 1 and 12 in Fig. 520 draw lines parallel with these two reactions, giving point 13 by their



intersection. Join 13 to the points on the load line and by parallel lines construct the curve of thrust on Fig. 519. The area of the section may be taken as 4·25 sq. ins., the distance of neutral axis from edge of web $y = 3{\cdot}4$ in., and from face of table $x = 5 - 3{\cdot}4 = 1{\cdot}6$ in. The moment of inertia $\mathrm{I} = 10{\cdot}45$ in. units. Then section modulus for web side

$$Z_y = \frac{\mathrm{I}}{y} = \frac{10{\cdot}45}{3{\cdot}4} = 3{\cdot}07,$$

and for table side $Z_x = \frac{\mathrm{I}}{x} = \frac{10{\cdot}45}{1{\cdot}6} = 6{\cdot}53$. The stress per sq. in. may

now be calculated by the formula $\frac{W}{A} \pm \frac{M}{Z}$, in this case W = length of 9–13 in Fig. 520 = 47·5 cwt., A = 4·25 sq. in., M = 47·5 × 11 = 522 cwt.-in., $Z_x = 6{\cdot}53$ and $Z_y = 3{\cdot}07$.

Then $$\frac{W}{A} + \frac{M}{Z_y} = \frac{47{\cdot}5}{4{\cdot}25} + \frac{522}{3{\cdot}07} = 11{\cdot}1 + 170 = 181{\cdot}1 \text{ cwt.}$$

or 9·05 tons per sq. in. compression in web, and

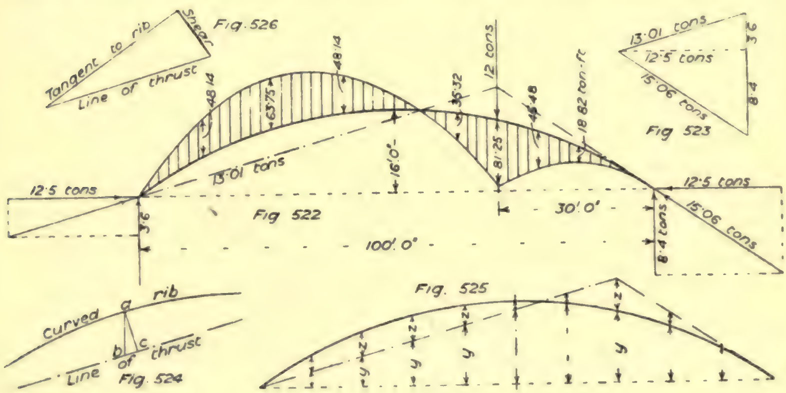
$$\frac{W}{A} - \frac{M}{Z_x} = \frac{47{\cdot}5}{4{\cdot}25} - \frac{522}{6{\cdot}53} = 11{\cdot}1 - 80 = -68{\cdot}9 \text{ cwt.}$$

or 3·45 tons per sq. in. tension in table; whereas if the section had been turned up the other way, web upwards, the stresses would have been 11·1 + 80 = 91·1 cwt., or 4·55 tons per sq. in. compression in table, and 11·1 − 170 = − 158·9 cwt., or 7·94 tons per sq. in. tension in the web, which would have been a more satisfactory arrangement.

An important and interesting question upon an arched rib was set in the Honours Practical Geometry, 1904, as follows:—"The form of an arched rib is a circular arc of 100-ft. span on 16 ft. rise, the supports being at the same level. It is hinged at the ends and loaded with a weight of 12 tons at a horizontal distance of 30 ft. from one end. The horizontal thrust of the arch is known to be 12·5 tons. Draw a diagram of bending moment for the arch. Indicate the places where the shearing force, thrust, and bending moment on the rib have their maximum values, and give these values." The solution of it given below is by R. E. Marsden. Looking at the loaded arched rib, Fig. 522, it can be seen that, if the ends are hinged as stated, the load tends to flatten the curve of that part of the arch beneath the load, and to increase the convexity of the opposite half upwards. To solve the problem, set out the load line, Fig. 523, and use a pole for vector polygon distant from this load line equal to 12·5 tons on the force scale. If this polar distance is set out on a horizontal line drawn 3·6 tons from the top end of the load line, it will save drawing two funicular polygons in Fig. 522. Next draw the line of thrust, as shown by chain-dotted lines in Fig. 522, this gives the form of structure required to give a horizontal thrust of 12·5 tons while supporting the given load of 12 tons. Where this line of thrust crosses the arched rib, there is a change in the bending moment from positive to negative; that is to say, this is a point of no bending moment.

To find the amount of bending moment on any point of the arch, drop a perpendicular on to the line of thrust, and measure this amount, as *ac* in Fig. 524 on the length scale, and multiply by the amount of thrust (13·01 tons) in the line of thrust. The same result will be obtained by dropping a vertical line *ab* from a point on the arched rib, and multiplying this length by the horizontal thrust 12·5 tons. In both cases the bending moment is given in ton-feet. The bending moments for various points on the arch have been calculated in this way, and plotted on verticals having one end on arched rib, and passing through the point at which the bending moment was calculated. In Goodman's

"Mechanics Applied to Engineering" a proof is given that if a number of equidistant spaces are marked off along the line of the arched rib, and verticals drawn through these as in Fig. 525, then when the length of the ordinate up to the arch rib y, Fig. 525, is multiplied by that portion lying between the line of thrust and the arched rib z, Fig. 525, the total of all the minus or negative quantities thus obtained should equal the total of all the plus or positive quantities. When the line of thrust passes below the rib, the ordinates between are minus quantities; and when it passes above, they are positive or plus. Tested in this way, Fig. 522 is proved to be correct. The shearing force is determined by drawing a tangent to the rib through the point, and resolving a force equal and parallel to the line of thrust immediately below the point along this line, and along one



perpendicular to the rib, as in Fig. 526. The shear is given by the component of thrust at right angles to the direction of the rib. The greatest bending moment is found under the load, which is also the place of greatest shear. The horizontal thrust could not have been determined had it not been given by the examiners, unless other data had been provided. The section of the arch, affecting its stiffness, etc., has something to do with the determination of the horizontal thrust, for the section may be so stiff in proportion to its length that it exerts no more thrust on the supports than does an ordinary straight girder. The condition of loading greatly affects the amount of horizontal thrust, as also the method of fixing. A good method of understanding some of the stresses that result in a curved beam from loading is to substitute a lattice girder of the same shape, and work out the stresses in the individual members. By doing this for two or three different systems of lattice work a check could be had on the result. A discussion of metal arches under all kinds of conditions will be found in Perry's "Applied Mechanics," Goodman's "Mechanics Applied to Engineering," Rankine's "Civil Engineering," and also in "Statically Indeterminate Structures and the Principles of Least Work" by Martin (35 and 36, Bedford Street, Strand, London). Other works will be found in the Patent Office library.

EXERCISES ON LECTURE XXV

Q. 81. The curved tie-bar of a roof is composed of a standard steel tee, 4-in. table and 5-in. web, both $\frac{3}{8}$ in. thick. The section area is 3·25 sq. in., the moment of inertia 7·77, neutral axis from edges of section 1·47 in. and 3·53 in. The bar is curved to a versine of 3 in. in a chord length of 5 ft., web upwards, and the tensile stress on the chord line is 5 tons. Find the maximum intensity of the

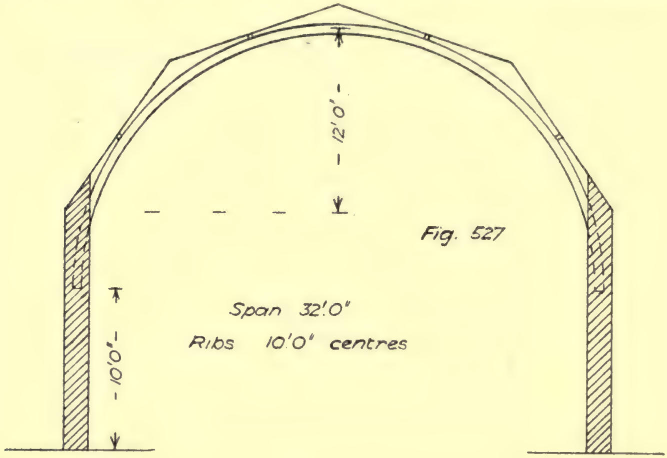
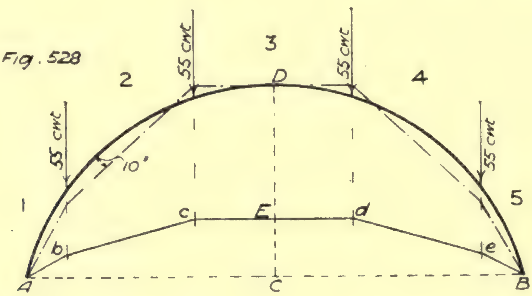


Fig. 527

stresses in tension and compression. Would any serious difference have occurred in the stresses if the web had been downwards?

Answer. The first step will be to find the section modulus for each side of the neutral axis. $Z_x = \frac{I}{x} = \frac{7 \cdot 77}{3 \cdot 47} = 5 \cdot 28$, $Z_y = \frac{I}{y} = \frac{7 \cdot 77}{3 \cdot 53} = 2 \cdot 2$. Then $f = \frac{W}{A} \pm \frac{M}{Z}$ will be the formula to use, or more correctly $-\frac{W}{A} + \frac{M}{Z_y}$ for the web, and $-\frac{W}{A} - \frac{M}{Z_x}$ for the table side. Then $-\frac{W}{A} + \frac{M}{Z_y} = -\frac{5}{3 \cdot 25} + \frac{5 \times 3}{2 \cdot 2} = -1 \cdot 538 + 6 \cdot 818 = 5 \cdot 28$ tons per sq. in. maximum compression in web, and

Fig. 528



$$-\frac{W}{A} - \frac{M}{Z_x} = -\frac{5}{3 \cdot 25} - \frac{5 \times 3}{5 \cdot 28} = -1 \cdot 538 - 2 \cdot 841 = 4 \cdot 38 \text{ tons per sq. in.}$$

maximum tension in table.

Web downwards the stresses would have been

$$-\frac{W}{A}-\frac{M}{Z_y}=-\frac{5}{2\ 5}-\frac{5\times 3}{2{\cdot}2}=-1{\cdot}538-6{\cdot}818=8{\cdot}356 \text{ tons per sq. in.}$$

maximum tension (in web), and

$$-\frac{W}{A}+\frac{M}{Z}=-\frac{5}{3{\cdot}25}+\frac{5\times 3}{5{\cdot}28}=-1{\cdot}538+2{\cdot}841=1{\cdot}303 \text{ tons per sq. in.}$$

maximum compression (in table), showing an increase in the maximum stress of $100\left(\frac{8{\cdot}356}{5{\cdot}28}\right)-100=58$ per cent.

Q. 82. Fig. 527 represents a curved rib, consisting of a (B.S.B. 9) 6 in. by 4½ in. by 20 lbs. rolled steel joist, supporting the roof purlins as shown. Assuming the ribs to be 10 ft. centres and the load to be ½ cwt. per ft. super on the roof, draw the curve of thrust to pass through centre of rib at crown and abutments, and calculate the stresses on the rib.

Answer. Draw the frame diagram, Fig. 528, and the load from each purlin will be $\frac{11\times 10}{2}=55$ cwt. as shown. Set down the load line, Fig. 529, and select any pole O, but as the loading is symmetrical O is taken on a line central between 1 and 5. Draw vectors to the points on the load line and parallel with these

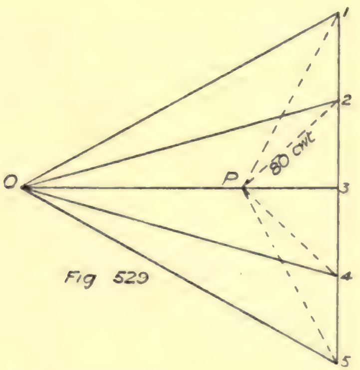


Fig 529

vectors construct the curve of thrust A*bcde*B. This curve does not pass through point D, another pole P must therefore be selected so that the distance

$$P3=O3\times\frac{CE}{CD}.$$

Then, as before, draw vectors and complete the curve of thrust passing through the points ADB. The maximum stress will then be across spaces 2 and 4, where the thrust is 80 cwt. = 4 tons and the leverage 10 in., then

$$\frac{W}{A}\pm\frac{M}{Z}=\frac{4}{5{\cdot}8}\pm\frac{4\times 10}{11{\cdot}54}=0{\cdot}68\pm 3{\cdot}47=4{\cdot}15 \text{ tons per sq. in.}$$

compression and 2·79 tons per sq. in. tension so that this section will be sufficient.

LECTURE XXVI

Vaulting—Groined Vaults—Ribs and Panels—Finding Elevation of Ribs—Curve of Thrust—Determination of Stresses.

In Roman vaulting, called barrel or wagon vaulting, where two cylindrical vaults intersected at right angles there was an elliptical groin, but as a rule no groin rib. The vaulting was of considerable thickness, say 2 to 3 ft., upon which it relied for stability. In Gothic vaulting the groin ribs at the intersection formed an independent diagonal arch from corner to corner, which supported the filling of the spandrils, say 6 in. thick, and transmitted the load directly to the abutments. The ribs or groins are thus a source of strength to the Gothic vaulting which is absent in the Roman. Fig. 530 shows a perspective view of simple Gothic vaulting.

It was stated in a recent article in *Engineering Record* that "the groined arch, though used centuries ago, is a purely empirical type of structure, designed, or, speaking more accurately, drawn without satisfactory mathematical analysis and used without much knowledge of the factor of safety." It may be the fact that in many cases the modern architect is unable to make the necessary calculations, and, instead of calling in expert assistance, copies as nearly as possible what has been done before, but it is impossible to believe that the ancient and mediæval stonemasons were not fully cognisant of the essential nature and value of the stresses brought into action in their structures.

Groined ceiling vaults such as occur in the crypt of a Gothic church, with a pendent stone in the centre, and on a larger scale at the junction of nave and transepts with or without an open eye in the centre, consist in the simplest cases of four ribs running diagonally to the four corner piers, with an arched wall rib or transverse rib from pier to pier, and between the ribs a thin panel filling, sometimes curved slightly in its cross section to help to carry its own weight as an arch, the weight of the panels being then entirely transmitted to the ribs. The voussoirs of the ribs are for two or three feet up from the springing, or where they branch out separately, worked out of the solid stones with horizontal beds, above the capitals of the piers or columns, in order to give a solid backing and reduce the arch thrust; this portion is called the "tas de charge." Besides arching in cross section, another method by which the failure of the panels was prevented was the introduction of intermediate ribs, reducing the width of the panels, and a later arrangement was the insertion of lierne ribs between the intermediate and main ribs. Fig. 531 shows the plan of a simple Gothic vault with the names of the parts marked on, and two methods of coursing the panels. The

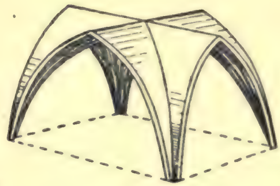


Fig. 530

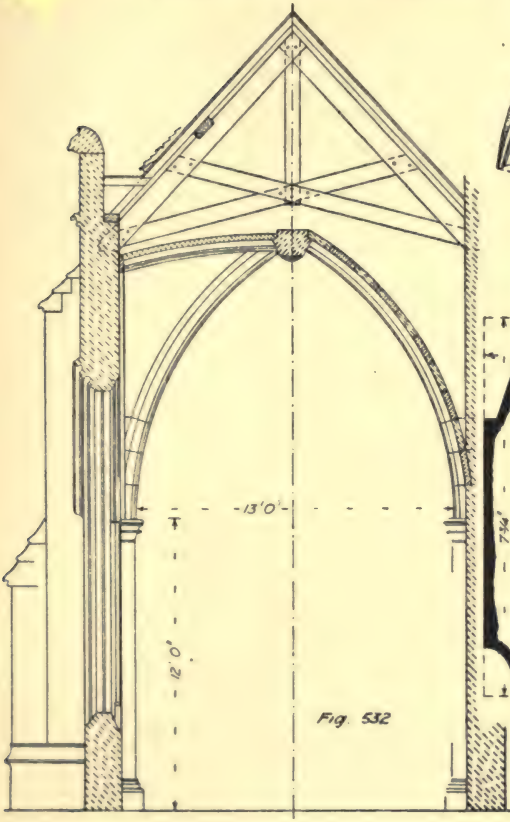


Fig. 532

Fig 534

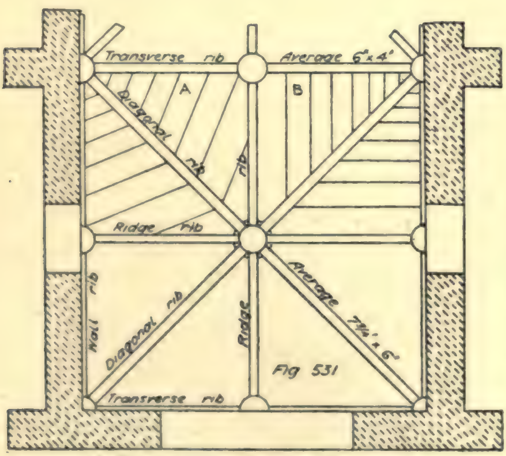


Fig 531

A . B = alternative coursing for panels

Fig 533

ribs are usually rebated to receive the panelling except in a few early examples where the filling simply rests on the back of the rib. The stresses in groined vaulting follow exactly the same principles as those laid down for arches and arched bridges, but they have only their own weight to carry, except in the few cases where the haunches are loaded with chalk rubble. Fig. 532 shows a section of the groined vault corresponding with the plan. Fig. 533 shows a section of the transverse ribs, the wall rib in section being similar to the portion on one side of the centre line. Fig. 534 shows a section of the diagonal ribs. The dotted lines show the rectangles taken in the calculations as the virtual sections.

When the filling-in is flat, it compares with a section of an ordinary

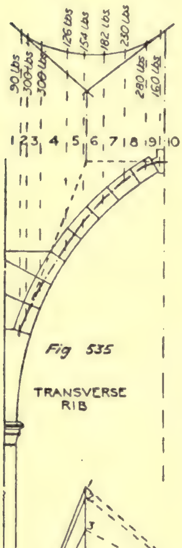


Fig 535

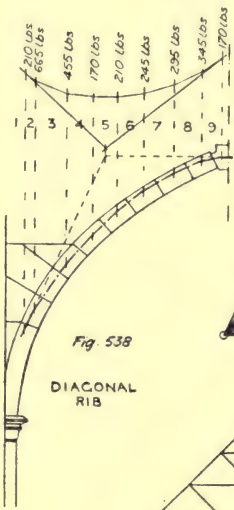


Fig. 538

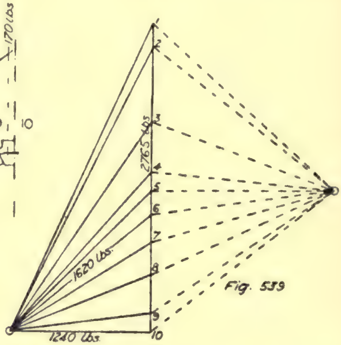


Fig. 539

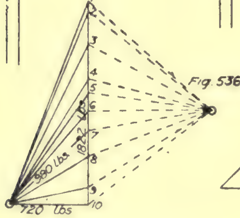


Fig. 536

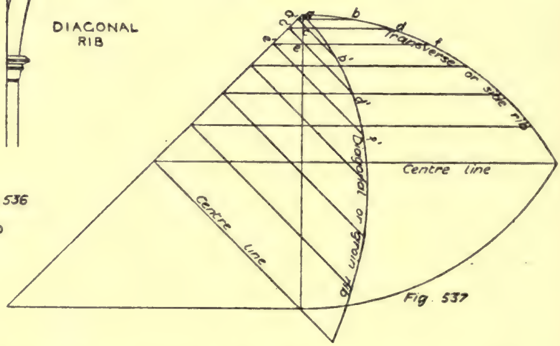


Fig. 537

barrel vault and its stability is therefore subject to the same limitations. Assuming no ridge ribs, one spandril is kept in position by the pressure of the one opposite, but on account of its want of thickness it has not the advantage of an ordinary arch with deeper voussoirs, and therefore considerable reliance has to be placed on the mortar between the joints.

In this form of vaulting very little pressure falls on the ribs from the filling-in, and unless the filling-in is carefully built, and with good mortar, it is liable to give way; this weakness, no doubt, was the reason for the introduction of intermediate ribs. When the filling-in is arched from rib to rib its stability is increased, because it is then

arched in both directions. Each ring forming a complete segmental arch need not necessarily be so carefully formed as when flat, and is therefore always stronger and not so liable to failure as the former. In this case the ribs are subjected to direct thrusts normal to the line of rib in plan, but inclined according to the curvature of web. It will be seen that the thrusts neutralise each other to a certain extent, and their resultant simply acts as dead weight on the rib. The half elevation of a transverse rib, Fig. 533, with the loads upon it is shown in Fig 535, and the curve of thrust is found in the usual manner from Fig. 536. The maximum stresses will occur across space 7, and will be

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{980}{6 \times 4} \pm \frac{980 \times 2}{\frac{1}{6}(4 \times 6^2)} = 41 \pm 82 = 123 \text{ lbs. per sq. in.,}$$

or 7·87 tons per sq. ft. compression, and 41 lbs. per sq. in. or 2·62 tons per sq. ft. tension, which is quite safe. The curve of the diagonal rib, Fig. 534, must next be found as follows. In Fig. 537 set up the elevation of the transverse rib, and also the springing line of the diagonal rib. From any point a in the springing line of the transverse rib draw a perpendicular ab. Produce ba to a', and from a' at right angles to springing line of diagonal rib set up $a'b'$ equal to ab. This process repeated with other points will give the curve of the diagonal rib, and the half-elevation of diagonal rib with loads will then be as Fig. 538. The curve of thrust will be obtained from Fig. 539, and the maximum stresses occur across space 6 ; then

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{1620}{6 \times 7{\cdot}75} \pm \frac{1620 \times 2{\cdot}5}{\frac{1}{6}(6 \times 7{\cdot}75^2)} = 35 \pm 67{\cdot}5 = 102{\cdot}5 \text{ lbs. per sq. in.,}$$

or 6·56 tons per sq. ft. compression, and 32·5 lbs. per sq. in., or 2·08 tons per sq. ft. tension, which is satisfactory, and the vaulting will therefore be perfectly safe.

EXERCISES ON LECTURE XXVI

Q. 83. Sketch the plan of one quarter of a bay, showing what is meant by "lierne rib vaulting." Name the parts shown.

Answer. See Fig. 540.

Q. 84. A semicircular barrel vault, 15 ft. span, intersects another of the same span at right angles. Find the section across the groin.

Answer. See Fig. 541.

Q. 85. In the example worked out in the Lecture find the stresses on the buttress from the vaulting ribs.

Answer. There will be the thrusts from the transverse rib and two diagonal ribs on each buttress. The horizontal component of the thrust from each diagonal rib (Fig. 539) = 1240 lbs. acting at 45° to face of wall, and combining the two, as in Fig. 543, gives a thrust of $\sqrt{2(1240)^2} = 1753$ lbs. perpendicular to wall. In addition to this there will be the horizontal component of thrust from transverse rib (Fig. 536) = 720 lbs., making the total thrust perpendicular to wall = 2473 lbs., as shown in elevation, Fig. 542. This must now be combined with the combined vertical components of the thrusts = (2 × 2765) + 1822 = 7352 lbs., and the resultant so found combined with the weight of wall, buttress and roof = 64,040 lbs., this latter step being drawn to a reduced scale. The final resultant is then found to cut the base at 1·39 feet from inner face of wall. The calculations about the section AB are as follows :—Distance of centre of gravity of base from inner edge

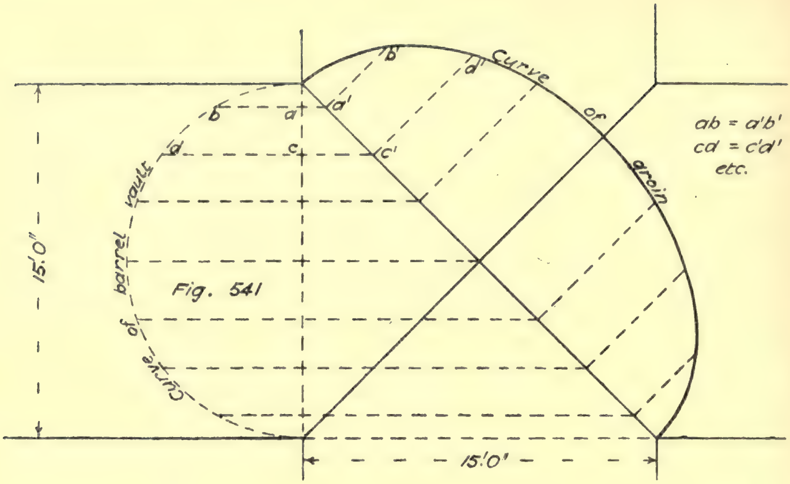


Fig. 541

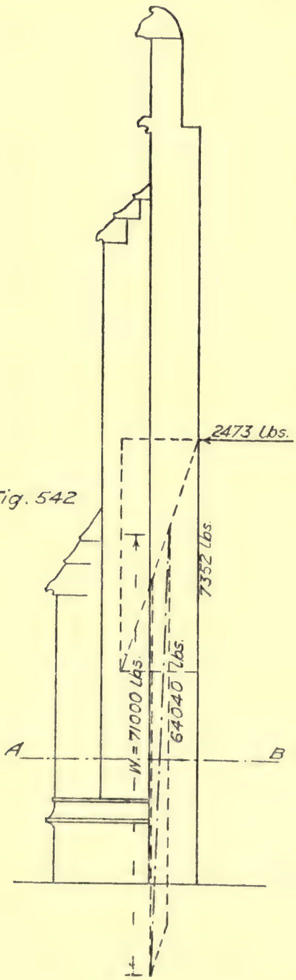


Fig. 542

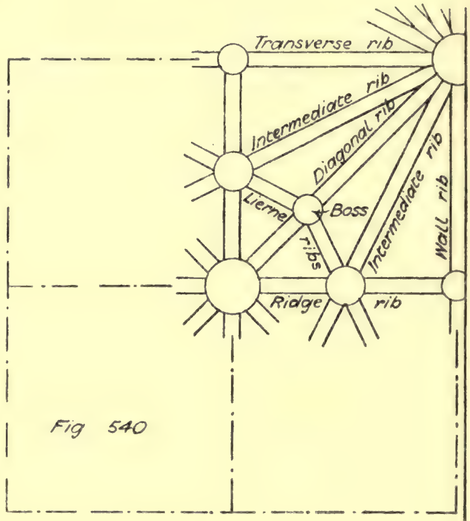


Fig 540

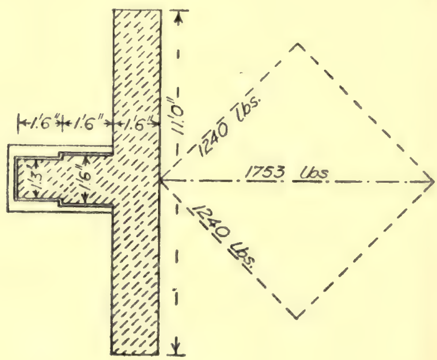


Fig. 543

$$= \frac{(11 \times 1\frac{1}{2} \times 0{\cdot}75) + (1\frac{1}{2} \times 1\frac{1}{2} \times 2{\cdot}25) + (1\frac{1}{2} \times 1\frac{1}{4} \times 3{\cdot}75)}{(11 \times 1\frac{1}{2}) + (1\frac{1}{2} \times 1\frac{1}{2}) + (1\frac{1}{2} \times 1\frac{1}{4})} = 1{\cdot}19 \text{ ft.}$$

$$Z = \frac{(11 \times 4\frac{1}{2}^2 - 9{\cdot}5 \times 3^2)^2 - 4 \times 11 \times 4\frac{1}{2} \times 9{\cdot}5 \times 3(1{\cdot}5)^2}{6(11 \times 4\frac{1}{2}^2 - 9{\cdot}5 \times 3^2)} = 7{\cdot}12 \text{ ft. units.}$$

$$\frac{W}{A} \pm \frac{M}{Z} = \frac{71{,}000}{20{\cdot}62} \pm \frac{71{,}000 \times \left(\frac{1{\cdot}39 - 1{\cdot}19}{12}\right)}{7{\cdot}12} = 3443 \pm 166 = 3609 \text{ lbs.}$$

or 1·61 tons per sq. ft. compression at outer edge and 3277 lbs. or 1·47 tons per sq. ft. compression at inner edge.

LECTURE XXVII

Theory of Domes—Joint of Rupture or point of Contra-flexure—Domes compared with Arches—Curve of Thrust in Dome giving pressure on Beds—Method of finding horizontal thrusts or tension and compression in Vertical Joints.

DOMES are not generally supposed to be reducible to the same principles as ordinary arches; we are told that the dome differs from the arch in having no thrust at the crown and when hemispherical having no thrust at the base, but this is a mistake. A similar statement about no thrust at the base is made regarding a semicircular arch, but we have seen that the true arch is the line of thrust, and as this is never vertical at the springing, there is of necessity always an outward thrust. A theory of the stability of domes will be found in Rankine's "Applied Mechanics," p. 265, and this is repeated in Gwilt's "Encyclopædia of Architecture." The same principles are adopted in Marsh and Dunn's "Reinforced Concrete," and may be stated as follows :—

Let the arc *ab*, Fig. 544, represent half the section through a thin hemispherical dome of radius $0a$ and $0b$, and uniform thickness. The weight of any segment or zone is proportional to its vertical height, because the surface area = circumference × height, therefore if the vertical axis be divided into say four equal parts the horizontal planes through these points will cut off equal weights. Take any point *c*, and draw a tangent to meet a horizontal line through the crown in *d*, complete the parallelogram *cdef* as shown, then *bf* represents the vertical load at *c*, *be* the radial thrust shown acting outwards at *c*, and really being the equilibrant to balance the inward tendency, and *ef* the direct compression in the material at point *c*. The outline *bef* gives the triangle of the forces acting at *c*, which are shown separately in Fig. 545, the ordinary force triangle for which would be as in Fig. 546. The joint of rupture, as it is called, but more correctly point of contra-flexure, in a thin hemispherical dome of uniform thickness, occurs at a height of $\frac{1}{2}(\sqrt{5} - 1) \times$ radius $= 0{\cdot}618$ radius, the point forming a centre angle with the vertical of $51° \, 49'$ as shown. The matter will perhaps be made clearer by taking out the force triangles from Fig. 540 and comparing them with each other. The triangle of forces at point *c* will be as Fig. 547. In a similar manner the triangle of forces at c_1 is given in Fig. 544, and will be as shown separately in Fig. 548. The force *fe* is the thrust in the dome from point *c*, ff_1 the weight of the segment from *c* to c_1, and f_1e the reaction at c_1, or direct thrust in the dome at that point, showing no horizontal thrust as points *c* and c_1 are approximately equidistant from the joint of rupture. At point c_2 the polygon of forces will be as Fig. 549, where ef_1 is the thrust from point

c_1, be the horizontal thrust acting inwards, or the reverse way to its action at point c, f_1f_2 the weight of the segment, and f_2b_1 the reaction at point c_2. At point c_3 at the base of dome, the polygon of forces will be as Fig. 550, where f_2b_1 is the thrust from point c_2, f_2f_3 the weight of segment, f_3b the reaction at point c_3, and bb_1 horizontal thrust acting

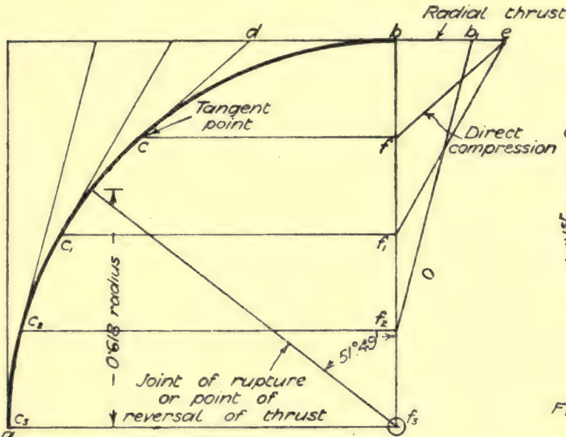


Fig. 544

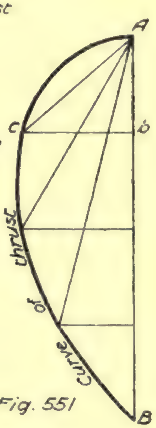


Fig. 551

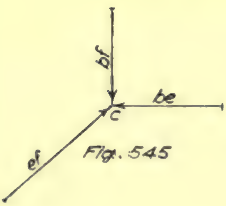


Fig. 545

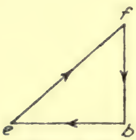


Fig. 546

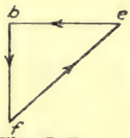


Fig. 547

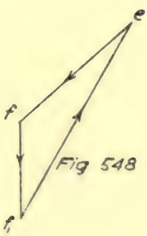


Fig 548

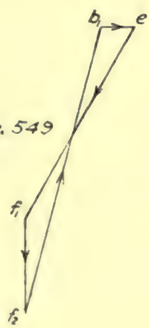


Fig. 549

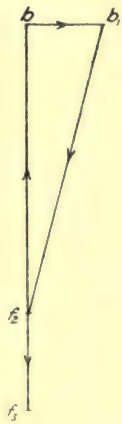


Fig 550

inwards. The horizontal force at c is shown to act outwards to balance the tendency of the dome to fall inwards at that point, while the horizontal forces at c_2 and c_3 are shown to act inwards to balance the tendency of the dome to spread outwards at those points. If these forces can be applied, the line of thrust will pass down the centre of

the thickness of the dome, but in practice this cannot easily be done. It is only on the supposition that it is effected, that it can be said there is no thrust at the base of a hemispherical dome.

The diagram of stresses may take another form, Fig. 551. Set off

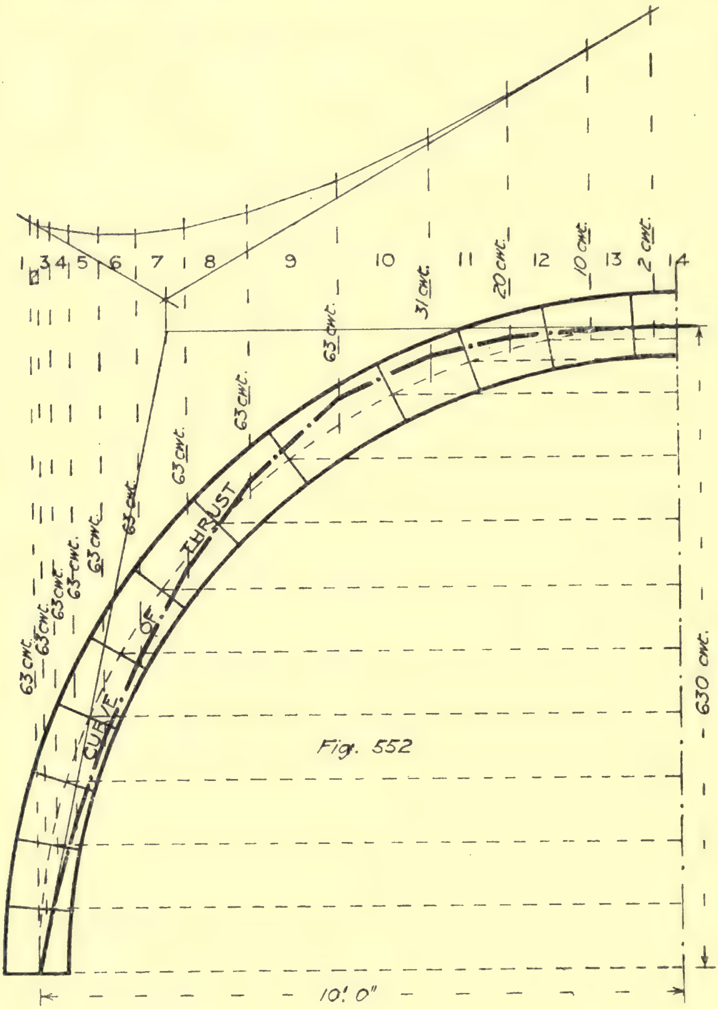


Fig. 552

the load line as before and through A draw lines parallel to those tangential to the surface of the dome at the various points to intersect with the horizontal lines. Through these intersections draw the curve from A to B. Then we have the stress diagram giving the force triangle Abc for point c, showing vertical load Ab, horizontal thrust bc, and direct compression Ac. Any number of divisions may be taken on

the vertical line, and the tangents and horizontal lines drawn in. Then the actual horizontal thrust at any point will be the difference between the length of line at that point and the length of line at the point above. Above the point of contra-flexure the horizontal thrust will act inwards and below it outwards, because the upper part tends to fall in and the lower part to be thrust out.

These radial thrusts being calculated upon the whole circumference require to be divided by the circumference at that point in order to give the local intensity, $\frac{\text{radial thrust}}{\text{units in circumference}} = \text{intensity}$. The "hoop tension," as the circumferential stress below the point of contra-flexure is called, is given by the formula $\frac{\text{radial thrust} \times \text{radius}}{\text{circumference}} = \frac{\text{radial thrust}}{6{\cdot}2832}$, or approximately 0·16 thrust.

The weight W of a thin dome, where r = mean radius of curvature of dome, v = mean rise, w = weight per unit area of surface, is

$$W = w \times 2\pi r \times v.$$

It will now be interesting to see how the stresses in a dome may be worked upon the same principles as the arch. Fig. 552 represents the half section of a dome 20 ft. mean diameter, and 1 ft. thick. Assuming the weight to be 112 lbs. per cubic foot, the weight of dome will be $W = w \times 2\pi r \times v = 1 \times 2 \times 3{\cdot}1416 \times 10 \times 10 = 630$ cwt. Dividing the mean rise into 10 equal parts and drawing horizontal lines across will give rings of voussoirs having equal weights $= \frac{630}{10} = 63$ cwt., but the top voussoir may be again divided for greater accuracy in obtaining the curve of thrust. The load line may now be set down as in Fig. 553, and the curve of thrust obtained in the usual manner. The maximum stresses will occur across space 5, where the thrust = 400 cwt., and distance from curve of thrust to centre line = 0·4 ft. The mean radius at this point = 9·1 ft., therefore the circumference $= 3{\cdot}1416 \times 9{\cdot}1 \times 2 = 57{\cdot}2$ ft., and the thrust per ft. run $= \frac{400}{57{\cdot}2} = 7$ cwt. The stresses will then be $\frac{W}{A} \pm \frac{M}{Z} = \frac{7}{1 \times 1} \pm \frac{7 \times 0{\cdot}4}{\frac{1}{6}(1 \times 1^2)} = 7 \pm 16{\cdot}8 = 23{\cdot}8$ cwt. or 1·19 tons per sq. ft. compression at inner edge, and 9·8 cwt. or 0·49 ton per sq. ft. tension at outer edge. The curve of thrust in this case does not allow for any tension hoops until the springing is reached, where the hoop tension will be $\frac{132}{6{\cdot}2832} = 21{\cdot}04$ cwt. These stresses are in the bed joints of the dome, and are independent of the stresses at the vertical joints. The latter stresses are caused directly by the tendency of the upper part of the dome to fall in, and the lower to push out. Taking the section of dome again in Fig. 554, and setting off the same divisions as before, construct a graphic diagram of thrusts, Fig. 555, on the principle of Fig. 551. Then at the centre of each division on Fig. 554 set off horizontally the *difference* between the successive thrust values in Fig. 555 divided by the vertical depth of the division, and join the points so found to give the horizontally shaded areas. Above the point of contra-flexure, or reversal of thrust, the joints will

be in compression, resisted simply by the strength of the stone. For instance, take the portion of the dome AB, Fig. 554, then the total inward horizontal thrust $= \frac{55 + 25}{2} \times 1 = 40$ cwt., and the hoop compression $= \frac{40}{6{\cdot}2832} = 6{\cdot}36$ cwt., and the area of the vertical joint between A and B being $1{\cdot}9 \times 1 = 1{\cdot}9$ sq. ft., the compression on this joint will be $\frac{6{\cdot}36}{1{\cdot}9} = 3{\cdot}35$ cwt. per sq. ft., and the curve for compression

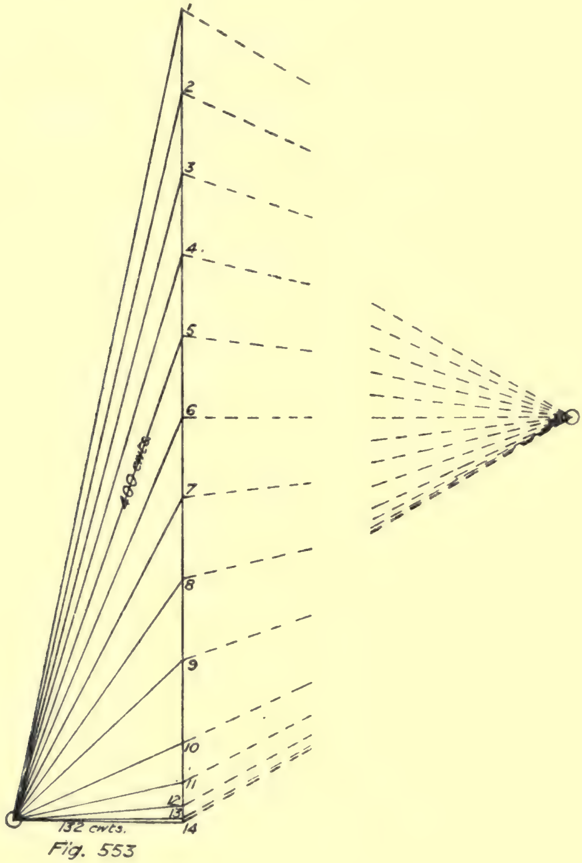


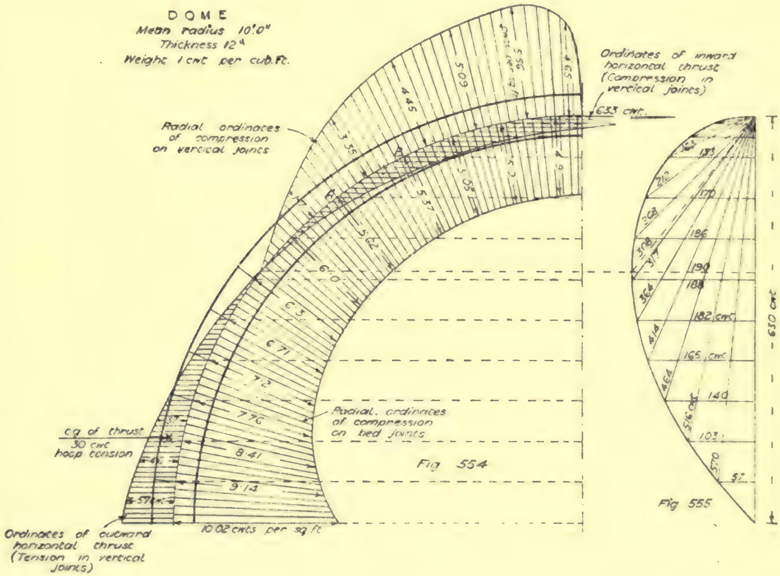
Fig. 553

on the vertical joints will be as shown. Below the point of contra-flexure the joints will be in tension which may be resisted by a single belt or chain round the dome at the level of the centre of gravity of the outward thrusts, say at X, the courses being dowelled together in order to transmit the effect of the belt throughout the required depth. The tensile resistance necessary in the belt will be found thus :—The total

outward thrust collected at X, or horizontally shaded area below point of contra-flexure, by taking area and mean pressure on each section, = say 188·36 cwt., and the hoop tension will therefore be $\frac{188 \cdot 36}{6 \cdot 2832} = 30$ cwt. or 1·5 tons. By a more accurate method, taking the area by planimeter = 1 sq. in. × 100 cwt. to 1 in. × 2 ft. to 1 in. = 1 × 100 × 2 = 200 cwt. total outward thrust, and hoop tension—

$$= \frac{200}{6 \cdot 2832} = 31 \cdot 8 \text{ cwt.} = 1 \cdot 59 \text{ ton.}$$

The lower radial ordinates, Fig. 554, represent the compression per



square foot on the bed joint, and to obtain this the total thrust must be divided by the circumference and the thickness of dome at the point under consideration. For instance, take the horizontal joint at springing, the total thrust is 630 cwt., circumference = 2 × 10 × 3·1416 = 62·832 ft., and thickness 1 ft., therefore the compression per square foot $= \frac{630}{62 \cdot 832 \times 1} = 10 \cdot 02$ cwt.

EXERCISES ON LECTURE XXVII

Q. 86. Find the curve of thrust on a concrete dome 20 ft. mean span, 7 ft. 6 ins. mean rise, 1 ft. thick, $1\frac{1}{4}$ cwt. per cub. ft., and state the hoop tension collected at base.

Answer. The outline of dome will be as in Fig. 556, and the weight will be $2w\pi rv = 2 \times 1 \cdot 25 \times 3 \cdot 1416 \times 10 \cdot 41\dot{6} \times 7 \cdot 5 = 615$ cwt. The load will then be divided up as shown and the curve of thrust obtained from Fig. 557, whence the hoop tension $= \frac{205}{6 \cdot 2832} = 32 \cdot 62$ cwt.

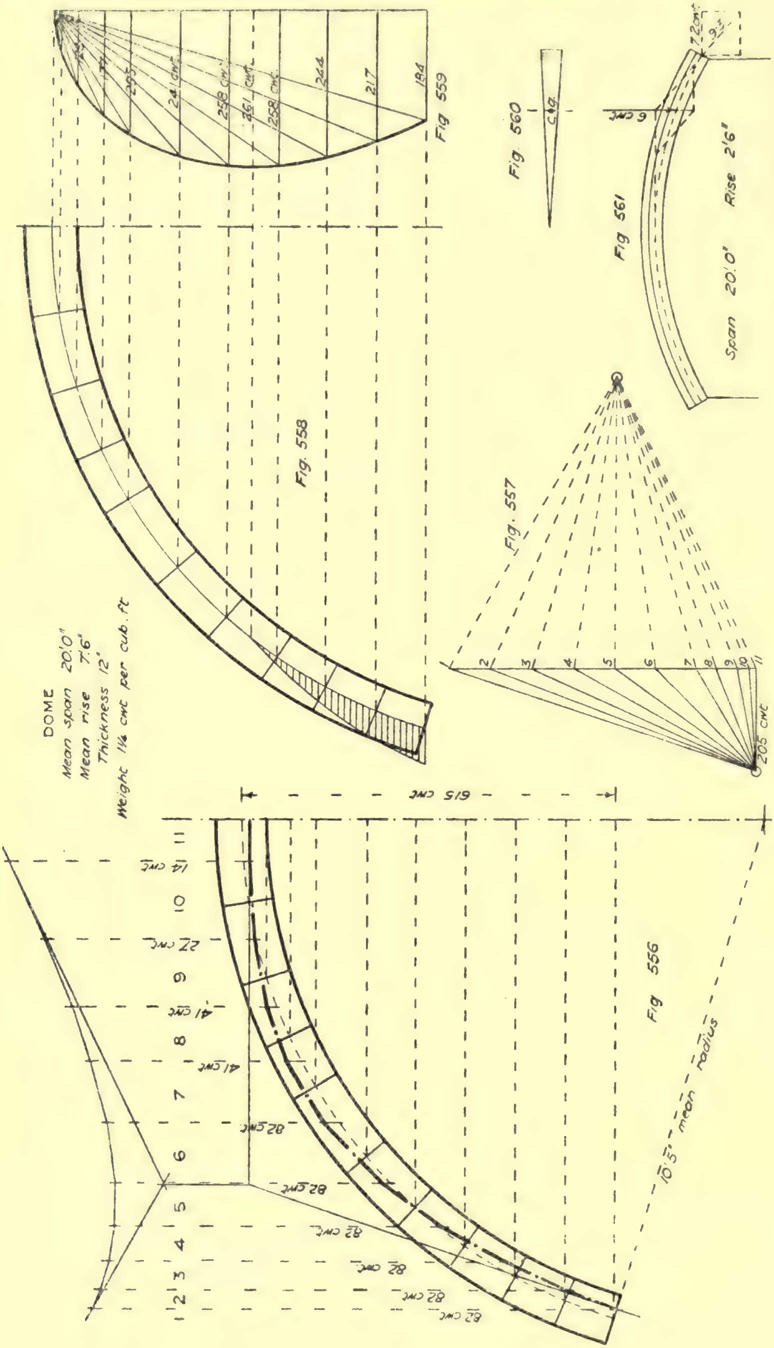


Fig 559
Fig. 560
Fig 561
Span 20'0" Rise 2'6"
Fig. 558
Fig. 557
205 cwt
DOME
Mean span 20'0"
Mean rise 7'6"
Thickness 12"
Weight 1¼ cwt per cub. ft
615 cwt
Fig 556
10'5" mean radius

By another method draw out the dome again as in Fig. 558, and from it set off the curve of thrusts in Fig. 559 and also the ordinates of outward horizontal thrust as shown by the shaded area in Fig. 558. The area of this shaded portion gives the total outward thrust = 75·5 cwt., and the hoop tension will therefore be $\frac{75 \cdot 5}{6 \cdot 2832} = 12$ cwt.

By Marsh and Dunn's formulæ, rise of tangent $= \frac{10 \times 10}{10 \cdot 41\dot{6} - 7 \cdot 5} = 34 \cdot 3$ ft., total radial thrust $= \frac{615 \times 10}{34 \cdot 3} = 180$ cwt., and the hoop tension $= \frac{180}{6 \cdot 2832} = 28 \cdot 64$ cwt.

Q. 87. The circular domed roof of a furnace chamber, 20 ft. mean diameter, 2 ft. 6 ins. rise, 18 ft. radius of curvature, 13½ ins. thickness of brickwork, collapsed after being heated to 450° F. and fractured the steel belt encircling it.

Find the hoop tension when cold by various methods.

Answer. An approximate idea of the thrust at the base of the dome may be obtained by taking a strip as Fig. 560 1 ft. wide at base, and nothing at crown, as a half-arch, the thrust being found as in Fig. 561. Then the thrust per foot run, multiplied by the mean diameter, will give the total tension to be resisted by two sides of the encircling band, and half of this will give the tension in the band; thus $\frac{7 \cdot 2}{20} \times \frac{20}{2} = 3 \cdot 6$ tons.

By Marsh and Dunn's formulæ (based on Rankine). Clear span 18 ft., rise 2 ft. 6 ins., thickness 13½ ins., virtual span 20 ft., radius 18 ft. Weight of dome $= 36 \times 3\frac{1}{7} \times 2 \cdot 5 \times 135 = 38{,}187$ lbs. $= 17 \cdot 05$ tons.

$$\text{Rise of tangent} = \frac{10 \times 10}{18 - 2 \cdot 5} = 6 \cdot 45 \text{ feet.}$$

$$\text{Total radial thrust} = \frac{17 \cdot 05 \times 10}{6 \cdot 45} = 26 \cdot 43 \text{ tons.}$$

$$\text{Hoop tension} = \frac{26 \cdot 43 \times 10}{20 \times 3\frac{1}{7}} = 4 \cdot 2 \text{ tons.}$$

By Godard's formulæ :—

$$Q = \frac{2WR}{3v} = \frac{17 \cdot 05 \times 10}{3 \cdot 25} = 22 \cdot 73 \text{ tons.}$$

$$T = \frac{Q}{2\pi} = \frac{22 \cdot 73}{2 \times 3\frac{1}{7}} = 3 \cdot 61 \text{ tons.}$$

By Rankine's formula :—

$$\text{Hoop tension } T = 0 \cdot 3wtr^2 = \frac{0 \cdot 3 \times 135 \times 1 \cdot 125 \times 18^2}{2240} = 6 \cdot 58 \text{ tons.}$$

By another formula :—

$$\text{Total radial thrust } R = \frac{Wd}{6v} = \frac{38186 \times 20}{6 \times 2 \cdot 5} = 50914 \cdot 7 \text{ lbs.} = 22 \cdot 73 \text{ tons.}$$

$$\text{Hoop tension at base } T = \frac{R}{2\pi} = \frac{22 \cdot 73}{6 \cdot 2832} = 3 \cdot 88 \text{ tons.}$$

By still another formula :—

$$\text{Radial thrust per foot run } R' = \frac{W}{6\pi v} = \frac{17 \cdot 047}{6 \times 3 \cdot 1416 \times 2 \cdot 5} = 0 \cdot 362 \text{ ton.}$$

Hoop tension $= R'(\frac{1}{2}d) = 0 \cdot 362 \times 10 = 3 \cdot 62$ tons.

LECTURE XXVIII

Principles of Shoring—Raking, Flying, Dead and Needle Shores—Formulæ and Scantlings—Foundations, Width and Depth—Supporting Power and Natural Slope of various Soils—Pile Driving and Supporting Power of Piles—Formulæ.

THE principles governing the stresses in shoring are very easily laid down, but the actual stresses in any given case are so difficult to determine as to be practically impossible of solution, and reliance has to be placed upon personal judgment and experience.

With an ordinary raking shore as A, Fig. 562, there is a certain thrust B exerted, or liable to be exerted, by the wall, and the weight of

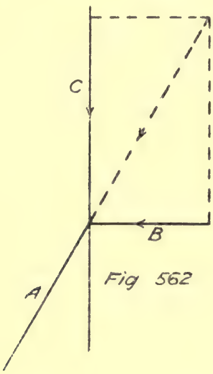


Fig 562

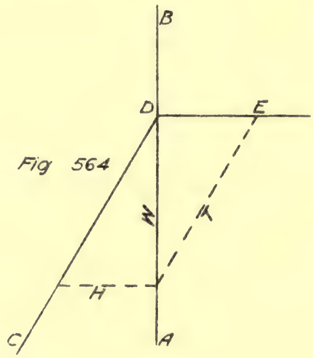


Fig 564

wall above the point of application of the shore, represented by the force C, must be such as, compounded with B, will produce a resultant in the direction of the length of A. It does not matter if the weight of wall exceeds the required amount as only so much of it will be brought into action as may be required. The weight of wall may be taken as that part within an angle of 60 degrees above the point of the shore, together with the weight of roof that may be resting on that portion. The wedging up of the shore brings the forces into play, and usually nothing more is done than wedging up until everything appears to be tight. Then any further tendency of the wall to fall out is met by the resistance of the shore and the weight above it. The head of a raking shore is fitted as shown in Fig. 563, the lower end of the shore being wedged up or prised along the sole piece to tighten it against the

needle. In Fig. 564 let AB be the face of wall, CD the line of the shore, and DE a horizontal line. Construct the parallelogram of forces as shown, where W is the weight of the wall DB contained in an equilateral triangle having its apex at D, including roof, floors, etc., H the horizontal thrust which can be resisted by the shore, and T the

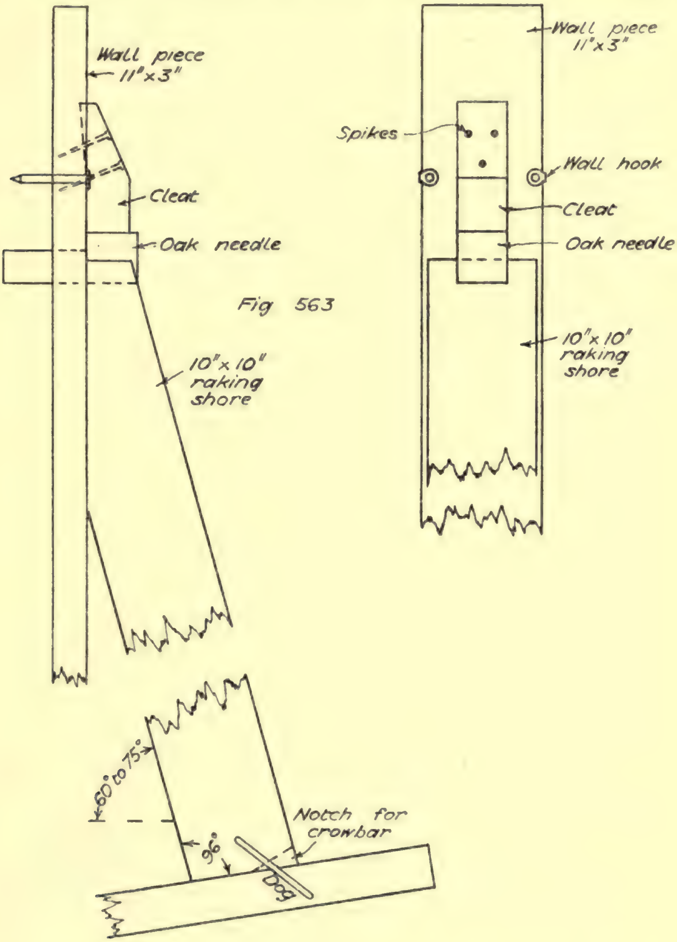


Fig 563

thrust or compression in the shore when in full action, that is, this diagram gives the maximum figures possible. In a system of raking shores as Fig. 565 the wall piece enables the lower needle to assist the upper one when the top shore is near the top of wall. It should be observed that the supporting power of the shores is transmitted entirely through the needles to the wall, and it will often be found in practice that the needles are the weakest points in the system, particularly in

heavy shoring. They should, therefore, be of oak or other hard wood. The raking shores may be calculated by Gordon's formula, or by similar means, taking the width as least diameter, but the following formulæ are sometimes adopted :

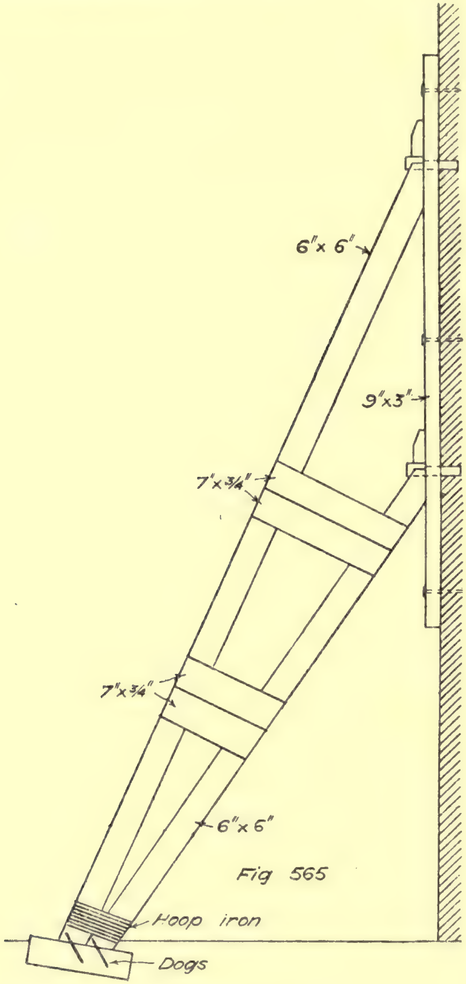


Fig 565

a = Area of wall supported in sq. ft.
t = Thickness of wall in inches.
w = Weight of wall per cubic foot, say 1 cwt.
h = Height to head of shore in feet.
w = Weight of shore in cwts.
θ = Angle of shore with horizon.
P = Upward vertical thrust of shore.
Q = Horizontal thrust at top of raking shore in cwts.

Cwts. horizontal thrust of wall at top of raking shore $= Q = \frac{wat^2}{288h}$.

Cwts. upward vertical thrust of shore $= P = Q \tan\theta - \frac{1}{2}w'$.

And to provide against this the distance from head of shore to top of wall should not be less than $\frac{8P}{wt}$, or with roof $= \frac{4P}{wt}$.

Cwts. compression in raking shore $= P \sin\theta + Q \cos\theta$.

Safe load cwts. on shore $= 15{\cdot}5d$ ins. $\frac{b^3 \text{ ins.}}{l^2 \text{ ft.}}$.

In these formulæ Q is the maximum horizontal thrust that the wall would stand without shoring, and the calculations are based upon that amount of thrust. The formulæ given in "Stock on Shoring" are equivalent to these.

Applying these methods to a given case, say a 9-in. wall 30 ft. high, the height to top of shore 20 ft., shore 6 ins. by 6 ins. inclined at 60 degrees. Then weight of wall coming on to shore

$$= \frac{8{\cdot}66 \times 10}{2} \times \tfrac{9}{12} \times 1 \text{ cwt. per cubic foot} = 32{\cdot}5 \text{ cwt.},$$

then the maximum horizontal thrust by the method shown in Fig. 564 $= 32{\cdot}5 \times \cot\ 60° = 18{\cdot}8$ cwt., and the maximum thrust in shore $= 32{\cdot}5 \operatorname{cosec} 60° = 37{\cdot}6$ cwt. By the above formulæ

$$Q = \frac{1 \times \left(\frac{8{\cdot}66 \times 10}{2}\right) \times 9^2}{288 \times 20} = 0{\cdot}61 \text{ cwt.},$$

assuming the shore to weigh 35 lbs. per cubic foot,

$$P = 0{\cdot}61 \times \sqrt{3} - \tfrac{1}{2} \times \tfrac{36}{144} \times \frac{20 \times 2}{\sqrt{3}} \times \tfrac{35}{112} = 0{\cdot}156 \text{ cwt.},$$

and the compression in shore

$$= 0{\cdot}156 \times \frac{\sqrt{3}}{2} + 0{\cdot}61 \times \tfrac{1}{2} = 0{\cdot}44 \text{ cwt.}$$

The usual scantlings for raking shores are as follows :

Height of wall in feet.	*No. of shores in each set.*		*Scantling of each shore.*
Up to 20	2	10 to 15 ft. between each set. Angle 60° to 75°	20 sq. in.
20 ,, 30	2		40 ,,
30 ,, 35	3		50 ,,
35 ,, 40	3		70 ,,
40 ,, 50	4		80 ,,
50 and upwards	4		100 ,,

Dead shores should be calculated as posts by Gordon's formula, and needle shores as beams with concentrated loads. Flying shores are hardly subject to calculation, but an empirical formula is sometimes used for the thrust on flying shores which has no apparent scientific foundation. It is $T = \frac{Wt}{2h}$, where T = thrust in cwts., W = weight of wall in cwts., t = thickness of wall in feet at ground line, h = height in

feet to principal strut. The usual scantlings for flying shores are as follows:

Span.	*Principal Strut.*	*Raking Strut.*	*Straining Pieces.*
Up to 15 ft.	6 × 4	4 × 4	4 × 2
,, 25 ,,	9 × 6	6 × 4	6 × 2
,, 35 ,,	9 × 9	9 × 4½	9 × 3

The maximum span is about 35 feet on account of the difficulty of getting longer timber. The sets of shores are placed from 10 to 15 ft. apart, and if there is only one principal strut in each set it should be placed about ⅔ height of wall; if two, then at ½ and ¾ with continuous wall pieces.

There are two points to particularly observe in connection with foundations—the width and the depth. The width depends primarily

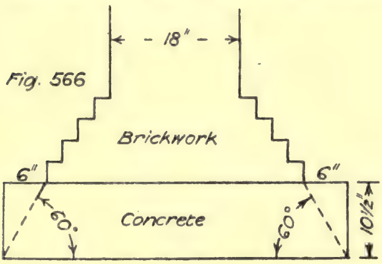


Fig. 566

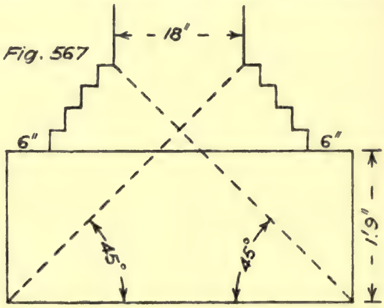


Fig. 567

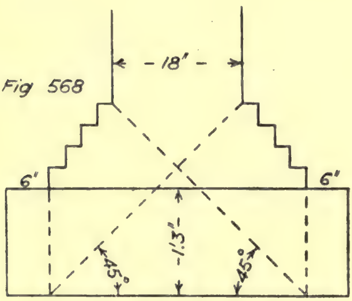


Fig 568

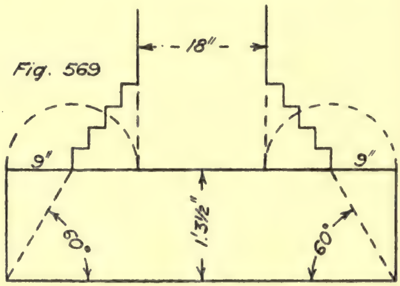


Fig. 569

upon the load imposed by the wall or pier and the supporting power of the soil. The load is usually a simple matter of calculation, but the resistance is matter for judgment. The text-books give tables of safe loads for different soils, but the classification of the given soil has to be discovered before use can be made of the data. In the north of England and in Scotland it is not usual to put footings to walls, but it is universal in the south. The rule is, as many courses of footings as there are half bricks in the thickness of wall, projecting 2¼ ins. each on each side, so that the bottom course will be twice the width of the wall. By the London Building Act and many local byelaws, there must be concrete under the footings not less than 6 ins. thick, and projecting beyond the brickwork not less than 4 ins. on each side. Architects, however, specify 6 ins. projection, because that is the width

of excavation beyond the bottom course of footings allowed for the bricklayer to work in, and they frequently make the thickness very variable but much greater than the minimum allowance ; in fact, they seem to have no rule for thickness. The 6 ins. projection requires a minimum depth of say 9 ins., *i.e.* half as much again, to prevent it breaking off, and it is only natural to give a proportionately wider base to a thick wall, and not limit it to 6 ins. projection. The following figures will therefore be of some interest. Fig. 566 shows the ordinary method for reasonably good work. Fig. 567 shows Mitchell's suggestion for proportioning the size of concrete which makes it too thick. Fig. 568 shows an improved modification of Mitchell's method, and Fig. 569 shows an improvement on all of them. The depth of the bottom of concrete from the surface of the ground will naturally vary in different soils independent of the question of basement rooms or cellarage. Generally speaking, a minimum depth of 2 ft. 6 ins. should be adopted on gravel soils and 5 ft. on clay soils, but in the London suburbs one often sees the concrete started on top of the grass, pegs being put in to hold up scaffold boards edgeways to contain it. Rankine's formula for depth of foundation is based upon the same theory as his formula for the pressure of earth against a retaining wall, and assumes to give a depth that will prevent the ground rising round the foundation by being squeezed out. The formula is as follows :—

Safe Load on Foundations (Rankine).

p = maximum vertical pressure in lbs. per sq. ft.
d = depth of underside of foundation below surrounding surface in feet.
w = weight of earth in lbs. per cubic ft.
θ = angle of repose of earth.

$$p = wd\left(\frac{1 + \sin\theta}{1 - \sin\theta}\right)^2 \qquad d = \frac{p}{w}\left(\frac{1 - \sin\theta}{1 + \sin\theta}\right)$$

	$\left(\frac{1 + \sin\theta}{1 - \sin\theta}\right)^2$	$\left(\frac{1 - \sin\theta}{1 + \sin\theta}\right)^2$
$\theta = 15°$	2·89	0·346
30	9·00	0·111
45	33·94	0·0295

Weight and Natural Slope of Various Soils.

Soil.	Lbs. per cub. ft. (w).	Angle of repose θ.
Vegetable earth	90	30
Sandy loam	100	34
Loamy clay	110	36
Firm gravel	120	40
Loose gravel	110	36
Stiff clay	128	45
Wet clay	120	16

SAFE LOADS ON MATERIALS.

Material.	Ordinary load. Tons per ft. sup.	Maximum load. Tons per ft. sup.
Granite	20	30
Portland and compact limestone	15	20
Hard York stone	12	15
Limestone (ordinary)	6	6
Blue brick in cement	9	12
Stock ,, ,,	6	8
,, ,, lias mortar	5	6
,, ,, grey lime mortar	3	4
Portland cement concrete (6 to 1)	5	10
Lias lime concrete (3 to 1)	3	5
Gravel and natural compact earth	3	3
Hard clay	$1\frac{1}{2}$	2
Made ground rammed in layers	$\frac{3}{4}$	1

SAFE LOAD ON MORTAR.

Stone lime mortar—

50 lbs. sq. in. or 3 tons sq. ft. compression.
25 ,, ,, $1\frac{1}{2}$,, tension.

Lias lime mortar—

150 lbs. sq. in., or 9 tons sq. ft. compression.
50 ,, ,, 3 ,, tension.

Portland cement mortar—

200 lbs. sq. in., or 12 tons sq. ft. compression.
75 ,, ,, 5 ,, tension.

There are several very interesting points for consideration in connection with pile-driving and the supporting power of piles, but time will not permit us to do more than glance at the question. The best known formula is that of Major Saunders, U.S.A., but it is only a rough approximation which assumes that the safe load on a timber pile is given by

$$L = \frac{WH}{8D}$$

where L = safe load in cwts.
W = weight of ram in cwts.
H = height ram falls in inches.
D = distance driven by last blow in inches ;

in other words, the safe load is taken at one-eighth of the mean resistance to penetration of the last blow. The monkey is the little slip-hook that runs up and down to lift and release the ram, although there are many persons who call the ram the monkey. The weight of the ram should be not less than the weight of the pile, and not more than $1\frac{1}{2}$ times that. Theoretically a 2-cwt. ram with 10-ft. fall will have the same effect as a 10-cwt. ram with 2-ft. fall, but the result will be very different. A light ram and long fall will expend all its work on the top of the pile and break it up, while a heavy ram with a low fall will

do little damage, expending its energy in pushing the pile down. For a full understanding we ought to know the nature of the resistance during the penetration. "Mean resistance" gives a rectangular diagram which is an impossibility. It may in reality be triangular or parabolic; at any rate, the resistance is small at the commencement of the movement and very great at the finish. For fuller discussion of the point see the author's paper on "Timber Piling in Foundations and other Works," 2nd edition, 6*d*.

Mr. A. C. Hurtzig, M.Inst.C.E., made some useful investigations into the formulæ for piles, and in a Paper read before the Liverpool Engineering Society in November, 1886, gave the following formula:

$$y = \frac{x}{P} - \frac{P}{625}$$

where P = extreme supporting power of pile in tons.
y = "set" of last blow in feet.
x = energy of last blows in foot-tons.

Let us now test these formulæ.

An experimental pile was driven and the following data were obtained:
Length of pile 30 feet.
Scantling $12\frac{1}{2}$ ins. by $12\frac{1}{2}$ ins. at top, $11\frac{1}{2}$ ins. by 11 ins. at bottom.
Weight of ram 910 lbs.
Fall of last blows 5 ft.
Depth driven at last blow $\frac{3}{8}$ in.
Actual load to just cause further sinking 62,500 lbs.

By Saunders' formula, safe load

$$P = \frac{Wh}{8d} = \frac{910 \times 60}{8 \times 0{\cdot}375} = 18200 \text{ lbs.} = 8{\cdot}128 \text{ tons.}$$

By Hurtzig's formula, extreme load

$$y = \frac{x}{P} - \frac{P}{625}, \text{ whence } P^2 + 625Py = 625x$$

which is a quadratic equation, but $y = 0{\cdot}03125$ and $x = 2{\cdot}03125$, then $P^2 + 19{\cdot}53125P = 1269{\cdot}53125$, and by adding the square of half the coefficient of P to both sides, and taking the square root of both sides of the equation, $P + 9{\cdot}765625 = 36{\cdot}94$, or $P = 27{\cdot}17$ tons, and allowing a factor of safety of 3, the safe load will be $\frac{27{\cdot}17}{3} = 9{\cdot}05$ tons, which is only slightly different from Saunders' result.

Another experimental pile was driven on another occasion and the following data were obtained:—
Pitch pine pile $12\frac{1}{2}$ ins. by $11\frac{1}{2}$ ins.
Weight of ram 18 cwt.
Fall of ram 8 ft.
Depth driven at last blow $\frac{2\frac{1}{4}}{4} = 0{\cdot}5625$ in.
Loaded with $56{\cdot}9$ tons sunk $\frac{1}{4}$ in.

By Saunders' formula, safe load

$$P = \frac{Wh}{8d} = \frac{2016 \times 96}{8 \times 0{\cdot}5625} = 43008 \text{ lbs.} = 19{\cdot}2 \text{ tons.}$$

By Hurtzig's formula, extreme load

$$P^2 + 625Py = 625x$$
$$y = 0{\cdot}046875,\ x = 7{\cdot}2$$
$$P^2 + 29{\cdot}3P = 4500$$
$$P + 14{\cdot}65 = \sqrt{4500 + 14{\cdot}65^2}$$
$$\therefore P = 54 \text{ tons,}$$

and with factor of safety 3,

$$P = 18 \text{ tons,}$$

which is not quite so close an agreement as before, but it should be noted how closely the latter formula agrees with the actual test load.

EXERCISES FOR LECTURE XXVIII

Q. 88. A needle beam and dead shores are required to carry a load estimated at 10 tons. What size timbers should be used? The dead shores are to be 4 ft. centre to centre and 13 ft. high.

Answer. $W = \frac{bd^2}{L}$ for safe distributed load cwts.; $bd^2 = 10 \times 20 \times 2 \times 4 = 1600$ say $b = 12$, then $d = \sqrt{\frac{1600}{12}} = 11{\cdot}5$, therefore say 12 ins. by 12 ins. for beam.

For dead shores try 9 ins. by 9 ins.; then by Gordon's formula taking ends as one fixed and one rounded, safe load

$$W = \frac{fS}{1 + \frac{m}{nq}\left(\frac{l}{d}\right)^2} = \frac{6 \times 81}{1 + \frac{2{\cdot}5}{1 \times 250}\left(\frac{13 \times 12}{9}\right)^2} = 121{\cdot}5 \text{ cwts.,}$$

while the actual load is $5 \times 20 = 100$ cwts., so that there appears to be good margin, but the bearing area of post on beam should be tested. The maximum safe load perpendicular to the grain of timber is 250 lbs. per sq. in. and the post must have a sectional area of at least $\frac{100 \times 112}{250} = 45$ sq. ins., while it actually has $9 \times 9 = 81$ sq. ins., so that it will be quite safe.

Q. 89. What depth is required for the foundation of a wall carrying a load of $1\frac{1}{2}$ tons per sq. ft. at underside of concrete on a soil with a natural slope of 30° and weighing 100 lbs. per cubic foot?

Answer. $d = \frac{p}{w}\left(\frac{1 - \sin\theta}{1 + \sin\theta}\right)^2 = \frac{1{\cdot}5 \times 2240}{100} \times 0{\cdot}111 = 3{\cdot}729$, say 3 ft. 9 ins., apart from any atmospheric considerations which would have to be taken into account.

Q. 90. A 14-in. by 14-in. pile is to carry 25 tons with a factor of safety of 3. The ram weighs 30 cwt. From what height must it fall when the last blow is to drive the pile $\frac{1}{4}$ in.?

Answer. By Hurtzig's formula

$$y = \frac{x}{P} - \frac{P}{625}$$

therefore $\qquad \frac{0{\cdot}25}{12} = \frac{x}{25 \times 3} - \frac{25 \times 3}{625}$ whence $x = 10{\cdot}56$ ft.-tons,

and $\qquad \frac{10{\cdot}56 \text{ ft.-tons}}{1{\cdot}5 \text{ ton ram}} = 7$ ft. fall.

LECTURE XXIX

Gin Poles—Derrick Poles—Guy Ropes—Shear Legs—Tripods—Cranes.

A GIN pole, or derrick pole, is usually a single stick of timber, round or square, but it may be a rolled steel joist, or a solid or hollow steel mast, which may be trussed on four sides to stiffen it. It is used as a support for lifting girders and roof trusses by means of a pulley wheel or pulley blocks attached to the top, the rope or chain being led down through a snatch block at the bottom and away to a crab winch worked by hand or steam. The top of the pole is kept in position by two or more guy ropes made fast at their lower ends. The pole may be upright or made to rake over in any required direction by slackening the guy ropes on one side and tightening them on the other side. Up to 20 ft. long there are often only two guy ropes, and the pole is kept steady by giving it a rake or inclination away from the guy ropes. The term "derrick" is generally used to signify something that rakes over or lifts at different distances from its foot, while a gin pole might be simply an upright pole carrying a gin- or jenny-wheel. Take now a simple case of a 9-in. by 9-in. fir pole 20 ft. long with a 5-ft. overhang, find what thrust it will safely bear, and then what load it will carry and what size guy ropes will be needed. All derrick poles and shear legs must be calculated as if with "ends rounded." The formulæ for the strength of poles give results that differ very considerably; three of these only will be considered.

$$\text{Gordon's formula } W = \frac{Af}{1 + a\left(\frac{l}{d}\right)^2}$$

where W = safe load in cwts., A = sectional area in square inches, f = working load in cwts. per square inch = 13 for fir, a = constant $= \frac{4}{250}$ for both ends rounded, l = length in inches, d = least width in inches.

Then

$$W = \frac{9 \times 9 \times 13}{1 + \frac{4}{250}\left(\frac{20 \times 12}{9}\right)^2} = \frac{1053}{1 + 0{\cdot}016 \times 711} = \frac{1053}{12{\cdot}375} = 85 \text{ cwt.}$$

Reuleaux's formula for timber posts (fir) is

$$W = 2000\frac{bd^3}{l^2}$$

where b = greatest width, d = least width, l = length, all in inches. Then $W = 2000\frac{9 \times 9}{(20 \times 12)^2} = 227$ cwt.

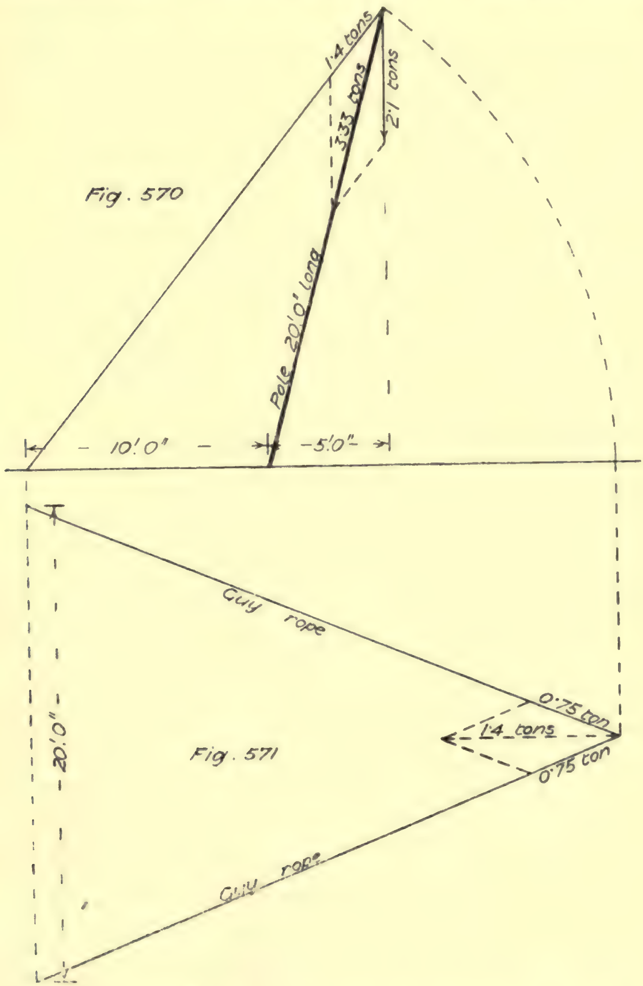
Ritter's formula is Gordon's formula with a different constant, viz. 0·0027 instead of $\frac{4}{250}$. Using this constant the calculation will be

$$W = \frac{9 \times 9 \times 13}{1 + 0{\cdot}0027\left(\frac{20 \times 12}{9}\right)^2} = \frac{1053}{1 + 0{\cdot}0027 \times 711} = \frac{1053}{2{\cdot}9} = 363 \text{ cwt.}$$

The last two are so greatly in excess of the strength by the ordinary Gordon formula that we should hesitate to adopt anything like the load indicated, and if we assume a 5-ton load we shall probably be putting as much on the pole as it ought to bear. If we allow 50 per cent. additional stress for the possible surging of the load, the thrust from the dead load should not be more than $\frac{5}{1\frac{1}{2}} = 3{\cdot}33$ tons. Now, assuming the guy ropes to be fixed 10 ft. back from the pole and 20 ft. apart, construct a parallelogram of forces (Fig. 570) with 3·33 tons down the pole, and on scaling off it will be found that the net load will be 2·1 tons. For stress in the guys the allowance of 3·33 tons in the pole must be taken, making the stress in the plane of the guys 1·4 tons. Projecting the real plan of the guys as shown by the dotted lines in elevation, Fig. 570, and plan, Fig. 571, the tension in each is obtained and found to be 0·75 ton. The safe load in cwts. on a good hemp rope fall is $\frac{2}{3}(\text{circumference})^2$, therefore $\frac{2}{3}(\text{circumference})^2 = 0{\cdot}75 \times 20$, whence circumference $= \sqrt{\frac{0{\cdot}75 \times 20 \times 3}{2}} = 4{\cdot}74$ ins. If wire ropes be used the safe load in cwts. will be $\frac{7}{8}(\text{circumference})^2 \times 6$, whence circumference $= \sqrt{\frac{0{\cdot}75 \times 20 \times 8}{6 \times 7}} = 1{\cdot}7$ in.

Shear legs consist of two derrick poles and a single guy rope. Assume two poles 35 ft. long, 8 ins. diameter at top and 12 ins. at bottom, the lower ends 10 ft. apart, the overhang at top 8 ft., and the guy rope making an angle of 45 degrees with the ground. Find the stresses in each part with a dead load of 1 ton. The elevation and plan will be as shown in Fig. 572. Let AB be the position of the lower ends and AC, BC the legs laid down on the ground, each 35 ft. long. Bisect AB in D, produce CD to E, making DE equal to the overhang 8 ft. From point D with radius DC strike an arc to cut a vertical from E in point F. Join DF and this will represent the side elevation of the legs. On DC set off point G, making angle EFG = 45°, then join GF to represent the elevation of the guy rope. As the load to be lifted is 1 ton, or allowing for surging, say 1·5 ton, set off a parallelogram

from point F as shown, from which it will be seen that a stress of 0·65 ton must be resisted by the guy, requiring a $\sqrt{\frac{0{\cdot}65 \times 20 \times 3}{2}} = 4{\cdot}41$ ins. circumference hemp rope, or $\sqrt{\frac{0{\cdot}65 \times 20 \times 8}{6 \times 7}} = 1{\cdot}58$ in. circumference wire rope. The thrust of 2 tons in the plane of the legs



must be resolved into an equivalent force in each leg. This is done by parallelogram as shown at G, where it is found that the thrust in each leg amounts to 1·03 ton. In addition to this thrust it has a cross strain due to its own weight, which will be appreciable owing to the

length and overhang. Taking the weight of fir as 35 lbs. per cubic foot, the weight of one leg will be $\frac{1}{3}h(\mathrm{A} + a + \sqrt{\mathrm{A}a})$

$$= \frac{1}{3} \times 35\left(\frac{0{\cdot}7854 \times 12^2 + 0{\cdot}7854 \times 8^2 + \sqrt{0{\cdot}7854 \times 12^2 \times 0{\cdot}7854 \times 8^2}}{144}\right)$$

$= 19{\cdot}5$ cubic ft., and $19{\cdot}5 \times 35 = 682$ lbs., say 0·31 ton, which, allowing for the taper, will produce a bending moment of about

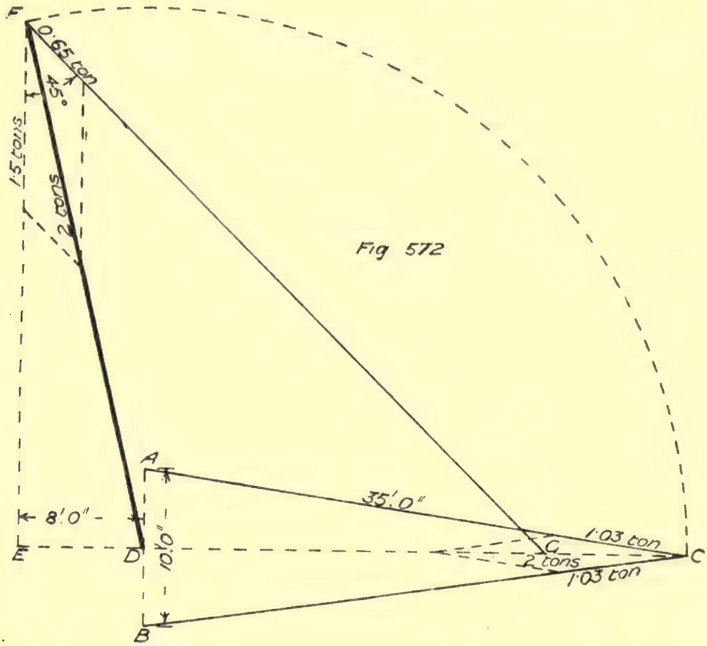


Fig 572

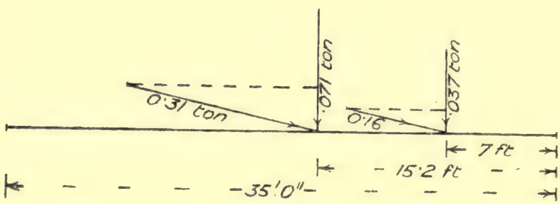


Fig. 573

0·305 ton-feet found as follows. The height of centre of gravity of pole from ground

$$= h\frac{2d^2 + \mathrm{D}^2}{3d^2 + 3\mathrm{D}^2} = 35 \times \frac{2(\frac{2}{3})^2 + 1^2}{3(\frac{2}{3})^2 + 3 \times 1^2} = 15{\cdot}2 \text{ feet,}$$

and the diameter at this point will be $12 - \dfrac{15{\cdot}2}{35}(12 - 8) = 10{\cdot}26$ ins.

The weight of the pole below the centre of gravity will therefore be

$$\tfrac{1}{3} \times 15{\cdot}2\left(\frac{0{\cdot}7854 \times 12^2 + 0{\cdot}7854 \times 10{\cdot}26^2 + \sqrt{0{\cdot}7854 \times 12^2 + 0{\cdot}7854 \times 10{\cdot}26^2}}{144}\right)\frac{35}{2240}$$

$= 0{\cdot}16$ ton, and the height of the centre of gravity of this part

$$= 15{\cdot}2 \times \frac{2\left(\frac{10{\cdot}26}{12}\right)^2 + 1^2}{3\left(\frac{10{\cdot}26}{12}\right)^2 + 3 \times 1^2} = 7 \text{ ft.}$$ From Fig. 573 it will be seen that the amounts of these two loads acting at right angles to the pole $= \frac{8}{35} \times 0{\cdot}31 = 0{\cdot}071$ ton and $\frac{8}{35} \times 0{\cdot}16 = 0{\cdot}037$ ton. The reaction at ground $= \dfrac{0{\cdot}071 \times (35 - 15{\cdot}2)}{35} = 0{\cdot}04$ ton, therefore the bending moment at centre of gravity of pole $= 0{\cdot}04 \times 15{\cdot}2 - 0{\cdot}037 \times (15{\cdot}2 - 7)$ $= 0{\cdot}305$ ton-ft. Then the maximum working strength of one leg taken as a pillar, reckoning from the mean diameter, will be

$$\frac{0{\cdot}65 \text{ ton per sq. in.}}{1 + \frac{4}{250}\left(\frac{35 \times 12}{10}\right)} = 0{\cdot}022 \text{ ton per sq. in.,}$$ and the actual stress with the thrust of $1{\cdot}03$ ton and the weight of leg $0{\cdot}31$ ton will be

$$\frac{W}{A} + \frac{M}{Z} = \frac{1{\cdot}03 + 0{\cdot}31}{0{\cdot}7854 \times 10{\cdot}26^2} + \frac{0{\cdot}305 \times 12}{0{\cdot}0982 \times 10{\cdot}26^3} = 0{\cdot}016 + 0{\cdot}034 = 0{\cdot}05 \text{ ton}$$ per sq. in.

This shows the poles to be insufficient in strength, although with care they might doubtless be used. Try 15 ins. bottom diameter, and 12 ins. top diameter, but not in quite such detailed calculations. The weight will now be $35 \times \frac{0{\cdot}8}{144}\left(\frac{15^2 + 12^2}{2}\right) \times \frac{35}{2240} = 0{\cdot}56$ ton, and the bending moment will be approximately $\frac{WL}{8} = \frac{0{\cdot}56 \times 8}{8} = 0{\cdot}56$ ton-ft.

$$\text{Then } \frac{W}{A} + \frac{M}{Z} = \frac{1{\cdot}03 + 0{\cdot}56}{0{\cdot}7854 \times 13{\cdot}5^2} + \frac{0{\cdot}56 \times 12}{0{\cdot}0982 \times 13{\cdot}5^3} = 0{\cdot}013 + 0{\cdot}028$$
$$= 0{\cdot}041 \text{ ton per sq. in.,}$$

whereas the maximum allowable

$$= \frac{0{\cdot}65}{1 + \frac{4}{250}\left(\frac{35 \times 12}{13{\cdot}5}\right)^2} = 0{\cdot}04 \text{ ton per sq. in.,}$$

so that this size is probably just sufficient, although the real overhang will be the diagonal distance from E to B in Fig. 572.

Taking the case of a tripod, suppose we have three poles having a vertical height of 20 ft., and the bases placed at the points of an equilateral triangle of 10 ft. side, the load to be lifted being 5 tons without any additional allowance. Fig. 574 will represent the plan and Fig. 575 the elevation projected from it. Draw the parallelogram of forces from the load in the elevation, and the thrust in each pole will be as shown in DC = 1·03 ton. The proof that this will be correct for

each of the poles will be to project the full length of the poles AB in plan, as shown by EB, EA. Then from E, along ED, set off the thrust shown in elevation on AB–D, and completing the parallelogram it will be found that the thrust shown on EB, EA, will be equal to that

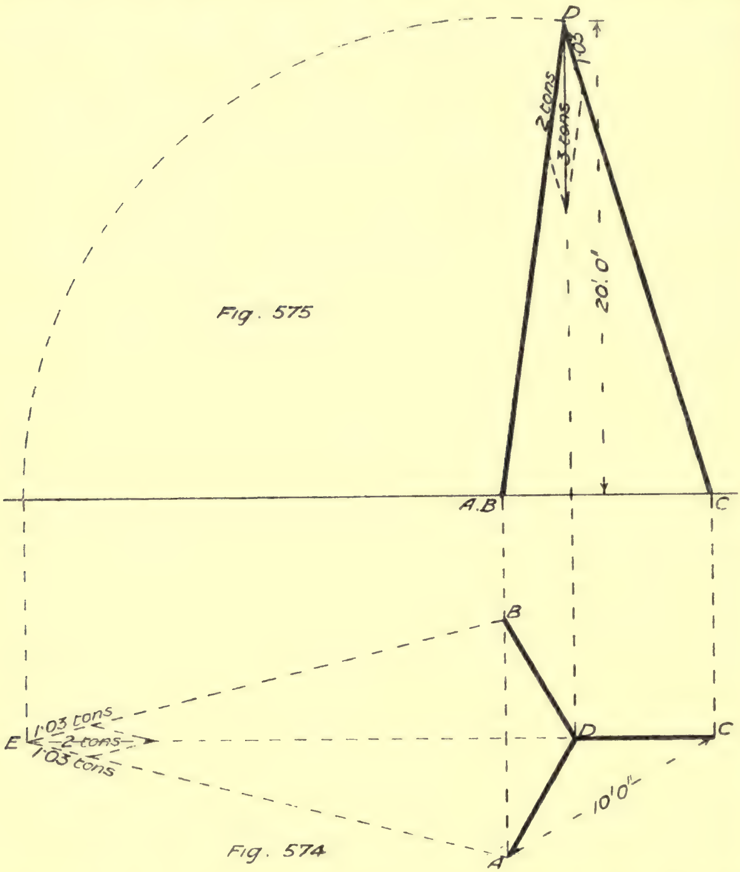


Fig. 575

Fig. 574

shown on CD in the elevation. Now having a thrust of 1·03 tons to provide for in poles 20 ft. long the problem is to find the scantling. By Gordon's formula $W = \dfrac{Af}{1 + a\left(\dfrac{l}{d}\right)^2}$. Try first 8 ins. by 8 ins., then taking the length from DC in the elevation,

$$\frac{8 \times 8 \times 13}{1 + \frac{4}{250}\left(\frac{20{\cdot}75 \times 12}{8}\right)^2} = 53{\cdot}6 \text{ cwt.} = 2{\cdot}68 \text{ tons}$$

as the safe load, against 1·03 tons the actual thrust, but this does not allow for the overhang. The weight of each pole

$$= 20{\cdot}75 \times \frac{8 \times 8}{144} \times \tfrac{35}{2240} = 0{\cdot}14 \text{ ton}$$

and the bending moment $= \dfrac{WL}{8} = \dfrac{0{\cdot}14 \times 6}{8} = 0{\cdot}105$ ton-ft.

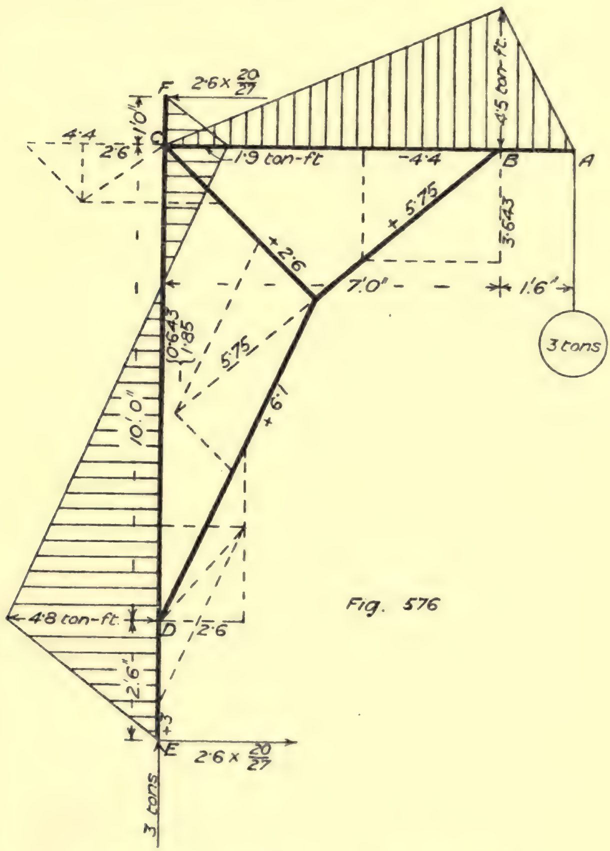


Fig. 576

Then
$$\frac{W}{A} \pm \frac{M}{Z} = \frac{1{\cdot}03 + 0{\cdot}14}{64} + \frac{0{\cdot}105 \times 12}{\frac{1}{6}(8 \times 8^2)}$$
$$= 0{\cdot}018 + 0{\cdot}015 = 0{\cdot}033 \text{ ton per sq. in.,}$$

and the maximum allowable

$$= \frac{0{\cdot}65}{1 + \frac{4}{250}\left(\frac{20{\cdot}75 \times 12}{8}\right)^2} = 0{\cdot}042 \text{ ton per sq. in.}$$

Some very difficult problems occur at times in finding the stresses

on cranes, but these varieties would hardly come under the head of building construction. One example of a wall crane to show the nature of the stresses will be taken. Fig. 576 shows an awkward case that may arise among the smaller cranes, where the outline of the crane is shown by thick lines and the force diagrams by dotted lines, and the bearings in the top and bottom brackets are at F and E. The points of application of the thrusts and resistances not being opposite each other, bending moments are caused in addition to the direct stresses, and are shown by the shaded areas. The load of 3 tons at A is equivalent to a load of $3 \times \frac{8 \cdot 5}{7} = 3 \cdot 643$ tons at B, and taking this latter load the stresses in the members may be found by parallelogram of forces as shown, and the thrust at D and pull at C are also given $= 2 \cdot 6$ tons each. The thrust at E and pull at F will therefore be

$$2 \cdot 6 \times \frac{10}{13 \cdot 5} = 2 \cdot 6 \times \tfrac{20}{27} \text{ tons each,}$$

producing a bending moment at D of $2 \cdot 6 \times \frac{20}{27} \times 2 \cdot 5 = 4 \cdot 8$ ton-ft., and at C, $2 \cdot 6 \times \frac{20}{27} \times 1 = 1 \cdot 9$ ton-ft. The 3 tons load at A produces a bending moment of $3 \times 1 \cdot 5 = 4 \cdot 5$ ton-ft. at B as shown in Fig. 576. In the foregoing calculations it has been assumed that the guy ropes are straight and without weight, but straight guys are impossible, they will always hang in a curve, and it is the tangent to the curve at the head of the poles that fixes the virtual anchorage of the guy rope.

EXERCISES ON LECTURE XXIX

Q. 91. From the conditions given in Figs. 577 and 578 find the stresses in the pole and ropes, and calculate the sizes required.

Answer. The elevation of the pole and ropes is given in Fig. 577, and by parallelogram of forces the thrust in pole is found to be 35 cwt. $= 1 \cdot 75$ tons, and the tension in plane of ropes 13 cwt. By setting off the true plan of ropes in Fig. 578 and constructing the parallelogram of forces the tension in each rope $= 7 \cdot 2$ cwt. Then the size of hemp rope required $= \sqrt{\frac{7 \cdot 2 \times 3}{2}} = 3 \cdot 3$ in. circumference, or $\sqrt{\frac{7 \cdot 2 \times 8}{6 \times 7}} = 1 \cdot 17$ in. circumference for wire rope. Assuming a 9-in. by 9-in. pole, the weight at 35 lbs. per cubic foot will be

$$\frac{9 \times 9}{144} \times 25 \times \tfrac{35}{2240} = 0 \cdot 22 \text{ ton,}$$

and the bending moment will be $\frac{\mathrm{WL}}{8} = \frac{0 \cdot 22 \times 6 \cdot 5}{8} = 0 \cdot 18$ ton-ft. The stresses produced will then be

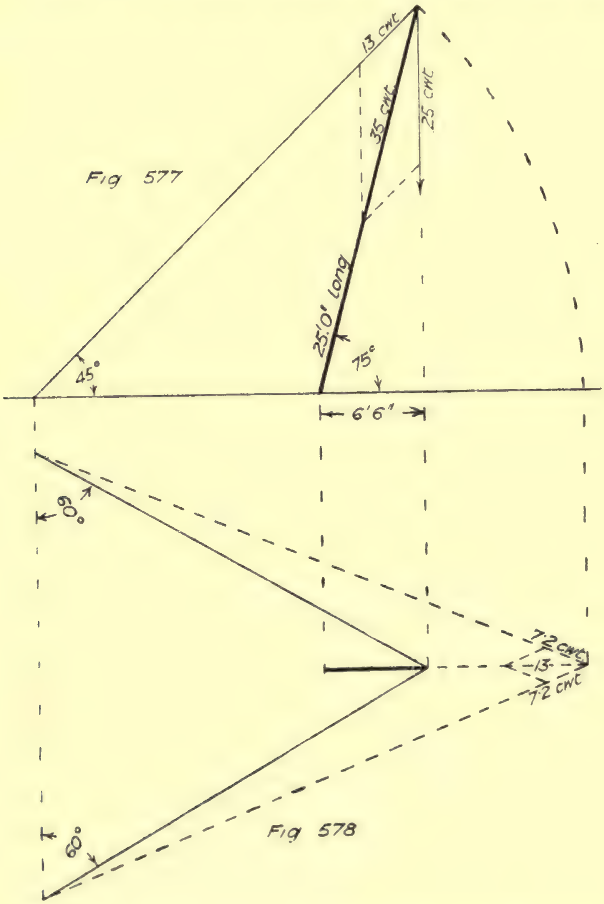
$$\frac{\mathrm{W}}{\mathrm{A}} + \frac{\mathrm{M}}{\mathrm{Z}} = \frac{1 \cdot 75 + 0 \cdot 22}{9 \times 9} + \frac{0 \cdot 18 \times 12}{\frac{1}{6}(9 \times 9^2)} = 0 \cdot 024 + 0 \cdot 018 = 0 \cdot 042 \text{ cwt. per sq. in.,}$$

whereas the maximum allowable stress $= \dfrac{0 \cdot 65 \text{ ton per sq. in.}}{1 + \frac{4}{250}\left(\frac{25 \times 12}{9}\right)^2} = 0 \cdot 035$ ton per sq. in.

This shows that the scantling is not quite enough, so a 10-in. by 10-in. pole may be

tried. The weight will be $\frac{10 \times 10}{144} \times 25 \times \frac{35}{2240} = 0{\cdot}27$ ton, producing a bending moment of $\frac{WL}{8} = \frac{0{\cdot}27 \times 6{\cdot}5}{8} = 0{\cdot}22$ ton-ft. The stresses produced will be $\frac{W}{A} + \frac{M}{Z} = \frac{1{\cdot}75 + 0{\cdot}27}{10 \times 10} + \frac{0{\cdot}\dot{2}\dot{2} \times 12}{\frac{1}{6}(10 \times 10^2)} = 0{\cdot}02 + 0{\cdot}016 = 0{\cdot}036$ ton per sq. in., and the allowable stress $= \dfrac{0{\cdot}65}{1 + \frac{1}{250}\left(\frac{25 \times 12}{10}\right)^2} = 0{\cdot}042$ ton per sq. in., so that this size will be quite sufficient.

Q. 92. A derrick consisting of two poles 15 ft. apart at the ground, joined together at a height of 25 ft. from the ground and raking over 8 ft., has a single



guy rope made fast at ground level 50 ft. in direct line from the head of derrick. Assuming the sag in the rope to give a versine of 2 ft. 6 ins., draw an elevation and plan and figure the parallelograms of forces at head of derrick for a load of 30 cwt.

and also for no load, making the size of rope suitable for the full load, and show for the given weight and position of rope what pull it will give on head of derrick. Discuss the relative bearing of the three results.

Answer. The elevation of the poles and rope will be as in Fig. 579 and the plan as Fig. 580. Draw *ab* tangent to the curve of rope at head of poles and *cd* tangent to curve at ground. Taking *ab* as the line of the rope, draw the parallelogram of forces for the full load, giving 18·5 cwt. tension in the rope, requiring a rope $\sqrt{\dfrac{18\cdot5 \times 8}{6 \times 7}} = 1\cdot87$ in. circumference. The 44 cwt. compression in the plane of the legs must be divided over the two legs as shown in Fig. 580,

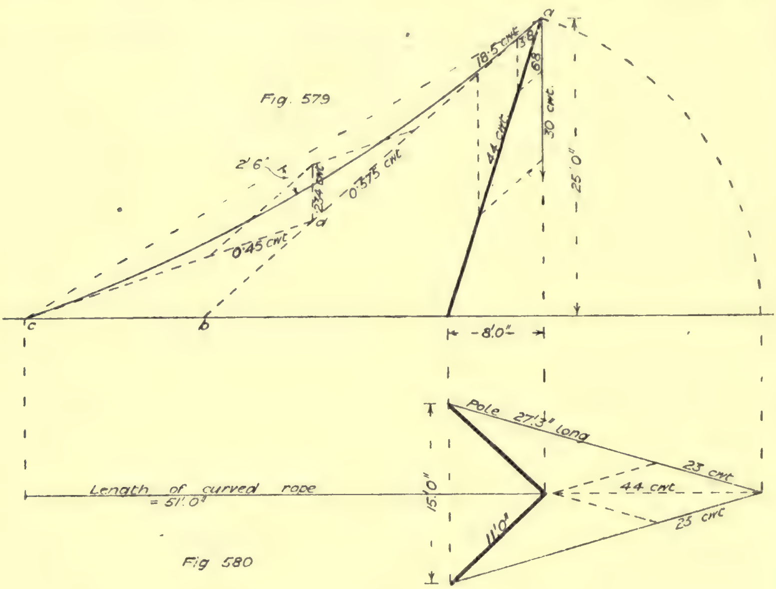


Fig. 579

Fig 580

giving 23 cwt. = 1·15 ton compression in each leg. Assume a pole 13 ins. diameter at base and 11 ins. at top, then the weight will be approximately $27\cdot25 \times \dfrac{0\cdot8}{144}\left(\dfrac{13^2 + 11^2}{2}\right) \times \frac{35}{2240} = 0\cdot34$ ton, and the bending moment

$$\frac{WL}{8} = \frac{0\cdot34 \times 11}{8} = 0\cdot47 \text{ ton-ft.}$$

Then

$$\frac{W}{A} + \frac{M}{Z} = \frac{1\cdot15 + 0\cdot34}{0\cdot7854 \times 12^2} + \frac{0\cdot47 \times 12}{0\cdot0982 \times 12^3} = 0\cdot013 + 0\cdot034 = 0\cdot047 \text{ ton per sq. in.},$$

whereas the maximum allowable $= \dfrac{0\cdot65}{1 + \frac{4}{250}\left(\dfrac{27\cdot25 \times 12}{12}\right)^2} = 0\cdot05$ ton per sq. in.

Half the weight of the poles may be set off as load in Fig. 579, and by parallelogram of forces it is found to produce 3·8 cwt. tension in the rope.

The steel rope will weigh $\dfrac{\frac{7}{8} \times 1\cdot87^2 \times \frac{51}{6}}{112} = 0\cdot234$ cwt., which by parallelogram

of forces at d in Fig. 579 produces a tension of 0·575 cwt. in the rope. It is evident that with a derrick of this kind the angle of the legs and the sag of the rope will vary with the load. With the sag shown the rope only gives a pull of 0·575 cwt., while without a load the weight of legs alone gives a pull of 3·8 cwt., so that the rope would yield and show a smaller versin. Still more will the full load tighten the rope and reduce the versin. In every case the plane of the legs will make such an angle as to cause the pull of the rope and load to balance each other.

LECTURE XXX

Reinforced Concrete—Beams—Floor Slabs—Tee Beams—Pillars—Formulæ and Calculations.

THE essential feature of reinforced concrete is that the cement concrete is strengthened by steel rods in such a manner that practically the whole of the compressive stress will be taken by the concrete, and the tensile stress by the steel. The shear is taken partly by the concrete and partly by the steel. The ordinary bending moment and shearing force diagrams for beams and girders apply equally to reinforced concrete construction, but owing to a doubt in some cases of the efficiency of the design and execution of the work certain allowances are made. For example, a beam with fixed ends and uniformly distributed load has a bending moment in the centre of $\frac{wl^2}{24}$, but to allow for possible defect in fixing the ends of a reinforced concrete beam it is taken as $\frac{wl^2}{12}$. Also when fixed one end and supported at the other the negative bending moment at support is taken as $\frac{wl^2}{8}$ both for ordinary and reinforced beams, but the maximum positive bending moment at $\frac{3}{8}l$ from supported end is taken as $\frac{9}{128}wl^2$ for ordinary beams and $\frac{1}{10}wl^2$ for reinforced concrete beams. In calculating the bending moment the effective span should be taken, as with ordinary girders, and this should not exceed twenty-four times the effective depth. For floor slabs supported direct by the walls, the effective span will be the clear span plus the thickness of slab. When supported by intermediate beams the effective span of the slab will be from centre to centre of the beams. The bending moment across the centre of a square slab supported on four edges and reinforced in both directions, with a load uniformly distributed, may be taken as $\frac{Wl}{16}$; and with the edges fixed the bending moment will be taken as $\frac{Wl}{48}$ in the centre, and $\frac{Wl}{24}$ at the edges. Other modifications will occur for other methods of fixing or difference of length and breadth, but the above will be sufficient to indicate the general result.

The modulus of elasticity for 1 : 2 : 4 cement concrete (*i.e.* 1 part British Standard Portland cement, 90 lbs. to a cubic foot, 2 parts clean

sharp sand, 4 parts broken stone or gravel, graded from $\frac{1}{4}$ in. to $\frac{3}{4}$ in.) averages one-fifteenth of that of mild steel or

$$\begin{aligned} & \text{lbs. per sq. in.} \\ \text{For concrete } E &= \ \ 2{,}000{,}000 \\ \text{,, mild steel } E &= 30{,}000{,}000 \\ \text{Modular ratio } m &= \frac{E_c}{E_s} = 15 \end{aligned}$$

The following are the allowable working stresses :—

	lbs. per sq. in.
Concrete in compression in beams	600
,, ,, columns under simple compression .	500
,, ,, shear in beams	60
Adhesion of concrete to metal	100
Steel in tension	16,000
,, ,, shear	12,000

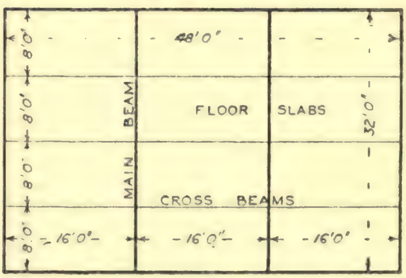
The grip length of a reinforcing bar, round or square, to prevent pulling out, is $2500td$, where t = tensile stress in tons and d = diameter in inches. The reinforcement should not be nearer to the face than $\frac{1}{2}$ in. in slabs, 1 in. in cross beams, $1\frac{1}{2}$ in. in main beams, and 2 ins. in pillars. There should be at least 1 in. between reinforcing bars horizontally and $\frac{3}{4}$ in. vertically.

The formulæ for reinforced concrete work are rather complicated, and in general require the neutral axis to be first determined. When the subject was first before the public the writer used an approximate empirical formula which has saved much time in dealing with simple beams and plain floor slabs. It is

$$W = (0{\cdot}37p + 0{\cdot}214)\frac{bd^2}{L}$$

where W = safe load in cwts. distributed including weight of beam itself, p = percentage of reinforcement (0 to $2\frac{1}{2}$) excluding $1\frac{1}{2}$ in. of concrete below reinforcement, b = breadth of concrete in inches, d = depth of concrete in inches to centre of reinforcement, L = clear span in feet.

Fig. 581



Nowadays more precise calculations have to be made, and full allowance given for the effect of the floor slab in converting the beams into tee beams. The best method of explaining the work will be to take the case of a floor as in Fig. 581 to carry an external load of 1 cwt. per ft. sup. The calculations may be based upon the following formulæ, where y = distance from neutral axis to extreme compressed fibre, y' = distance from neutral axis to extreme fibre in tension, m = modular ratio = 15, A = area of tensile reinforcement in square inches,

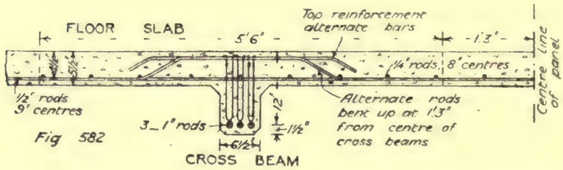
b = breadth in inches of tee beam, d = depth in inches from upper surface to centre of reinforcement, e = total depth of floor slab, I = moment of inertia in inch units, c = compression on the concrete in lbs. per sq. in., t = tension in the steel in lbs. per sq. in., M = bending moment in lb.-inches.

Then for floor slabs taking 1 ft. width, y may be obtained from the formula

$$6y^2 + mA_t y - mA_t d = 0,$$
$$I = 4y^3 + mA_t(d - y)^2,$$
$$c = \frac{My}{I} \text{ and } t = \frac{mMy'}{I}.$$

For tee beams when the neutral axis falls within the depth of slab,

$$by^2 + 2mA_t y - 2mA_t d = 0,$$
$$I = \frac{by^3}{3} + mA_t(d - y)^2,$$
c and t as before.



These formulæ may also be used for rectangular beams, taking b as breadth of beam.

For tee beams when the neutral axis falls outside the depth of slab,

$$y = \frac{2mA_t d + be^2}{2(mA_t + be)},$$
$$I = \frac{be^3}{12} + \frac{be(y - \frac{1}{2}e)^2}{4} + mA_t(d - y)^2.$$

It will be on the safe side and not cause any appreciable waste of strength if the whole of the slab panels and the beams be assumed to be simply supported at the ends, giving for distributed load a bending moment of $\frac{1}{8}wl^2$.

Taking first the floor slab, assuming the sizes shown in Fig. 582, the load per ft. run $= 112 + \left(\frac{5\frac{1}{2}}{12} \times 150\right) = 180$ lbs.

$$\text{and } M = \frac{wl^2}{8} = \frac{180}{12} \times \frac{(8 \times 12)^2}{8} = 17280 \text{ lb.-ins.}$$
$$\text{then } 6y^2 + mA_t y - mA_t d = 0$$
$$\text{or } 6y^2 + 15 \times 0{\cdot}26y = 15 \times 0{\cdot}26 \times 4\tfrac{1}{2}$$
$$\text{or } y^2 + 0{\cdot}65y = 2{\cdot}92$$
$$\text{or } y^2 + 0{\cdot}65y + \left(\frac{0{\cdot}65}{2}\right)^2 = 2{\cdot}92 + \left(\frac{0{\cdot}65}{2}\right)^2$$
$$\text{or } y + 0{\cdot}325 = 1{\cdot}735$$
$$\text{whence } y = 1{\cdot}41$$

$$\begin{aligned} \text{I} &= 4y^3 + m\text{A}_t(d - y)^2 \\ &= 4 \times 1{\cdot}41^3 + 15 \times 0{\cdot}26(4{\cdot}5 - 1{\cdot}41)^2 \\ &= 48{\cdot}44 \text{ inch-units} \end{aligned}$$

$$c = \frac{\text{M}y}{\text{I}} = \frac{17280 \times 1{\cdot}41}{48{\cdot}44} = 503 \text{ lbs. per sq. in.}$$

$$t = \frac{m\text{M}y'}{\text{I}} = \frac{15 \times 17280(4{\cdot}5 - 1{\cdot}41)}{48{\cdot}44} = 16540 \text{ lbs. per sq. in.}$$

so that this size will just do. To resist what are known as temperature cracks light rods are laid at right angles to the main reinforcement of the slab, say, $\frac{1}{4}$-in. rods at 8-in. centres. For the cross beams, as shown

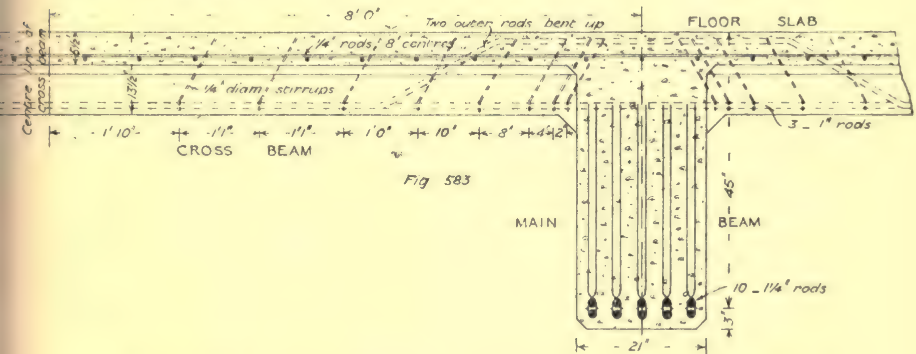


Fig 583

in Figs. 582 and 583, the width of floor slab to be taken into calculations is 12 times the thickness = 12 × 5·5 = 5 ft. 6 ins.

$$\text{Weight per ft. run} = \left(\frac{66 \times 5\frac{1}{2}}{144} + \frac{8 \times 6\frac{1}{2}}{144}\right)150 = 429 \text{ lbs.}$$

$$\text{External load per ft. run} = 112 \times 5\tfrac{1}{2} = 616 \text{ lbs.}$$

$$\text{Total } 1045 \text{ lbs.}$$

$$\text{then M} = \frac{wl^2}{8} = \tfrac{1045}{12} \times \frac{(16 \times 12)^2}{8} = 401280 \text{ lb.-ins.}$$

$$\text{and } by^2 + 2m\text{A}_t y = 2m\text{A}_t d$$

$$\text{or } 66y^2 + 2 \times 15 \times 2{\cdot}35y = 2 \times 15 \times 2{\cdot}35 \times 12$$

$$\text{or } y^2 + 1{\cdot}06y = 12{\cdot}72$$

$$\text{or } y^2 + 1{\cdot}06y + \left(\frac{1{\cdot}06}{2}\right)^2 = 12{\cdot}72 + \left(\frac{1{\cdot}06}{2}\right)^2$$

$$\text{or } y + 0{\cdot}53 = 3{\cdot}6$$

$$\text{whence } y = 3{\cdot}07$$

$$\begin{aligned} \text{I} &= \frac{by^3}{3} + m\text{A}_t(d - y)^2 \\ &= \frac{66 \times 3{\cdot}07^3}{3} + 15 \times 2{\cdot}35(12 - 3{\cdot}07)^2 \\ &= 3448 \text{ inch-units} \end{aligned}$$

$$c = \frac{\text{M}y}{\text{I}} = \frac{401280 \times 3{\cdot}07}{3448} = 357 \text{ lbs. per sq. in.}$$

$$t = \frac{m\text{M}y'}{\text{I}} = \frac{15 \times 401280 \times (12 - 3{\cdot}07)}{3448} = 15590 \text{ lbs. per sq. in.}$$

The shear reinforcement may be calculated by the formula

$$a_s = \frac{bd}{48} = \frac{6{\cdot}5 \times 12}{48} = 1{\cdot}625 \text{ sq. in.}$$

$$\text{say 8 stirrups in half span} = \frac{1{\cdot}625}{8} = 0{\cdot}203 \text{ sq. in. each,}$$

$$\text{or 6 stirrup bars} = \frac{0{\cdot}203}{6} = 0{\cdot}034 \text{ sq. in. each,}$$

therefore say $\frac{1}{4}$-in. diameter rods.

For the main beam, as shown in Figs. 583 and 584, the total concentrated load from each cross beam $= 16 \times 1045 = 16720$ lbs., producing a bending moment in the centre of

$(25080 \times 16) - (16720 \times 8) = 267520$ lb.-ft., or 3210240 lb.-inches. The weight of beam $= \frac{21 \times 48}{144} \times 150 \times 32 = 33600$ lbs., producing a bending moment of $\frac{33600 \times (32 \times 12)}{8} = 1612800$ lb.-ins., making a total of $3210240 + 1612800 = 4823040$ lb.-ins., as in Fig. 585.

$$\text{Then } by^2 + 2m\mathrm{A}_t y = 2m\mathrm{A}_t d$$
$$\text{or } 21y^2 + 2 \times 15 \times 12{\cdot}3y = 2 \times 15 \times 12{\cdot}3 \times 45$$
$$\text{or } y^2 + 17{\cdot}6y = 792$$
$$\text{or } y^2 + 17{\cdot}6y + \left(\frac{17{\cdot}6}{2}\right)^2 = 792 + \left(\frac{17{\cdot}6}{2}\right)^2$$
$$\text{or } y + 8{\cdot}8 = 29{\cdot}5$$
$$\text{whence } y = 20{\cdot}7$$

$$\mathrm{I} = \frac{by^3}{3} + m\mathrm{A}_t(d-y)^2$$
$$= \frac{21 \times 20{\cdot}7^3}{3} + 15 \times 12{\cdot}3(24{\cdot}3)^2$$
$$= 171000 \text{ inch-units}$$
$$c = \frac{\mathrm{M}y}{\mathrm{I}} = \frac{4823040 \times 20{\cdot}7}{171000} = 584 \text{ lbs. per sq. in.}$$
$$t = \frac{m\mathrm{M}y'}{\mathrm{I}} = \frac{15 \times 4823040 \times 24{\cdot}3}{171000} = 10280 \text{ lbs. per sq. in.}$$

$$\text{The shear reinforcement } a_s = \frac{bd}{48} = \frac{21 \times 45}{48} = 20 \text{ sq. ins.,}$$

$$\text{say 16 rows of stirrups} = \tfrac{20}{16} = 1{\cdot}25 \text{ sq. in. in each row,}$$

$$\text{10 stirrup bars} = \frac{1{\cdot}25}{10} = 0{\cdot}125 \text{ sq. in. area each, say } \tfrac{5}{16} \text{ in. square bars.}$$

The points at which the rods may be bent up can be ascertained as follows. For instance, take the main beam, the bending moment diagram must be drawn out as in Fig. 585, then the upper middle rod may be turned up at the points where the bending moment is one-tenth less, as at *ab*, or 12 ft. from either end. The two upper inner rods may be turned up at the points where the bending moment is $\frac{3}{10}$ less, as at *cd*, or 7 ft. 6 ins. from either end. The two upper outer rods may be turned up where the bending moment is reduced to $\frac{5}{10}$, or half the total, as at *ef*, or 5 ft. from either end.

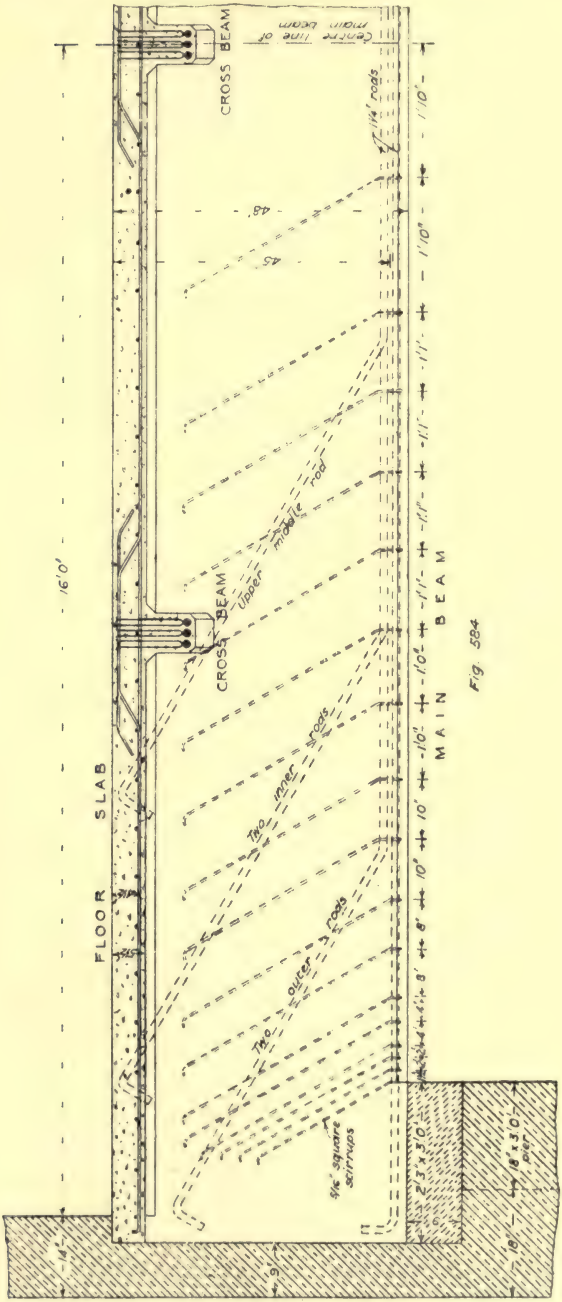

CROSS BEAM
Centre line of main beam
1¼" rods
48"
45"
Upper middle rod
CROSS BEAM
Two inner rods
Two outer rods
5/16" square stirrups
FLOOR SLAB
MAIN BEAM
16'0"
2'3" x 3'0"
18" x 3'0" pier
18"
14"
9"
6"
1'10"
1'10"
1'1"
1'1"
1'1"
1'1"
1'0"
1'0"
10"
10"
8"
8"

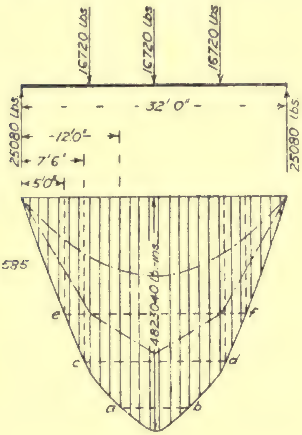
Fig. 584

As a rough check upon the section adopted it may be useful to note that approximately the bending moment in cwt.-ins. on the simple reinforced concrete beam = bd^2. In this case

$$M = \frac{4823040}{112} = 43063 \text{ cwt.-ins., as already found,}$$
$$\text{and } bd^2 = 21 \times 45^2 = 42525, \text{ showing a very close agreement.}$$

In calculating the loads on columns in buildings with a height of more than two stories, with the exception of warehouses, the superimposed loads from roof and top story must be calculated in full, but for each story below a reduction of 10 per cent. of the load may be made up to a maximum of 50 per cent. The maximum permissible stress on the concrete in columns under simple compression should not exceed 500 lbs. per sq. in., and in the steel 12,000 lbs. per sq. in. The vertical rods in pillars should not be placed nearer than $1\frac{1}{2}$ in. to the face of the column, or 2 ins. in important cases, and if hooping is employed there must be four lines of vertical reinforcement with straight laterals and six lines with curved laterals. The straight laterals must not be less than $\frac{3}{16}$ in., and curved laterals $\frac{1}{8}$ in. in diameter, and the pitch must not exceed $\frac{6}{10}$ of the effective diameter of the pillars, that is, the diameter to the outside of the vertical reinforcement.

Fig. 585



In rectangular pillars, where the ratio of the greater and lesser diameters exceeds $1\frac{1}{2}$, or square pillars with more than four vertical rods, the cross-section of the pillar must be subdivided by cross ties, and the distance between the rods on the longer side must not exceed that between the rods on the shorter side. The total area of the vertical reinforcement in pillars must not be less than 0·8 per cent. of the hooped core, and the volume of lateral reinforcement must not be less than 0·5 per cent. of the volume of the hooped core. When the column has fixed ends, the following stresses may be allowed according to the ratio of length to effective diameter :—

$\frac{l}{d}$	Stress allowed (P)
18	Full stress
22	0·8 of ,, ,,
24	0·6 ,, ,, ,,
27	0·4 ,, ,, ,,
30	0·2 ,, ,, ,,

If the pillar has one end fixed and one hinged allow $\frac{1}{2}$P, both ends hinged $\frac{1}{4}$P, and one end fixed and one free $\frac{1}{16}$P. The stress on the steel

may be worked by the formula $t = \frac{mW}{A + A_v(m - 1)}$ and on the concrete by the formula $c = \frac{W}{A + A_v(m - 1)}$, where t = the stress in the steel in lbs. per sq. in., c = the stress on the concrete in lbs. per sq. in., W = total axial load on the pillar in lbs., A = effective area of pillar in square inches, *i.e.* the area bounded by the lateral reinforcement, A_v = area of vertical reinforcement in square inches, m = modular ratio = 15. The stress thus found should not exceed the safe stress given by the formula $kC(1 + fsr)$ where C = the ultimate crushing resistance of concrete at 3 months, S = the safety factor = say 4, k = the reciprocal of the safety factor $= \frac{1}{S_f}$, f = form factor, depending upon the form or type of laterals (see table), s = spacing factor, depending upon the spacing or pitch of the laterals (see table), r = ratio of the volume of lateral reinforcement to the volume of the hooped core in any given length of pillar.

Form of lateral reinforcement.	Form factor $= f$.	Spacing of laterals in terms of diameter of hooped core.	Spacing factor $= s$.
		Diameter.	
Helical	1·0	0·2	32
,, ...	1·0	0·3	24
,, ...	1·0	0·4	16
Circular hoops . .	0·75	0·2	32
,, ,, . .	0·75	0·3	24
,, ,, . .	0·75	0·4	16
Rectilinear . . .	0·5	0·2	32
,, . . .	0·5	0·3	24
,, . . .	0·5	0·4	16
,, . . .	0·5	0·5	8
,, . . .	0·5	0·6	0

When the pillar is eccentrically loaded the maximum stress will be given by the formula $f = \frac{W}{A} \pm \frac{W'x}{Z}$, where f = maximum stress in lbs. per sq. in. at edge of section, W = total load from all sources in lbs. A = equivalent area in square inches $= A + (m - 1)A_v$, W′ = eccentric, load in lbs., x = distance in inches from neutral axis of pillar to centre of application, Z = section modulus of pillar in inch units, and the stress so found should not exceed the allowable stress given by the previous formula.

For a rectangular reinforced concrete pillar, $Z = \frac{1}{6}A_c h + \frac{1}{2}(m-1)A_v\frac{h_t^2}{h}$, where h is the whole diameter of the pillar at right angles to the neutral axis, and h_t is the diameter from centre to centre of reinforcement.

For a circular pillar reinforced with four bars,

$$Z = \tfrac{1}{8}A_c h + \tfrac{1}{2}(m - 1)A_v\frac{h_t^2}{h}$$

and for a circular pillar reinforced with bars arranged in a circle

$$Z = \tfrac{1}{8}A_c h + \tfrac{1}{4}(m - 1)A_v \frac{h_t^2}{h}$$

The following limits of stress should be observed in pillars :—

(1) The stress on the steel should not exceed one-fourth of the ultimate tensile strength, or 0·5 of the elastic limit, or the value of mc.

(2) The working stress on the concrete of pillars must not exceed 0·5C with rectilinear laterals, 0·58C with independent circular hoops, and 0·66C with helical reinforcement.

EXERCISES ON LECTURE XXX

Q. 93. Calculate and design the cross section and elevation of one of the main beams of the given floor if supported by a reinforced concrete pillar in the centre. Also calculate and give a cross section of the pillar assuming the height to be 14 ft. from floor to floor.

Answer. Assuming the general dimensions of the beam to be as shown in Figs. 586 and 587,

$$\text{the weight of beam} = \frac{12 \times 26}{144} \times 150 \times 16 = 5200 \text{ lbs.}$$

$$\text{and M} = \frac{5200 \times (16 \times 12)}{8} = 124800 \text{ lb.-ins.}$$

$$\text{M from load} = \tfrac{3}{16}\text{W}l = \frac{3 \times 16720 \times 16 \times 12}{16} = 601920 \text{ lb.-ins.}$$

making a total of 726,720 lb.-ins.

$$\text{Then } 12y^2 + 2 \times 15 \times 2{\cdot}4 \times y = 2 \times 15 \times 2{\cdot}4 \times 24$$
$$\text{or } y^2 + 6y = 144$$
$$\text{or } y^2 + 6y + (\tfrac{6}{2})^2 = 144 + (\tfrac{6}{2})^2$$
$$\text{or } y + 3 = 12{\cdot}4$$
$$\text{whence } y = 9{\cdot}4$$
$$\text{I} = \frac{12 \times 9{\cdot}4^3}{3} + 15 \times 2{\cdot}4(14{\cdot}6)^2 = 10998$$
$$c = \frac{726720 \times 9{\cdot}4}{10998} = 620 \text{ lbs. per sq. in.}$$
$$t = \frac{15 \times 726720 \times 14{\cdot}6}{10998} = 14460 \text{ lbs. per sq. in.}$$

The points at which to turn the rods up and to provide for reverse stresses may be found from the bending moment diagrams, Figs. 588 and 589. As the loads in this case are concentrated the shear stirrups should be equally spaced, the shear at left-hand abutment will be $\frac{5}{16} \times 16720 + \frac{3}{8} \times 5200 = 7175$ lbs., and at 5 tons per sq. in. this requires $\frac{7175}{5 \times 2240} = 0{\cdot}64$ sq. in. per ft. run, say stirrups 8 in. centres, then area of each $= \frac{0{\cdot}64}{\frac{12}{8} \times 8} = 0{\cdot}05$ sq. in., say $\frac{1}{4}$-in. rods. At the column the shear will be $\frac{11}{16} \times 16720 + \frac{5}{8} \times 5200 = 14745$ lbs., and at 5 tons per sq. in., requires $\frac{14745}{5 \times 2240} = 1{\cdot}32$ sq. in. per ft. run, say stirrups 4-in. centres, then area of each $= \frac{1{\cdot}32}{\frac{12}{4} \times 8} = 0{\cdot}05$ sq. in., say $\frac{1}{4}$-in. rods.

The load on pillar will be $\frac{5}{8}$ of total distributed load on both spans and $\frac{11}{16}$ of central concentrated loads, to which must be added the direct load over the pillar $= \frac{5}{8}(2 \times 5200) + \frac{11}{16}(2 \times 16720) + 16720 = 46210$ lbs. Assume the pillar to

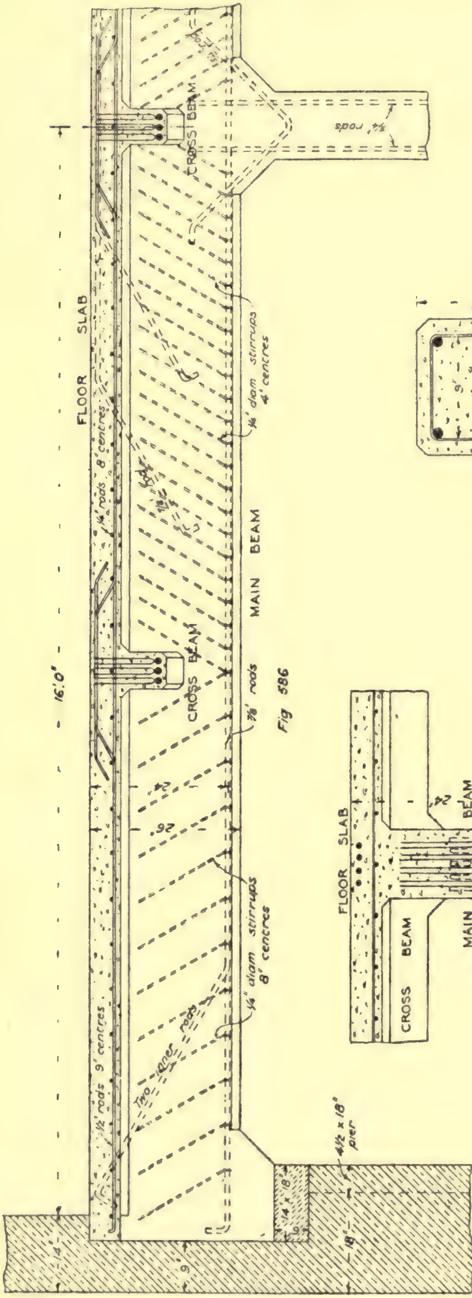


Fig 586

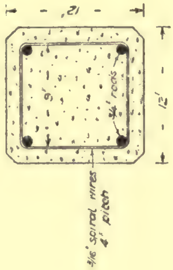


Fig 590

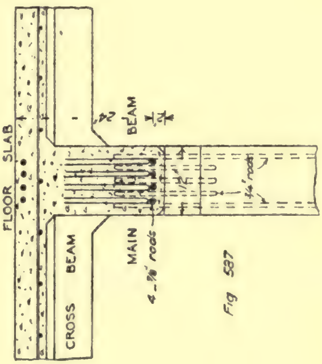


Fig 587

be 12 ins. square with four $\frac{3}{4}$-in. vertical rods, bound together spirally at intervals of 4 ins. with $\frac{3}{16}$-in. diameter wires, as in Fig. 590.

$$\text{The ratio of length to effective diameter} = \frac{(14 \times 12) - 26}{9} = 15{\cdot}8,$$

so that the full stress may be allowed on the pillar.

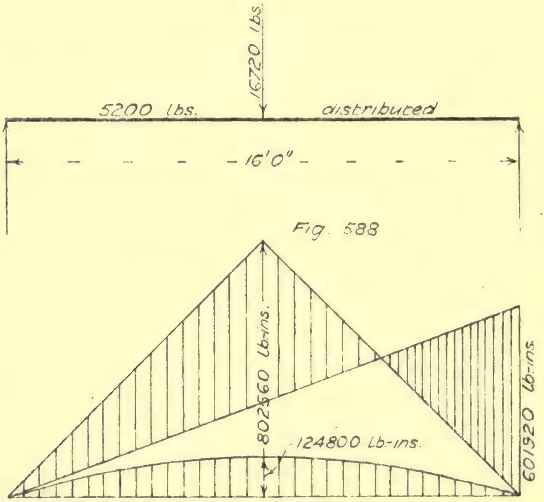


Fig. 588

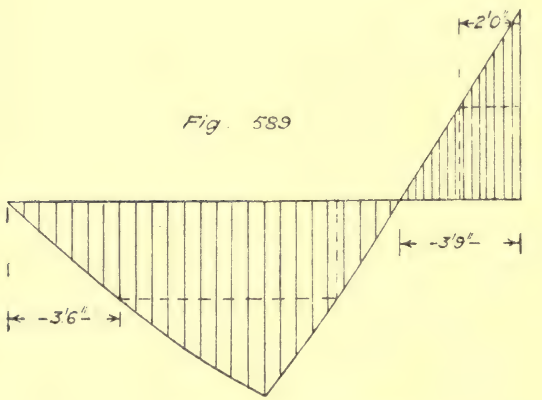


Fig. 589

$$\text{Then stress in concrete } c = \frac{\text{W}}{\text{A}+\text{A}_v(m-1)} = \frac{46210}{81+1{\cdot}76(15-1)} = 440 \text{ lbs. per sq. in.}$$

Then taking $k\text{C} = 500$ lbs. per sq. in., the maximum allowable $= kc(1 + fsr)$ $= 500(1 + 1 \times 16 \times 0{\cdot}003) = 525$ lbs. per sq. in.

$$\text{The stress in the steel} = \frac{m\text{W}}{\text{A}+\text{A}_v(m-1)} = \frac{15 \times 46910}{81+1{\cdot}76(15-1)} = 6600 \text{ lbs. per sq. in.}$$

LABORATORY WORK.

The laboratory work accompanying this course of instruction consisted of testing rectangular beams of white pine to breaking point by transverse stress, noting the deflections under given loads, plotting the results and calculating the numerical coefficients in the formulæ; also making up and testing Portland cement pats and briquettes, neat and in various combinations, and at different ages. A visit was made to a large building in course of construction, and various matters of interest therein were pointed out.

INDEX

THE END

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